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The Spectrum and Effective Action of SUSY Gluodynamics *

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We study the low-energy spectrum of SUSY gluodynamics using the generating functional for Green's functions of composite fields. Two dual formulations of the generating functional approach are given. Masses of the bound states are calculated and mixing patterns are discussed. Mass splittings of pure gluonic states, in the case when supersymmetry is softly broken, are consistent with predictions of conventional Yang-Mills theory. The results can be tested in lattice simulations of the SUSY Yang-Mills model.

Supersymmetric gluodynamics, the theory of gluons and gluinos, is an extremely useful testing ground for various nonperturbative phenomena occuring in conventional QCD. The Witten index of the $SU(N_c)$ SUSY gluodynamics equals to N_c [1]. Thus, the ground state of the model consists of at least N_c different vacua parametrized by the imaginary phase of a nonzero gluino condensate [1, 2]. The different vacua are related by discrete Z_{2N_c} transformations of gluino fields. Once one of the N_c vacua is chosen, the Z_{2N_c} symmetry group spontaneously breaks down to the Z_2 subgroup.

In analogy with QCD, one expects that in each of those vacua, the spectrum of the model consists of colorless bound states of gluinos and gluons. Among those are: pure gluonic bound states (glueballs), gluino-gluino mesons and gluon-gluino composites. These states fall into the lowest-spin representations of the $N = 1$ SUSY algebra written in the basis of parity eigenstates [3, 4]. The masses and mixings of these bound states can be given within the effective Lagrangian approach. The effective action for $N = 1$ SYM was proposed by Veneziano and Yankielowicz (VY) [5]. The VY action [5] involves fields for gluino-gluino and gluino-gluon bound states. However, it does not include dynamical degrees of freedom which would correspond to pure gluonic composites (glueballs).

At this stage we would like to make a digression and comment on the physical meaning of the VY effective action. This is not an effective action in the Wilsonian sense. In ref. [6] the VY action was constructed as a generating functional of one-particle-irreducible (1PI) Green's functions [7]. That means that the VY action, being written in terms of composite colorless fields of SYM theory, can be used to calculate various Green's functions of those composite variables. Performing those calculations, however, one is not supposed to take into account diagrams with composite fields propagating in virtual loops. Loop effects are already included in effective vertices and propagators occuring in the action. The simplest kind of Green's function one might be interested in is a two point correlator. As we mentioned above, the composite operators entering the VY action are the interpolating fields for the bound states of $N = 1$ SYM theory. Thus, a two point correlator (or simply a propagator)

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of those fields can be used to determine the mass of the corresponding bound state. Hence, the effective action, or more exactly the generating functional of 1PI diagrams, which we deal with in this paper can readily be used to deduce masses of composite bound states of the theory. In what follows the effective action (and effective Lagrangian) discussed will be understood in the sense specified above.

In this report we describe briefly how one can generalize the VY Lagrangian in order to incorporate the glueball states into the description [3, 4].

The classical action of $N=1$ SYM theory is invariant under chiral, scale and superconformal transformations. Once quantum effects are taken into account, these symmetries are broken by the chiral, scale and superconformal anomalies respectively. Composite operators that appear in the expressions for the anomalies can be gathered into a composite chiral supermultiplet $\mathrm{Tr}W^\alpha W_\alpha$ [8]

The effective action of the model can be a functional of the superfield S

$$S \equiv \frac{\beta(g)}{2g}\Big\langle \mathrm{Tr}W^\alpha W_\alpha \Big\rangle_Q \equiv A(y) + \sqrt{2}\theta\Psi(y) + \theta^2 F(y),$$

where the VEV is defined for nonzero value of an external (super)source Q [6]. $\beta(g)$ stands for the SYM beta function which is known exactly [9]. The lowest component of the S superfield, A, is bilinear in gluino fields and has the quantum numbers of the scalar and pseudoscalar gluino-gluino bound states. The fermionic component in S is related to the gluino-gluon composite and the F component of the chiral superfield includes operators corresponding to both the scalar and pseudoscalar glueballs ($G^2_{\mu\nu}$ and $G_{\mu\nu}\tilde{G}^{\mu\nu}$ respectively).

Assuming that the effective action of the model can be written in terms of the single superfield S, and requiring also that the effective action respects all the global continuous symmetries and reproduces the anomalies of the SYM theory, one derives the Veneziano-Yankielowicz effective superpotential [5]

$$\mathcal{W}_{\mathrm{VY}}(S) = \gamma\ S\ \ln\frac{S}{e\mu^3}, \tag{1}$$

where $\gamma \equiv -(N_c g/16\pi^2\beta(g)) > 0$, μ stands for the dimensionally transmuted scale of the model and $e \simeq 2.71$. If one uses this superpotential along with the simplest Kähler potential $(S^+S)^{1/3}$, one finds that no glueball operators are present in the Lagrangian. Indeed, all the glueball fields enter through the F components of the superfield S. These components have no dynamics and are thus integrated out from the action. We would like to argue below that one needs to use a bigger representation of supersymmetry in order to accomodate also the glueball degrees of freedom in the effective Lagrangian.

In order to determine how glueballs can be included in the action let us concentrate our attention on the expression for the F field. Using the equation of motion for the gluino field and for the D component one gets¶

$$F \equiv \Sigma + iQ \equiv \frac{\beta(g)}{4g}[G^2_{\mu\nu} + iG_{\mu\nu}\tilde{G}_{\mu\nu}]. \tag{2}$$

As we have already mentioned, the F field appears in the VY action without a kinetic term. The term bilinear in the F field is proportional to $F^+F = \Sigma^2 + Q^2$. Besides that, there are terms linear in the F field in the expression for the effective action, thus, the F field can easily be integrated out by means of its algebraic equations of motion [5].

In order to reveal subtleties of this procedure let us write down the following relation

$$Q = \frac{1}{4!}\varepsilon_{\mu\nu\alpha\beta}H^{\mu\nu\alpha\beta}, \tag{3}$$

¶In general, one is not allowed to use the equation of motion if the VEV of the F field is considered.

where $H^{\mu\nu\alpha\beta}$ is a field strength for a three-form potential $C_{\nu\alpha\beta}$, $H_{\mu\nu\alpha\beta} = \partial_\mu C_{\nu\alpha\beta} - \partial_\nu C_{\mu\alpha\beta} - \partial_\alpha C_{\nu\mu\beta} - \partial_\beta C_{\nu\alpha\mu}$. The $C_{\mu\nu\alpha}$ field itself is defined as a color singlet composite operator of colored gluon fields A^a_μ, $C_{\mu\nu\alpha} = \frac{\beta(g)}{64 g\pi^2}(A^a_\mu \overleftrightarrow{\partial}_\nu A^a_\alpha - A^a_\nu \overleftrightarrow{\partial}_\mu A^a_\alpha - A^a_\alpha \overleftrightarrow{\partial}_\nu A^a_\mu + 2 f_{abc} A^a_\mu A^b_\nu A^c_\alpha)$, with f_{abc} being structure constants of the corresponding $SU(N_c)$ gauge group. The right-left derivative in this expression acts as $A\overleftrightarrow{\partial}B \equiv A(\partial B) - (\partial A)B$ ††.

Using these definitions one finds that the expression bilinear in the F field acquires the following form

$$F^+ F = \Sigma^2 - \frac{1}{4!} H^2_{\mu\nu\alpha\beta}.$$

The second term in this expression is a kinetic term for the three-form potential $C_{\mu\nu\alpha}$. As before, the Σ field can be integrated out, however one should be careful in dealing with the $C_{\mu\nu\alpha}$ field.

In ref. [11] it was argued that the three-form field $C_{\mu\nu\alpha}$ plays an important role in the description of the pseudoscalar glueball. The glueball can be coupled to the QCD η' meson by means of the $C_{\mu\nu\alpha}$ field [12]. In the case of SYM theory the analog of the η' meson is the gluino-gluino bound state which acquires mass due to the anomaly in the $U(1)_R$ current within the VY approach. Thus, it is natural to attempt to couple the pseudoscalar glueball to the pseudoscalar gluino-gluino bound state within the VY action using the three-form potential $C_{\mu\nu\alpha}$.

To elaborate this approach, let us rewrite the SUSY transformations for the components of the S superfield in terms of Σ and $C_{\mu\nu\alpha}$ (instead of F and F^+) [10]

$$\delta_\zeta A = \sqrt{2}\zeta\Psi, \qquad \delta_\zeta \Psi = i\sqrt{2}\sigma^\mu\bar{\zeta}\partial_\mu A + \sqrt{2}\zeta(\Sigma + \frac{i}{6}\varepsilon_{\mu\nu\alpha\beta}\partial^\mu C^{\nu\alpha\beta}),$$
$$\delta_\zeta \Sigma = \frac{i}{\sqrt{2}}(\bar{\zeta}\bar{\sigma}^\mu\partial_\mu\Psi + \zeta\sigma^\mu\partial_\mu\bar{\Psi}), \qquad \delta_\zeta C_{\nu\alpha\beta} = \frac{1}{\sqrt{2}}\varepsilon_{\nu\alpha\beta\mu}(\bar{\zeta}\bar{\sigma}^\mu\Psi - \zeta\sigma^\mu\bar{\Psi}).$$

The set of fields given above forms an irreducible representation of supersymmetry algebra. All these fields can be assigned to a supermultiplet introduced in ref. [10]. That supermultiplet is called a constrained three-form supermultiplet [10]. The easiest way to present this multiplet is to introduce the following real tensor superfield U

$$U = B + i\theta z - i\bar{\theta}\bar{z} + \frac{1}{16}\theta^2 A^* + \frac{1}{16}\bar{\theta}^2 A + \frac{1}{48}\theta\sigma^\mu\bar{\theta}\varepsilon_{\mu\nu\alpha\beta}C^{\nu\alpha\beta} +$$
$$\frac{1}{2}\theta^2\bar{\theta}\left(\frac{\sqrt{2}}{8}\bar{\Psi} + \bar{\sigma}^\mu\partial_\mu z\right) + \frac{1}{2}\bar{\theta}^2\theta\left(\frac{\sqrt{2}}{8}\Psi - \sigma^\mu\partial_\mu\bar{z}\right) + \frac{1}{4}\theta^2\bar{\theta}^2\left(\frac{1}{4}\Sigma - \partial^2 B\right). \tag{4}$$

It is a matter of a straightforward calculation to check that the real superfield U satisfies the relation‡

$$\bar{D}^2 U = -\frac{1}{4}S. \tag{5}$$

††The quantity Q can also be expressed through the Chern-Simons current K_μ as $Q = \partial_\mu K_\mu$. Using this equation one can deduce the relation between the Chern-Simons current and the three-form potential $C_{\nu\alpha\beta}$, these two quantities are Hodge dual to each other: $K^\mu = \frac{1}{3!}\varepsilon^{\mu\nu\alpha\beta}C_{\nu\alpha\beta}$.

‡Despite a seeming similarity, the tensor multiplet U should not be interpreted as a usual vector multiplet. The vector field which might be introduced in this approach as a Hodge dual of the three-form potential $C_{\mu\nu\alpha}$ would give mass terms with the wrong sign in our approach (see section 2), thus, the actual physical variable is the three-form potential $C_{\mu\nu\alpha}$ rather than its dual vector field (the Chern-Simons current).

Thus, the real tensor multiplet U, defined by the expression (5), includes all the components of the chiral supermultiplet S. Consequently, using the relation (5) the VY action can be rewritten in terms of the bigger multiplet U. In addition, the multiplet has also a scalar B and fermion z. We will show below that this allows one to include glueball operators in the effective action.

First, let us notice some features of SUSY transformations of the components of the U field. The components which are shared by the tensor multiplet U and the chiral multiplet S (namely A, Ψ, Σ and C) transform among themselves, while other fields (B and z) are connected by SUSY rotations to the other four components. Furthermore, one can define a "gauge" transformation of the U field as the following shift $U \to U + Y$, where the superfield Y satisfies the relation $\bar{D}^2 Y = 0$. It is important to notice that by means of this " gauge " transformation one can get rid of the B and z fields in the expression for the U multiplet. This is the analog of the Wess-Zumino gauge for the tensor multiplet U. Thus, any Lagrangian written in terms of the S field only, if reexpressed in terms of the U field, is necessarily invariant under the "gauge" transformation defined above. As a result, the B and z fields can always be "gauged" away from that Lagrangian. Thus in order to be able to retain the B and z fields as dynamical variables one must include terms in the Lagrangian which breaks this "gauge" invariance. The simplest term of this type is the quadratic term $(U^2)|_D$. Once such a term is included in the Lagrangian, the "gauge" symmetry becomes explicitly broken and the B and z components of the superfield U survive as dynamical variables.

Let us now apply the U field formalism to the VY action. In the case at hand the chiral symmetry is spontaneously broken by the gluino condensate. In terms of the chiral superfield this corresponds to the existence of a nonzero VEV of the S field $\langle S \rangle = \mu^3$. With that in mind the appropriate relation between the U field and the chiral multiplet is

$$\bar{D}^2 U = -\frac{1}{4}(S - \langle S \rangle). \tag{6}$$

The only result of this modification is that the field A in eq. (4) gets replaced by the quantity $A - \langle A \rangle$.

Now use the relation (6) to write the action in terms of the U field. In order to break the "gauge" invariance of the VY action we add a term proportional to U^2 to the VY Lagrangian. An appropriate term with zero R-charge and correct dimensionality is

$$\left(-\frac{U^2}{(S^+ S)^{1/3}} \right) |_D . \tag{7}$$

Below, we are going to show that once this term is added to the VY action, the following fields become dynamical:

- The B field propagates and it represents one massive real scalar degree of freedom (identified later with the scalar glueball).
- The three-form potential $C_{\mu\nu\alpha}$, which becomes massive, also propagates. It represents one physical degree of freedom (identified with the pseudoscalar glueball).
- The complex field A, being decomposed into parity eigenstates, describes the massive gluino-gluino scalar, s, and pseudoscalar, p, mesons.
- z and Ψ describe the massive gluino-gluon fermionic bound states.

Relations between masses of these states will be given in the next section.

Based on the arguments given abowe one can write down the effective Lagrangian for the lowest-spin multiplets of the $N = 1$ SUSY YM theory in the following form

$$\mathcal{L} = \frac{1}{\alpha}(S^+S)^{1/3}|_D + \gamma[(S\log\frac{S}{\mu^3} - S)|_F + \mathrm{h.c.}] + \frac{1}{\delta}\left(-\frac{U^2}{(S^+S)^{1/3}}\right)|_D, \tag{8}$$

where α and δ are arbitrary positive constants. One obtains the VY Lagrangian in the limit $\delta \to \infty$. In general, higher powers of U can also be added to this Lagrangian. Those terms would introduce new quartic, quintic and other higher interaction terms. However, the quadratic part of the action which defines two-point Green's functions and masses will not be affected. In that respect, the effective Lagrangian (8) can be considered as the one describing small perturbations of fields about a vacuum state.

Let us determine the SUSY vacuum state defined by the Lagrangian (8). The potential for the model is a complicated function of the variables present in the U superfield. After integration over the auxiliary Σ field, the bosonic part of the potential is

$$V_0 = \frac{2}{\delta(16)^2}\frac{|\phi|^6 + \mu^6 - 2\mu^3|\phi|^3\mathrm{cos}3\rho}{|\phi|^2} + \frac{3}{\delta(48)^2}\frac{C^2_{\mu\nu\tau}}{|\phi|^2} + \\ + \frac{9\alpha|\phi|^4}{4}\frac{1}{1-\frac{\alpha}{\delta}\frac{B^2}{|\phi|^4}}\left(\frac{B}{24\delta|\phi|^2}\{1+\frac{2\mu^3}{|\phi|^3}\mathrm{cos}3\rho\} - 3\gamma\log\frac{|\phi|^2}{\mu^2}\right)^2, \tag{9}$$

where we introduced the notations $\phi \equiv A^{1/3}$ and $\rho \equiv \arg\phi$.

In order to find the vacuum state one should find the absolute minimum of the potential (9). Since we are dealing with a supersymmetric model, the value of the potential in that minimum has to be zero. As a result of Lorentz invariance, the VEV of the three-form field is zero, i.e. $\langle C_{\mu\nu\tau}\rangle = 0$. The VEV of Q is also zero due to the CP invariance of the model. After some algebra one finds that the only global, CP invariant minimum of the potential (9) is given by: $\langle\phi\rangle = \mu$, $\langle B\rangle = \langle C\rangle = \langle\rho\rangle = 0$. The effective Lagrangian (8) describes small perturbations of fields about the vacuum state defined by these VEV's.

We would like to make a comment here. The singularity in the potential at $\delta|\phi|^4 = \alpha B$ indicates that for large perturbations the higher dimensional terms omitted in (8) should become important. The same multiplier $1 - \frac{\alpha}{\delta}\frac{B^2}{|\phi|^4}$ appears in front of kinetic terms for the scalar fields, so the physical potential is always positively defined. As we mentioned above, we are mainly concerned with the mass spectrum of the model which can be studied using small perturbations about the ground state, so that the approximation given in (8) is good enough for our goals.

The actual physical states described by the action are mixed states. Below, we deduce the masses of these mixed physical states. Let us write down the mass and mixing terms of the Lagrangian separately. One finds the following pairs of bosonic variables being mixed with one another

$$\begin{aligned} &B-s \ \text{ system}: \quad \frac{9\alpha}{\delta(16)^2}\mu^2 s^2 + \frac{81}{2}\alpha^2\gamma^2\mu^2 s^2 + \frac{9\alpha}{\delta(16)^2}\mu^2 B^2 - \frac{27\sqrt{2}}{16}\sqrt{\frac{\alpha}{\delta}}\alpha\gamma\mu^2 Bs; \\ &C-p \ \text{ system}: \quad \frac{9\alpha}{\delta(16)^2}\mu^2 p^2 + \frac{27\alpha}{(48)^2\delta}\mu^2 C^2_{\mu\nu\tau} + \frac{9\sqrt{2}}{6}\alpha\gamma\mu p\varepsilon_{\mu\nu\tau\sigma}\partial^\mu C^{\nu\tau\sigma}; \end{aligned} \tag{10}$$

In order to find the physical masses one must diagonalize the corresponding mass matrices. Concentrate, for instance, on the first row of these expressions which describes the mixed state of scalar meson, s, and scalar meson-glueball, B. The former gets mass both from the superpotential and U^2-term while the latter gets mass only from the U^2-term. When the mixing term is switched on, the initially heavier state (s) gets even heavier, and the initially lighter state (B) becomes even lighter than they were originally. Performing the diagonalization, one finds that the physical eigenstates are mixed states with the following mass eigenvalues

$$\frac{1}{2}M_{\pm}^2 = \frac{9\alpha}{\delta(16)^2}\mu^2 + \frac{81}{4}\alpha^2\gamma^2\mu^2 \pm \frac{81}{4}\alpha^2\gamma^2\mu^2\sqrt{1 + \frac{1}{288}\frac{\alpha}{\delta}\frac{1}{(\alpha\gamma)^2}}. \tag{11}$$

Here, the subscript "+" refers to the heavier state $\tilde{s}$ which, without mixing, would have been a pure gluino-gluino bound state (the s particle). The subscript "−" refers to the lighter state $\tilde{B}$ (B in the absence of mixing).

Studying the potential of the model, we find that the physical eigenstates fall into the two different "multiplets". Neither of them contain pure gluino-gluino, gluino-gluon or gluon-gluon bound states. Instead, the physical excitations are mixed states of these composites. The heavier set of states contains:

- A pseudoscalar meson, which without mixing reduces to the 0^{-+} ($l = 0$) gluino-gluino bound state (the analog of the QCD η' meson).
- A scalar meson that without mixing is an 0^{++} ($l = 1$) gluino-gluino excitation.
- A mixed fermionic gluino-gluon bound state.

These heavier states become the chiral supermultiplet described by the VY action in the limit that the additional term we have added to the effective Lagrangian is removed. The new states which appear as a result of our generalization forms a lighter multiplet:

- A scalar meson, which for small mixing becomes a 0^{++} ($l = 0$) glueball.
- A pseudoscalar state, which for small mixing is identified as a 0^{-+} ($l = 1$) glueball.
- A mixed fermionic gluino-gluon bound state.

We call the reader's attention to an interesting feature of the effective action introduced here. Although the physical states fall into multiplets whose J^P quantum numbers correspond to two chiral supermultiplets, the action is written in terms of one real tensor supermultiplet U. The natural question arises whether the whole action can be rewritten in terns of two different chiral multiplets. The relation between a real tensor and chiral supermultiplets (the so called chiral-linear duality) was established in ref. [10]. For SYM theory the chiral-linear duality was used in refs. [13]. Applied to our problem the results of refs. [10] and [13] can be stated as follows. One introduces into the effective Lagrangian a new chiral superfield, let us denote it by χ

$$\chi(y,\theta) \equiv \phi_\chi(y) + \sqrt{2}\theta\Psi_\chi(y) + \theta^2 F_\chi(y). \tag{12}$$

One can find an effective Lagrangian written in terms of two chiral superfields, S and χ which is equivalent to the expression given in (8). In our case

$$\begin{aligned} \mathcal{L} \;=\; & \frac{1}{\alpha}(S^+ S)^{1/3}\Big|_D \;+\; \frac{\delta}{4}(S^+ S)^{1/3}(\chi \;+\; \chi^+)^2\Big|_D \;+ \\ & \Big[\gamma\,(S\,\log\frac{S}{\mu^3} \;-\; S)\Big|_F \;+\; \frac{1}{16}\chi\,(S \;-\; \mu^3)\Big|_F \;+\; \text{h.c.}\Big]\;. \end{aligned} \tag{13}$$

Comparing this expression to the VY Lagrangian one notices that both the Kähler potential and the superpotential are modified by new terms. The multiplets S and χ are independent.

We would like to relate this expression to the Lagrangian of the theory written in terms of the U field (8). If the U field is postulated as a fundamental degree of freedom, then the S field is a derivative superfield $S = \mu^3 - 4\bar{D}^2 U$. Using this relation the Lagrangian (13) can be rewritten as

$$\begin{aligned} \mathcal{L} \; &= \; \frac{1}{\alpha}\, (S^+ S)^{1/3}\Big|_D \; + \; \frac{\delta}{4}\, (S^+ S)^{1/3}\, (\chi \; + \; \chi^+)^2\Big|_D \; + \\ &\Big[\gamma\, (S\, \log \frac{S}{\mu^3} \; - \; S)\Big|_F \; + \; \mathrm{h.c.}\Big] \; + \; U(\chi \; + \; \chi^+)\Big|_D \, . \end{aligned} \tag{14}$$

This expression depends on two superfields U and χ (S is expressed through U in accordance with (6)). However, the dependence on the chiral superfield χ is trivial, the combination $\chi + \chi^+$ can be integrated out from the Lagrangian (14). As a result one derives

$$\chi + \chi^+ = -\frac{2U}{\delta (S^+ S)^{1/3}}. \tag{15}$$

Substituting this expression back into the Lagrangian (14) one arrives at the original expression (8) where the S field is a derivative field satisfying the relation (6).

Let us stress again that the descriptions in terms of the Lagrangian (8) and (13) are equivalent on the mass-shell. In the Lagrangian (8) the dynamical degrees of freedom are assigned to the only superfield U, while in the Lagrangian (13) the physical degrees of freedom are found as components of two chiral supermultiplets S and χ. The peculiarity of the expression (14) is that the chiral superfield χ enters only through the real combination $\chi + \chi^+$. That is why it was possible to formulate the action in terms only of the real superfield U. It is essential from a physical point of view since the component glueball field must be real.

In order to make contact with the results of lattice simulations of SUSY YM model [14] one needs to consider the model with soft SUSY breaking term introduced via gluino mass.

The potential of the softly broken model consists of two parts , V_0 defined in (9) and an additional SUSY breaking term [4]

$$V = V_0 + \widetilde{m}_\lambda \, Re\Big(\frac{\mu^3}{16}\, \phi_\chi \; + \; \gamma\, \phi^3\Big), \tag{16}$$

where $\widetilde{m}_\lambda \equiv \frac{32\pi^2}{g^2 N_c}\, m_\lambda$.

One calculates minima of the full scalar potential V. Explicit though tedious calculations yield the following results. The VEV of the ϕ field does not get shifted when the soft SUSY breaking terms are introduced. Thus, even in the broken theory $\langle\phi\rangle = \mu$. However, the ϕ_χ (and B) fields acquire nonzero VEVs in the broken case

$$\langle\phi_\chi\rangle = \frac{8}{9\alpha\mu}\widetilde{m}_\lambda \qquad \text{and} \qquad \langle B\rangle = -\frac{8\delta}{9\alpha}\widetilde{m}_\lambda \mu. \tag{17}$$

The shift of the vacuum energy causes the spectrum of the model to be also rearranged. Explicit calculations of the masses of all lowest-spin states yield the following results

$$M^2_{scalar\,\pm} \;\; = \;\; M^2_\pm \; - \; \frac{3}{4}\, \alpha\gamma\, \mu\, \widetilde{m}_\lambda \Big(1 \; \pm \; \sqrt{1+x}\, \Big) \Big(2 \; \pm \; \frac{1}{\sqrt{1+x}}\, \Big), \tag{18}$$

$$M^2_{fermion\,\pm} \;\; = \;\; M^2_\pm \; - \; \frac{3}{4}\, \alpha\gamma\, \mu\, \widetilde{m}_\lambda \Big(1 \pm \sqrt{1+x}\, \Big) \Big(3 \; \pm \; \frac{1}{\sqrt{1+x}}\, \Big), \tag{19}$$

$$M^2_{p-scalar\,\pm} \;=\; M^2_\pm \;-\; \frac{3}{4}\,\alpha\gamma\,\mu\,\widetilde{m}_\lambda\Big(1\pm\sqrt{1+x}\,\Big)\Big(4\;\pm\;\frac{1}{\sqrt{1+x}}\,\Big), \tag{20}$$

where $M^2_\pm$ denote the masses in the theory with unbroken SUSY [4] and $x\equiv\frac{1}{288}\frac{\alpha}{\delta}\frac{1}{(\alpha\gamma)^2}$. In these expressions the plus sign refers to the heavier supermultiplet and the minus sign to the lighter set of states. One can verify that these values satisfy the mass sum rule to leading order in $O(m_\lambda)$:

$$\sum_j(-1)^{2j+1}\,(2j+1)\,M^2_j \;=\; 0\;,$$

where the summation goes over the spin j of particles in the supermultiplet.

Let us discuss the mass shifts given in eqs. (18-20). Consider the light supermultiplet. In accordance with eqs. (18-20), the masses in the light multiplet are increased in the broken theory. The biggest mass shift is found in the pseudoscalar channel. The smallest shift is observed in the scalar channel. The fermion mass falls in between these two meson states. Thus, the lightest state in the spectrum of the model is the particle which without mixing would have been the scalar glueball. There is a fermion state above that scalar. Finally, the pseudoscalar glueball is heavier than those two states.

Let us now turn to the heavy supermultiplet. In the broken theory the masses in that multiplet get pulled down. However, all states of the heavy multiplet are still heavier than any state of the light multiplet in the domain of validity of our approximations. The ordering of the states in the heavy supermultiplet is just the opposite as in the light supermultiplet: the lightest state is the pseudoscalar meson, the heaviest is the scalar, and the fermion, as required, falls between them. The qualitative features of the spectrum are shown in fig. 1.

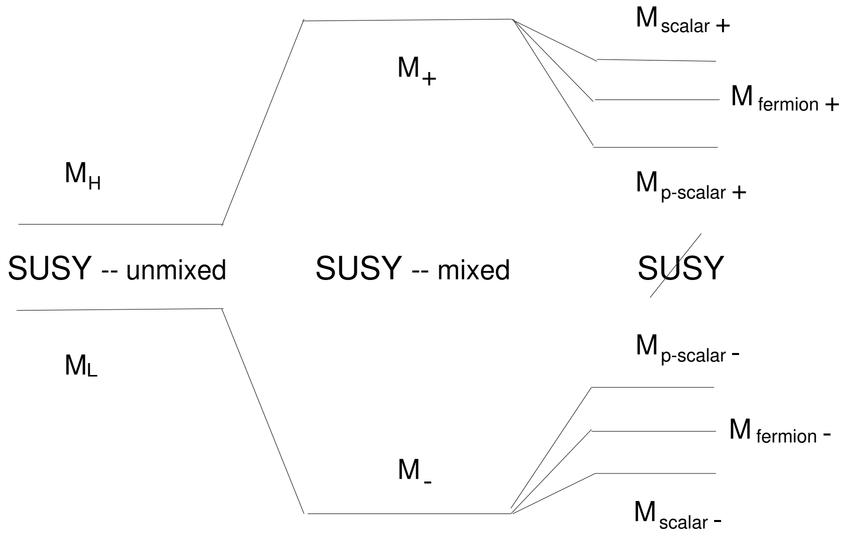

Figure 1: Qualitative behavior of mass spectrum when passing from SYM to softly broken model.

It is not surprising that the lowest mass state obtained in (18-20) is a scalar particle. This is in agreement with the result of ref. [15] where it was shown that the mass of the lightest state which couples to the operator $G^2_{\mu\nu}$ is less than the mass of the lightest state that couples to $G\tilde{G}$, in pure Yang-Mills theory. As a result, the lightest glueball turns out to be the scalar glueball [15]. One can apply the method of ref. [15] to the SYM theory as well. Due to the positivity of the gluino determinant (see ref. [16]) one also deduces that the lightest state in softly broken SYM spectrum should be a scalar particle. The pseudoscalar of that multiplet is therefore heavier.

Our result that the multiplet containing glueballs is split in such a way that the scalar is lighter than the pseudoscalar, and vice versa for the multiplet containing gluino-gluino bound states, is consistent with expectations from quark-model lore. In ordinary mesons the $l = 1$ states are heavier than their $l = 0$ counterparts and the $l = 0$ gluino-gluino bound state is a pseudoscalar, while an $l = 0$ gluon-gluon bound state is a scalar. It is interesting that in SYM with massless gluinos the $l = 0$ and $l = 1$ bound states are degenerate, but when the gluino masses are turned on one recovers the expected ordering seen in $q\bar{q}$ states.

Summarizing, we have shown that the generalized VY effective action can be written in two different ways. In one case the fundamental superfield upon which the action is constructed is the real tensor superfield U. In another approach all degrees of freedom of the model are described by two chiral superfields χ and S. In both cases the spectrum consists of two multiplets which are not degenerate in masses even when SUSY is unbroken. The spin-parity quantum numbers of these multiplets are identical to those of certain chiral supermultiplets.

We introduced a soft SUSY breaking term in the Lagrangian of the $N = 1$ SUSY Yang-Mills model. The spurion method was used to calculate the corresponding soft SUSY breaking terms in the generalized VY Lagrangian. These soft breaking terms cause a shift of the vacuum energy of the model. The physical eigenstates which are degenerate in the SUSY limit, are split when SUSY breaking is introduced. We studied these mass splittings in detail. We have confirmed that the spectrum of the broken theory is in agreement with some low-energy theorems [15], namely the scalar glueball turns out to be lighter than the pseudoscalar one. The results of the present paper can be directly tested in lattice studies of $N = 1$ supersymmetric Yang-Mills theory [14].

Even when SUSY is unbroken, the physical mass eigenstates are not pure gluon-gluon, gluon-gluino or gluino-gluino composites; rather, the physical particles are mixtures of them. The multiplet which without mixing would have been the glueball multiplet is lighter. As a result, those states cannot be decoupled from the effective Lagrangian. This means that comparisons of lattice results to analytic predictions based on the original VY action are not justified.

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