1. Three short questions
(a) (5 pts) State the steady current condition. Does the current distribution \( \mathbf{j}(r, t) = ax \hat{i} \) satisfy the steady current condition? Show your work.

\[
\text{current distribution is independent of time and change in distribution is also time independent.}
\]

\[
\text{equation of continuity: } \nabla \cdot \mathbf{j} = -\frac{\partial \rho}{\partial t} = 0
\]

\[
\nabla \cdot (ax \hat{i}) = a \text{ is not zero in general, so NO}
\]

(b) (5 pts) State the definition of resistivity (preferably in the form of an equation). Which, if any or more than one, of the following is a unit of resistivity?

\[
\begin{align*}
E &= pj \\
P &= \frac{E}{j} = \frac{Nm^2}{CA} = \frac{Nm^2 S}{C^2}
\end{align*}
\]

(c) (5 pts) The infinitesimal segment of length \( ds \) carrying current \( I \) in the \( \hat{j} \) direction is shown in the figure. What is its contribution to the magnetic field at points \( P_1 \), \( P_2 \) and \( P_3 \)?

\[
\vec{B}_{167} = \frac{\mu_0 I ds \times \hat{r}}{4\pi r^2}
\]

\[
d\vec{B}_1 = \frac{\mu_0 I ds \hat{k}}{4\pi a^2}
\]

\[
d\vec{B}_2 = 0 \quad (ds \parallel \hat{r})
\]

\[
d\vec{B}_3 = \frac{\mu_0 I ds \cdot \frac{1}{\sqrt{2}} \hat{K}}{4\pi (\sqrt{2}a)^2} \sin\theta
\]
2. A current I flows counterclockwise in the square loop of side \( a \) shown in the figure. This loop is centered at the origin and is partly in a region of nonzero magnetic field \( \vec{B} \) perpendicular to the page, shown as the shaded area.

(a) (2 pts) If the field is uniform with \( \vec{B}(x,y) = B_0 \hat{k} \), what is the magnetic force on the vertical segment CD?

(b) (4 pts) If the field is nonuniform with \( \vec{B}(x,y) = B_0(x/a)\hat{k} \), what is the magnetic force on the vertical segment CD?

(c) (4 pts) If the field is nonuniform with \( \vec{B}(x,y) = B_0(y/a)^2\hat{k} \), what is the magnetic force on the vertical segment CD?

\[
\vec{F} = I \vec{L} \times \vec{B} \quad \text{if} \quad \vec{B} \quad \text{is uniform on segment}
\]

\[
(a) \quad \vec{F} = Ia B_0 \hat{\lambda}
\]

\[
(b) \quad \vec{B} \quad \text{is uniform on segment}
\]

\[
\vec{F} = Ia \frac{B_0}{2} \hat{\lambda}
\]

\[C) \quad \vec{B} \quad \text{is NOT uniform on segment} \quad y = \frac{a}{2}
\]

\[
\vec{F} = \int I d\vec{s} \times \vec{B} = (\int I B_0 y^2 dy) \hat{\lambda}
\]

\[
y = -\frac{a}{2}
\]

\[
= \hat{\lambda} \frac{I B_0}{a^2} \frac{1}{3} [\frac{a^3}{2} - (\frac{a}{2})^3]
\]

\[
= \hat{\lambda} \frac{I B_0}{a^2} \frac{1}{2} \cdot 2a^3
\]

\[
= (\frac{IB_0}{a^2} \frac{1}{2} \cdot 2a^3) \frac{\hat{\lambda}}{12}
\]

\[
\vec{F} = \frac{IB_0 a}{12} \hat{\lambda}
\]
3. When two bulbs A and B are connected in parallel, bulb A is twice as bright as bulb B. (a) (5 pts) What is the resistance $R_B$ of bulb B in terms of the resistance $R_A$ of bulb A? 
(b) (5 pts) If the two bulbs are placed in the circuit shown in the figure with $R = 2R_A$, what is the brightness of bulb B in terms of the brightness of bulb A?

**OPTIONAL** If the brightnesses of the two bulbs when placed in the circuit shown are the same, what is $R$ in terms of $R_A$?

\[ P_A = \frac{\varepsilon^2}{R_A} = 2P_B = \frac{2\varepsilon^2}{R_B} \]

\[ R_B = 2A \]

\[ \frac{1}{2} 2RA \quad \frac{1}{2} 2RA \]

different ways to do this.

easiest - current through B is $\frac{1}{2}$ current through A

\[ \text{brightness of B} = i_B^2 R_B = \left( \frac{i_A}{2} \right)^2 \cdot 2R_A = \frac{1}{2} \left( \text{brightness of A} \right) \]
4. An infinitely long cylinder of radius 2a centered on the z axis has

\[ \vec{j}(\vec{r}) = j_0 \left( \frac{r}{a} \right) - \left( \frac{r}{a} \right)^2 \hat{k} \]

(a) (2 pts) Where inside the cylinder is \( \vec{j}(\vec{r}) \) equal to zero?
(b) (6 pts) Find the magnetic field at all points on the positive x axis.
(c) (2 pts) Where, if anywhere, inside the cylinder is \( \vec{B}(\vec{r}) \) equal to zero?

\[ \frac{2a}{r} \]

\( \vec{j}(\vec{r}) = 0 \) at \( r \) s.t. \( \left( \frac{r}{a} \right) - \left( \frac{r}{a} \right)^2 = 0 \)

\[ r = 0, a \]

b) Use Ampere's law:

\[ \vec{B} \text{ magnitude depends only on } r \]

\[ \text{along } \hat{\theta}, \quad r' = r \]

\[ B(r) 2\pi r = \mu_0 \int j(\vec{r}) 2\pi r' dr' \]

\[ r' = 0 \]

\[ = \frac{2\pi \mu_0}{a} \int j_0 \left( \frac{r^2 - r'^2}{a} \right) dr' \]

\[ B(r) 2\pi r = \frac{2\pi \mu_0}{a} j_0 \left( \frac{1}{3} r^3 - \frac{r^4}{4a} \right) \quad \text{for } r < 2a \]

\[ B(\vec{r}) = \frac{\mu_0 j_0}{a} \left( \frac{1}{3} \frac{r^2 - r'^2}{2a} \right) \]

\[ B(\vec{r}) = \frac{\mu_0 j_0}{a} (\frac{r^2}{3} - \frac{1}{4}(\frac{r}{a})) \]

\[ \left\{ \begin{array}{l}
\text{for } r < 2a, \quad r = 0, \quad r = \frac{r}{2} \quad a \\
\text{for } r > 2a
\end{array} \right. \]

\[ B(\vec{r}) = \frac{1}{2\pi r} \frac{2\pi \mu_0}{a} j_0 \left( \frac{1}{3} (2a)^3 - \frac{r^4}{4a} \right) \]

\[ = \mu_0 a^2 j_0 \left( \frac{8}{3} - 4 \right) = \mu_0 a^2 j_0 \left( -\frac{4}{3} \right) \]
5. (10 pts) What is the Thévenin equivalent of the network shown in the figure if all resistors have resistance 200Ω? Draw it and label it, being sure to show the points A and B.

\[ R_{eq} = \text{take out emf sources} \]
\[ = \left( \frac{1}{400} + \frac{1}{400} \right)Ω = 200Ω \]

\[ V_{eq} = \text{find open circuit voltage difference} \]
\[ 3 - 200i + 3 - 200i - 2 - 200i - 200i = 0 \]
\[ 4 - 800i = 0 \]
\[ i = \frac{1}{200} \text{A} \]

From A to B: \(-1V + 3V - 2V + 3V = +4V\)

\[ V_{eq} = 4V \]

\[ A \quad 200Ω \quad 4V \quad B \]
6. (10 pts) A system contains two infinite straight current-carrying wires as shown in the figure. What is the magnetic field at the point P?

\[ \vec{B}_p \text{ from left hand wire is} \quad \frac{\mu_0 I}{2\pi a} \hat{j} \]

\[ \vec{B}_p \text{ from right hand wire is} \quad \frac{\mu_0 I}{2\pi a} \hat{k} \]

\[ \vec{B} \text{ is} \quad \frac{\mu_0 I}{2\pi a} (\hat{j} + \hat{k}) \]
7. A system consists of two conducting spheres of equal radius $a$ separated by $R > 2a$ as in the figure. If I put a charge of 2.0 C on sphere 1 and zero charge on sphere 2, the potential on sphere 1 is 5.0 V and the potential on sphere 2 is 3.0 V.

(a) (4 pts) If I put 2.0 C on sphere 2 and zero charge on sphere 1, what is the potential on each sphere?

(b) (6 pts) Find the capacitance matrix for this system.

\[ \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \]

\[ \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \]

\[ 3(5C_{11} + 3C_{12} = 2) \]
\[ -5(3C_{11} + 5C_{12} = 0) \]

\[(9 - 25)C_{12} = 6 \]
\[ C_{12} = \frac{-6}{16} = -\frac{3}{8} \text{ F} \]
\[ C_{11} = \frac{5}{8} \text{ F} \]

\[ \begin{bmatrix} 5/8 \text{ F} & -3/8 \text{ F} \\ -3/8 \text{ F} & 5/8 \text{ F} \end{bmatrix} \]