An infinitely long non-conducting cylindrical shell is charged with 2 nC/m. The inner radius is 5 cm and the outer radius is 10 cm.

a.) What is the magnitude of the electric field at a point 20 cm away from the axis of the shell?

b.) How much is the volume charge density on the shell?

c.) How much is the magnitude of the electric field at a point which is at 8 cm from the axis of the cylindrical shell? (Please note that this point will lie in the cylindrical shell, since the distance from the axis is greater than the inner radius, and lesser than the outer radius).

$$20 \text{ cm} > \text{radius of cylinder. (easy)}$$

Construct Gaussian Cylinder of radius 20 cm and length L.

Gauss's Law

$$\int \mathbf{E} \cdot d\mathbf{A} = \frac{\Phi_{\text{enc}}}{\varepsilon_0}$$

Top & Bottom Circular Faces have 0 flux through them.

(Because Electric Field in RADIAL direction)

$$E \text{(Area of curved surface)} = \frac{\Phi_{\text{enc}}}{\varepsilon_0} \rightarrow \frac{\lambda \text{(charge per length)}}{2 \pi r L}.$$  

$$\Rightarrow E(r) = \frac{\lambda}{2 \pi \varepsilon_0 r L}.$$  

$$\Rightarrow E(r) = \frac{\lambda}{2 \pi \varepsilon_0 \cdot 20 \text{ cm}}.$$  

$$\Rightarrow E(r=20 \text{ cm}) = \frac{\lambda}{2} \frac{2 \times 10^{-9} \text{ C/m}}{\varepsilon_0 \cdot 0.2 \text{ m}} \approx 180 \text{ N/C}$$
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b) Charge per unit length = \(2 \times 10^{-9} \text{ C/m}\).

To find volume charge density use

\[ s = \frac{Q}{V} \]

\[ Q = \lambda L \]

Volume of this shell = \(\pi (r_{out}^2 - r_{in}^2) L\)

\[ s = \frac{Q}{V} = \frac{2L}{\pi (r_{out}^2 - r_{in}^2)} \]

\[ s = 85 \text{ nC/m}^3 \]
(c) For point inside the cylinder consider a Gaussian cylinder with radius $r_i$.

This Gaussian cylinder does not enclose the entire of the cylindrical shell, but only a part. If the length of the Gaussian cylinder is $L$ & radius $r$, then the net charge enclosed

$$\Phi_{enc} = S \times (\text{Volume enclosed})$$

$$\Rightarrow \Phi_{enc} = S \times (\pi r^2 - \pi r_i^2) L.$$ 

For Gauss's law $\oint \vec{E} \cdot d\vec{A} = E(r) 2\pi r L$ (same as part (a))

Thus $E(r) 2\pi r L = \frac{\Phi_{enc}}{\epsilon_0} = \frac{S}{\epsilon_0} \pi (r^2 - r_i^2) L$

$$\Rightarrow E(r) = \frac{S}{2 \epsilon_0} \left( \frac{r^2 - r_i^2}{r} \right).$$