

## Exercise 20 : Practice in Drawing Feynman Diagrams for Interacting Theories

Consider the Hamiltonian of interacting spinless electron gas

$$\mathcal{H} = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} c_{\mathbf{k}}^{\dagger} c_{\mathbf{k}} + \frac{1}{2} \sum_{\mathbf{k}, \mathbf{k}', \mathbf{q}} V(\mathbf{q}) c_{\mathbf{k}+\mathbf{q}}^{\dagger} c_{\mathbf{k}'-\mathbf{q}}^{\dagger} c_{\mathbf{k}'} c_{\mathbf{k}}.$$

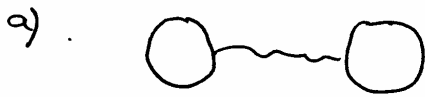
1. Draw all the connected Feynman graphs for free energy to :
  - a) first order in the interaction.
  - b) second order in the interaction.
  - c) third order in the interaction.
2. Draw all the one particle irreducible Feynman graphs for self energy to :
  - a) first order in the interaction.
  - b) second order in the interaction.
  - c) third order in the interaction.
3. Draw all the Feynman graphs contributing to the two particle Green's function to :
  - a) first order in the interaction.
  - b) second order in the interaction.
  - c) third order in the interaction.
4. Draw all the Feynman graphs contributing to the two particle vertex function to :
  - a) first order in the interaction.
  - b) second order in the interaction.
  - c) third order in the interaction.

Which one of these diagrams are two particle irreducible?

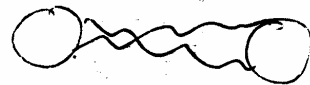
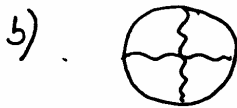
*Solutions by insoul 2001*

Free energy:

1st Order:

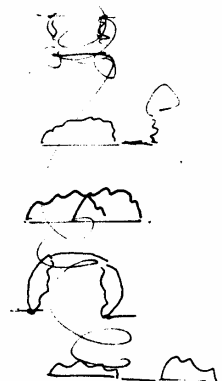
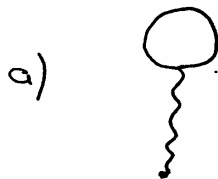


2nd order:

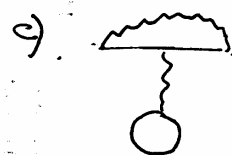
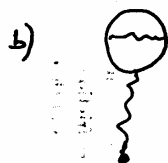
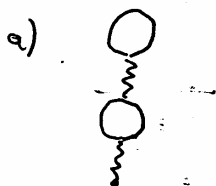


# Self Energy (1PI)

1st Order :



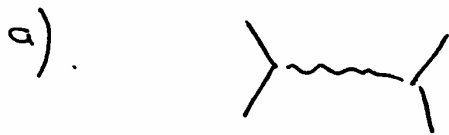
2nd Order :



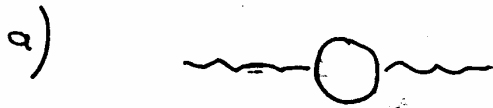


Vertex :

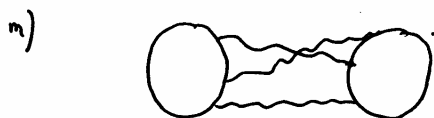
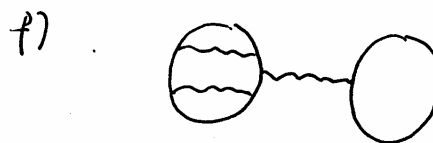
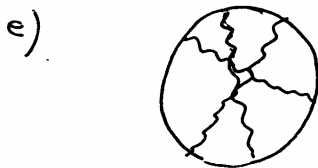
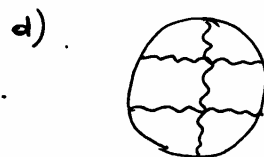
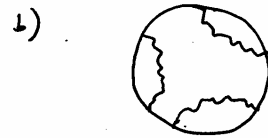
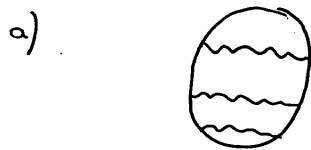
1st Order :



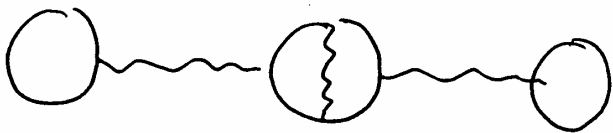
2nd Order :



# Free Energy (3rd order)



g)



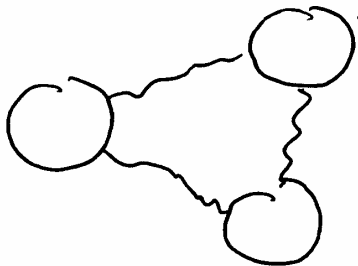
h)



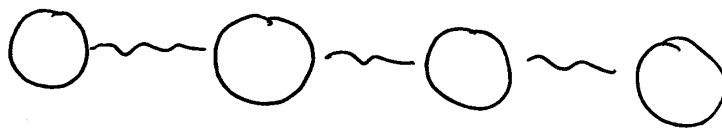
i)



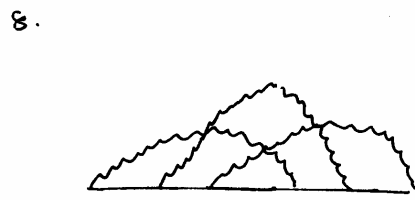
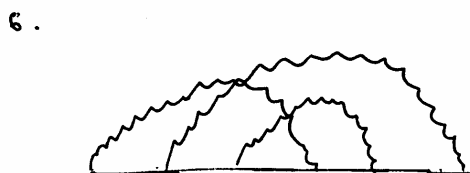
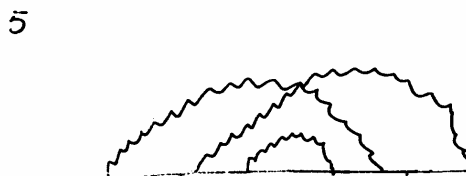
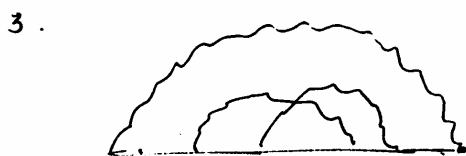
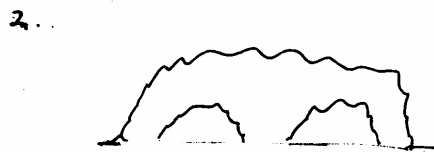
j)



k)



# Self Energy ( $\Pi$ ) (3rd order)





9.



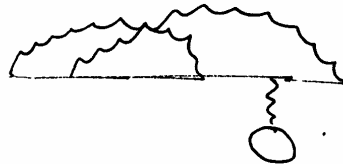
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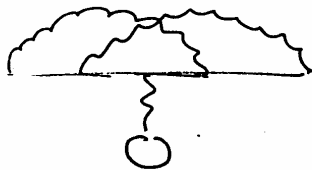
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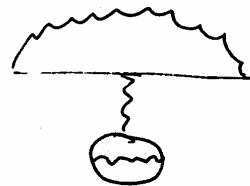
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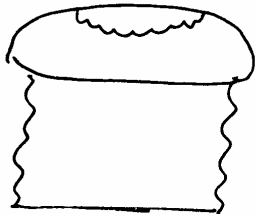
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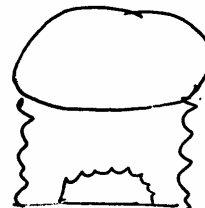
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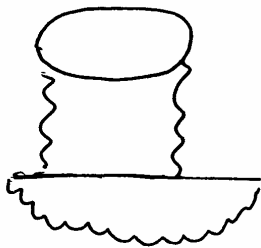
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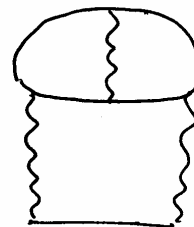
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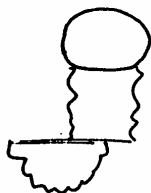
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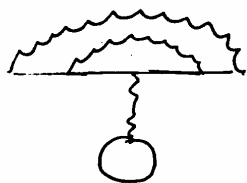
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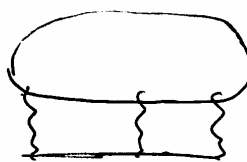
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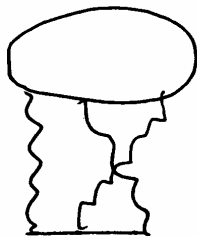
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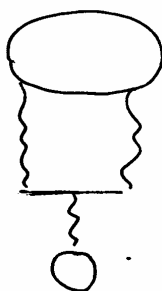
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24.



25.



26.



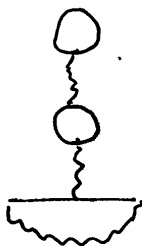
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28.



29.



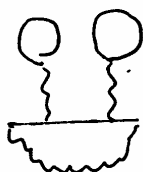
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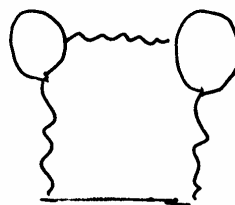
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31.

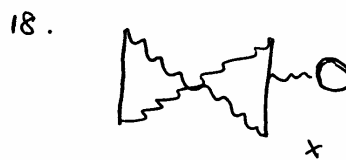
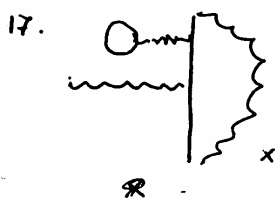
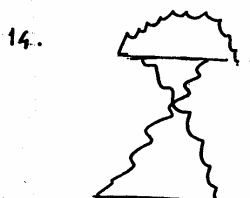
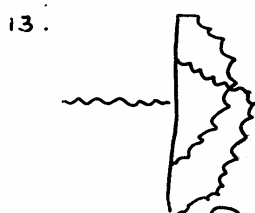
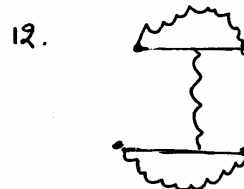
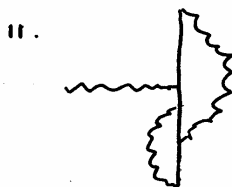
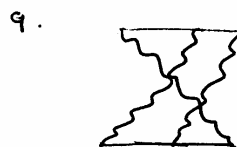
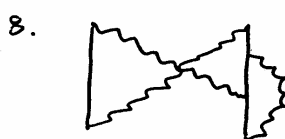
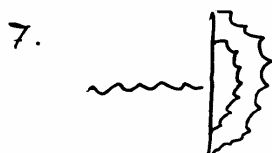
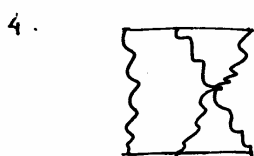
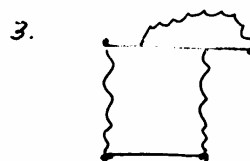
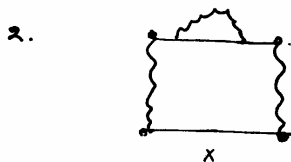
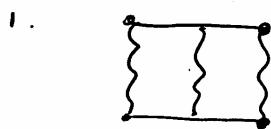


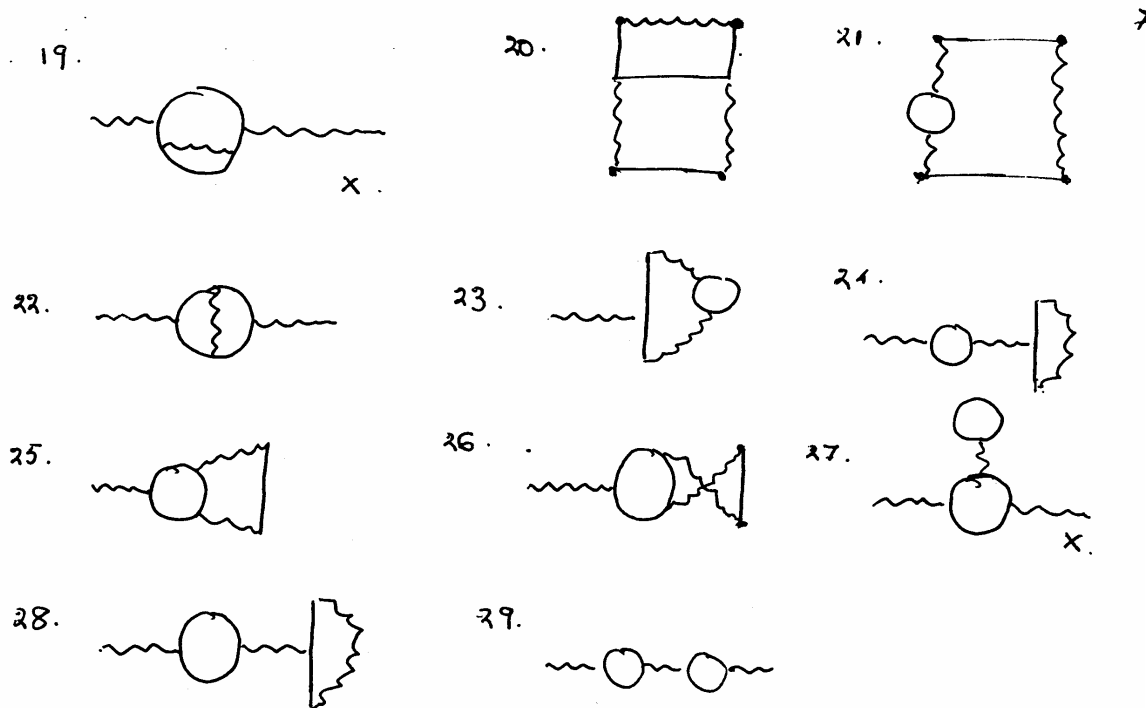
32.



# Vertex. (3rd order)

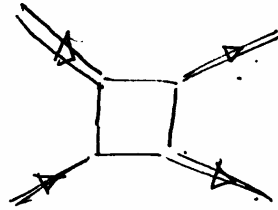
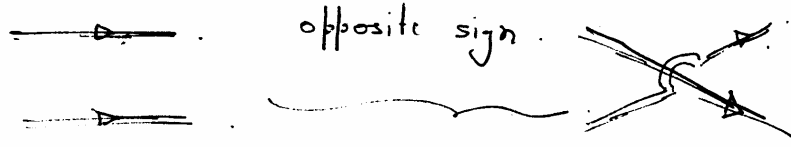
6



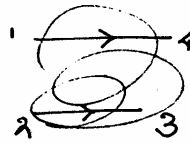
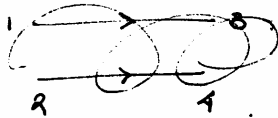


Note : Those with x sign are 2 particle reducible.

2-ptc G.

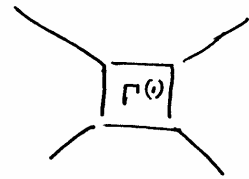
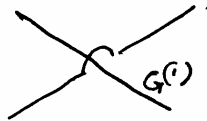


Zenth order



1st order

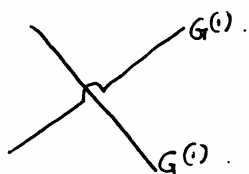
$G^{(1)}$



2nd order

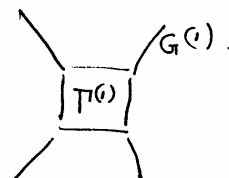
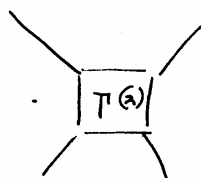
$G^{(2)}$

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$G^{(1)}$

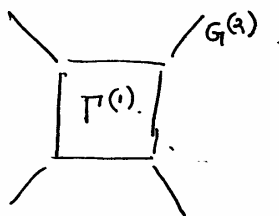
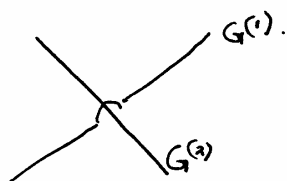
$G^{(1)}$



3rd order

$G^{(3)}$

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$G^{(2)}$

$G^{(1)}$

