Homework 2

The goal of this homework is to gain proficiency in evaluating explicitly the spectral functions and the Green's functions in a perturbative expansion in U to second order in U.

1) Consider the symmetric ($\epsilon_f = -\frac{U}{2}$) Anderson impurity model:

$$H = \sum_{k\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \sum_{k\sigma} (V_k c^\dagger_{k\sigma} f_\sigma + V_k^* f^\dagger_\sigma c_{k\sigma}) + \epsilon_f (n_\uparrow + n_\downarrow) + U n_\uparrow n_\downarrow$$

Take the gybridization function

$$\Delta(i\omega_n) = \sum_k \frac{|V_k|^2}{i\omega_n - \epsilon_k} = -i\Gamma \mathrm{sign}\omega_n$$

where $\Gamma = V^2\rho = 1$. Evaluate the Matsubara Green's function $G(\tau)$ at $\beta = 1$, $1/10$ and $1/50$ at $U = 2$. Evaluate the $T = 0$ spectral function for $U = .5$, 1, 1.5, 2, and 3. Carry out the calculation in two schemes:

a) bare perturbation $\Sigma = \Sigma(G^0, U)$

a) self-consistent perturbation theory $\Sigma = \Sigma_{skel}(G, U)$.

This calculation requires a numerical computation of the self energy. You can do it either in the time domain or the frequency domain, and you may want to do it both and compare the results.

2) Consider the $U = \infty$ Anderson impurity model:

$$H = \sum_{k\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \sum_{k\sigma} (V_k X_{\sigma 0} c_{k\sigma} + V_k^* c^\dagger_{k\sigma} X_{0\sigma}) + \epsilon_f \sum_\sigma X_{\sigma\sigma}$$

Compute the Green's function on an expansion of V (take again $V_k = V$, $\sum_k \frac{|V_k|^2}{i\omega_n - \epsilon_k} = -i\Gamma \mathrm{sign}\omega_n$) to order V^2.

Hint: use the equation of motion to get an expansion relating $\langle\langle X_{0\sigma} X_{\sigma 0}\rangle\rangle$ to a higher order Green's function, which can then be evaluated when $V = 0$ to get results valid to order V^2.