Chapter 1

REVIEW of QUANTUM MECHANICS

Key Concepts:

Hilbert space of states;
Observables;
Evaluation operator;
Along a contour;
Imaginary time;
Chronological Order;
Schrödinger-Heisenberg interaction;
Picture representation;
Schröedinger picture, states evolve in time according:

$$i\partial|\psi_s(t) > \frac{}{\partial t} = H_s(t)|\psi_s(t) >$$

$$|\psi_s(t) >= U_s(t, t_0)|\psi_s(t_0) >$$

$U_s(t, t_0)$ evelution operator plays an essential role in the theroy.

$$i\partial U(t, t_0 \frac{}{\partial t} = H_s(t)U(t, t_0)$$

$$U(t, t_0) = I$$

For mixed states, use density matrix:

$$id\rho_s \frac{}{dt} = [H_s, \rho_s]$$

Figure 1.1:

Formal solution of the evolution equation:
$U(t,t_0) = Te^{\frac{1}{i}\int_{t_0}^{t} H_s(\tau)d\tau}$
$= \sum_{n=0}^{\infty} \frac{(1}{i)^n} \int_{t_0}^{t} \ldots \int_{t_0}^{t_{n-1}} H_s(t_1) \ldots H_s(t_n) dt_1 \ldots dt_n$
The operator
orders operatorrs putting the latter ones on the right.

Eqn.( )–( ) can be in fact formulated for an arbituary oriented contour C in the condex plane if we assume that H_s is defined there.

If C is a contour going from Z_1 to Z_2 :

$$U(Z_2, Z_1) \equiv T_{\mathcal{C}} e^{\frac{1}{i}\int_{\mathcal{C}} H_s(z)\, dz}$$

obeys

$$i\partial U(t_2,t_1)\overline{\partial Z_2} = H_s(Z_2)U(Z_2,Z_1)$$

$T_{\mathcal{C}}$ orders operators along the contour C putting latter operators on the right.

Heisenberg picture.

Here one performs a unitary rotation to keep the state vectors fixed,*i.e.*

Figure 1.2: figure 2

$$|\psi_H(t) >= U(t,t_0)^{-1}|\psi_s(t) >$$

t_0 is an arbituary time where all the pictures ? made. The observables evolve according to $O_H(t) = U(t,t_0)^{-1}O_s(t)U(t,t_0)$

$$\dot{O}_H = \partial O_H \frac{}{\partial t} + \frac{1}{i}[O_H, H_H]$$

where $\partial O_H \frac{}{\partial t} \equiv U(t,t_0)_{-1}\partial O_S \frac{}{\partial t} U(t,t_0)$

The density matrix obeys $d\rho_H \frac{}{dt} = 0$

The notion of evolution along a contour is useful because it allows to unify different formulations of many body methods. A particular useful contour is [0, -iβ], $[i.e.] alongtheimagingaxis.(seeFig.2)$

Example: The evolution operator from 0 to $-i\beta$along the contour shown in Fig. 2 is given by:

$$U_S(-i\beta,0) = T_{\mathcal{C}} e^{\frac{1}{i}\int_{\mathcal{C}} H_S(\tau)\, d\tau'}$$

and equals to the Boltzmann density matrix $e^{-\beta H_S}$ if H_S does not depend on time.

Interaction picture:

This representation is defined with reference to a ?thing of the hamitonian into two parts $H = H_0 + V$. Roughly speaking, the onservables and evolved which is usually simple while the state vectors are driven by V.

$$|\psi_I(t) >= e^{iH_0(t_1-t_0)}|\psi_S(t) >$$

The observables evolve according to $V_I(t) = e^{iH_0(t-t_0)}V_S(t)^{-iH_0(t-t_0)}$. The evolution operator in this picture is given by:

$$U_I(t_i,t) = e^{iH_0(t_1-t_0)}U_S(t_1,t)e^{-iH_0(t-t_0)}$$

and obeys:

$$i\partial U_I(t_1,t_2)\overline{\partial t} = V_I(t_1)U_I(t_1,t_2)$$

$$U(t,t) = I$$

The formal solution:

$$U_I(t_1,t_2) = Te^{\frac{1}{i}\int_{t_2}^{t_1} V_I(\tau)\, d\tau}$$

similarly:

$$i\dot{O}_I = [O_I, H_0]$$