Homework 2

Spectral functions and Green's functions in a perturbative expansion in U to second order in U.

1) Consider the symmetric ($\epsilon_f = -\frac{U}{2}$) Anderson impurity model:

$$H = \sum_{k\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \sum_{k\sigma} (V_k c^\dagger_{k\sigma} f_\sigma + V^*_k f^\dagger_\sigma c_{k\sigma}) + \epsilon_f(n_\uparrow + n_\downarrow) + U n_\uparrow n_\downarrow$$

Take the gybridization function

$$\Delta(i\omega_n) = \sum_k \frac{|V_k|^2}{i\omega_n - \epsilon_k} = -i\Gamma \text{sign}\omega_n$$

where $\Gamma = V^2 \rho = 1$.

1) Evaluate numerically the Matsubara Green's function $G(\tau)$ and plot it at $\beta = 1$, $1/10$ and $1/50$ at $U = 2$.

2) Evaluate numerically the $T = 0$ spectral function and plot it for $U = .5$, 1, 1.5, 2, and 3.

Carry out the calculation for parts 1 and 2 in two schemes:

a) bare perturbation $\Sigma = \Sigma(G^0, U)$

a) self-consistent perturbation theory $\Sigma = \Sigma_{skel}(G, U)$.

Comment on the differences between the results of the two schemes.

3) Derive an expression for the spectral function at finite temperatures and evaluate it for the same parameters as in part 2.