Quiz 3 Solutions

1. A charged particle with charge 10C and mass 1g is travelling at velocity 200 m/s in the positive x direction. If you want to bring it to rest in 10 sec, what would be the magnitude and direction of acceleration? If this acceleration is provided by an electric field, what would be the magnitude and direction of this electric field?

Solution

Using \( v_f = v_i + at \) and \( v_f = 0 \text{m/s}, v_i = 200 \text{m/s}, \) and \( t = 10 \text{s} \), we get \( a = -20 \text{m/s}^2 \), which implies the magnitude of acceleration is \( 20 \text{m/s}^2 \) and direction is negative x. Then we can calculate the Force on particle \( F = ma = 1 \text{g} \times (-20 \text{m/s}^2) = -0.02 \text{N} \), and using \( F = qE \), we get the Electric Field, \( E = -(0.02 \text{N})/(10 \mu \text{C}) = -2000 \text{N/C} \). Thus, magnitude of electric field is \( 2000 \text{N/C} \) and direction is again negative x.

2. Quadrupole Field: Three charges are fixed on the x-axis. Charge 1 has a charge of \(+q \text{C}\) and is at \( x = -a \), charge 2 is \(-2q \text{C}\) and at \( x=0 \), and charge 3 is again \(+q \text{C}\) and is at \( x=+a \). Find the direction and magnitude of electric field at a point on the x-axis at \( x = +r \) where \( r \gg a \). (This setup is called a quadrupole and as we saw for an electric dipole, this electric field is different than Coulomb’s law at large distances.)

Solution

Total electric field at \( x = +r \) is the sum of individual electric fields of each charge. Note that electric field points away from positive charge and towards negative charge.

\[
\overrightarrow{E} = \overrightarrow{E}_1 + \overrightarrow{E}_2 + \overrightarrow{E}_3 \\
= \frac{kq}{(r + a)^2} \hat{i} + \frac{k(2q)}{r^2} (-\hat{i}) + \frac{kq}{(r - a)^2} \hat{i} \\
= kq \left( \frac{1}{(r + a)^2} - 2 \frac{1}{r^2} + \frac{1}{(r - a)^2} \right) \hat{i}
\] 

Since \( r \gg a \), if you set \( a = 0 \), you will get \( \overrightarrow{E} = 0 \). Similarly, if you assume that from far away all these charges look like they are at the same point and you say that \( q_{total} = q + (-2q) + q = 0 \), you will once again get \( \overrightarrow{E} = 0 \). Both these are wrong because they are bad approximations! This argument is equivalent to saying that \( \sin(\theta) = 0 \) for small \( \theta \), when \( \sin \theta \approx \theta \) for small \( \theta \). The idea is that if your function is 0 when your variable, say \( \theta = 0 \), then you need to use the next term in the Taylor series (of first power in \( \theta \)) to approximate it. Of course at \( \theta = 0 \) we do get 0, but for small \( \theta \) which is not 0, you will get a small but non-zero answer.
So, we then realize that the right way to go ahead is to not set \( a = 0 \) right away, but to expand in powers of \( a \) and then use the term with the lowest power of \( a \) with non-zero coefficient. Let’s work some more on the equation we have. The idea will be to expand in powers of \( a \) in the numerator, and see which one has the lowest non-zero coefficient.

\[
\vec{E} = kq \left( \frac{1}{(r + a)^2} - \frac{2}{r^2} + \frac{1}{(r - a)^2} \right) \hat{i} \]  
(4)

\[
= kq \left( \frac{r^2 (r - a)^2}{(r + a)^2} - 2 \frac{(r + a)^2}{r^2} (r - a)^2 + \frac{(r + a)^2}{r^2} r^2 \right) \hat{i} \]  
(5)

\[
= kq \left( \frac{r^2 (r^2 - 2ar - a^2)}{(r + a)^2} - 2 \frac{(r^2 - a^2)^2}{r^2 (r - a)^2} \right) \hat{i} \]  
(6)

\[
= kq \left( \frac{(r^4 - 2ar^3 + a^2 r^2)}{(r + a)^2} - 2 \frac{(r^4 - 2a^2 r^2 + a^4)}{r^2 (r - a)^2} \right) \hat{i} \]  
(7)

\[
= kq \left( \frac{6a^2 r^2 - 2a^4}{(r + a)^2 r^2 (r - a)^2} \right) \hat{i} \]  
(8)

\[
(9)
\]

Now we notice that in the numerator \( a^2 r^2 \gg a^4 \), since \( r \gg a \), and hence the numerator becomes just \( 6a^2 r^2 \). In the denominator we can ignore \( a \) since we get \( r^6 \) when we set \( a = 0 \). Thus putting it together for \( r \gg a \) we get

\[
\vec{E} \approx \frac{6kqa^2 r^2}{r^6} \hat{i} = \frac{6kqa^2}{r^4} \hat{i} \]  
(10)

Notice that as \( r \gg a \) this field falls off as \( 1/r^4 \). This field is known as a “Quadrupolar Field”. Compare this with electric field fall-off for a single charge which \( 1/r^2 \), and for a dipole, which we saw in the recitation, falls as \( 1/r^3 \).