1. The waveform for a standing wave on a string is $y(t) = 1\text{cm} \sin[(\pi \text{ m}^{-1}) x] \sin[(12 \pi \text{ sec}^{-1}) t]$. The length of the string is 3m.
   a. Which harmonic ($n$) does the equation describe?
   b. Find the frequency of the $n=5$ harmonic.

   \[ k_n = \frac{2\pi}{n} , \quad n = 1, 2, \ldots \]
   \[ \Rightarrow \quad \frac{2\pi}{n} = \frac{n\pi}{L} \]
   In this case $k = \frac{\pi}{3\text{ m}^{-1}} = \frac{n\pi}{3\text{ m}}$  \[ \Rightarrow \quad n = 3 \]

   (Remember $y = A\sin(kx)\sin(\omega t)$)

   b) $\omega$ for given wave function is $\omega_3 = 12\pi\text{ sec}^{-1} \rightarrow 3^{rd}$ Harmonic

   \[ \omega_n = \sqrt{\frac{k_n}{L}} \]
   \[ \Rightarrow \quad \omega_5 = \frac{5}{3} \quad \Rightarrow \quad \omega_5 = \frac{5 \times 12\pi\text{ sec}^{-1}}{3} \]
   \[ \Rightarrow \quad f_5 = \frac{\omega_5}{2\pi} = \frac{5 \times 12\pi}{3 \times 2\pi} = 10\text{ Hz} \]

2. You are standing on a platform with two trains moving towards the platform from opposite directions. One train is moving at 10 m/s and the other is moving at 20 m/s. Both of them are sounding their horns at frequency $f=200\text{Hz}$. Speed of sound in air is 343 m/s.
   a. Find the frequencies that you hear of the two trains. What will be the beat frequency?
   b. For a traveller on the train that is moving at 10m/s, what would be the beat frequency that she hears?

   a) Since both trains are travelling towards the platform, you will hear higher frequency horns for both trains and in both cases the source are moving.

   \[ f_1' = \frac{f_1 \cdot V_{\text{sound}}}{V_1 - V_{\text{sound}}} \quad f_2' = \frac{f_2 \cdot V_{\text{sound}}}{V - V_2} \]

   \[ f_1' = \frac{200 \times 343}{343 - 10} = 200 \times \frac{343}{333} = 206 \text{ Hz} \]

   \[ f_2' = \frac{200 \times 343}{343 - 20} = \frac{200 \times 343}{323} = 212.38 \text{ Hz} \]

   Beat frequency \[ = \left| f_1' - f_2' \right| = 212.38 - 206 = 6.38 \text{ Hz} \]
b) For traveller on train 1 (moving at 10 m/s) there is no Doppler shift for the horn of train 1 (no relative velocity of train and traveller on the same train).

\[ f'_1 = f_1 = 200 \text{ Hz} \]

For train 2, the detector is traveller on train 1. Thus \( V_D = 10 \text{ m/s} \). The source which is train 2 is also moving and thus \( V_S = 20 \text{ m/s} \).

\[ f'_2 = f_2 \frac{V + V_D}{V - V_S} \rightarrow (>0 \text{ since detector moving towards source}) \]

\[ f'_2 = \frac{200 \times (343 + 10)}{343 - 20} \]

\[ f'_2 = 218.6 \text{ Hz} \]

\[ \Rightarrow \text{ Beat Frequency} = |f'_1 - f'_2| = 218.6 - 200 = 18.6 \text{ Hz} \]