The Shapes of X-ray Galaxy Clusters: Effect on the Hubble Constant from the SZ Effect and the Distribution of Intrinsic Axial Ratios

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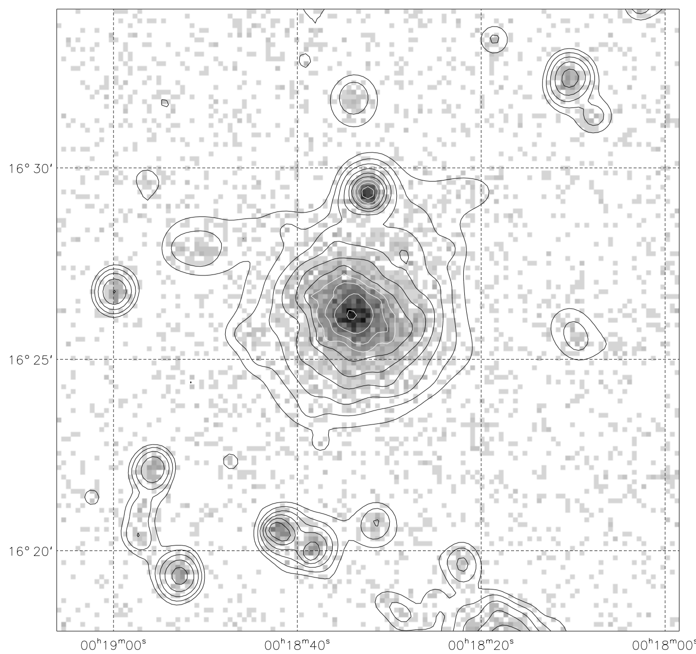
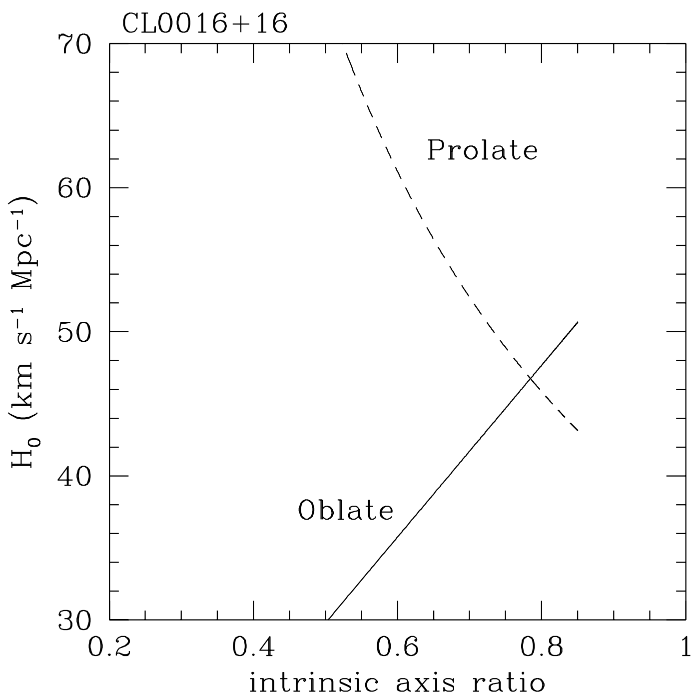
With thanks to David Merritt

Paper presented at the workshop on *Cosmological Constraints from X-Ray Clusters* held 9–11 December, 1998, in Strasbourg, France

ROSAT PSPC Image of CL0016+16 (Hughes & Birkinshaw 1998, ApJ, 501, 1) shows an apparent axial ratio of 0.85.



Derived value of H_0 from the Sunyaev-Zel'dovich (SZ) effect depends strongly on the assumed geometry and inclination angle.



Can one quantify the uncertainty on the intrinsic axial ratio of the cluster and thereby obtain an estimate of the error on H_0?

A Bayesian Formulation of the Problem

[See Statler, 1994, ApJ, 425, 500 for a detailed discussion of these issues in the context of determining the intrinsic shapes of elliptical galaxies]

Assumptions

- ⋆ Axisymmetric hypothesis
- ⋆ Random orientations

Probability of a specific observed axial ratio

$$f(i, o_0) \propto G_P(i)\, g(o_0, i)$$

[constant of proportionality set by normalization condition $\int f(i, o_0)\, di = 1$]

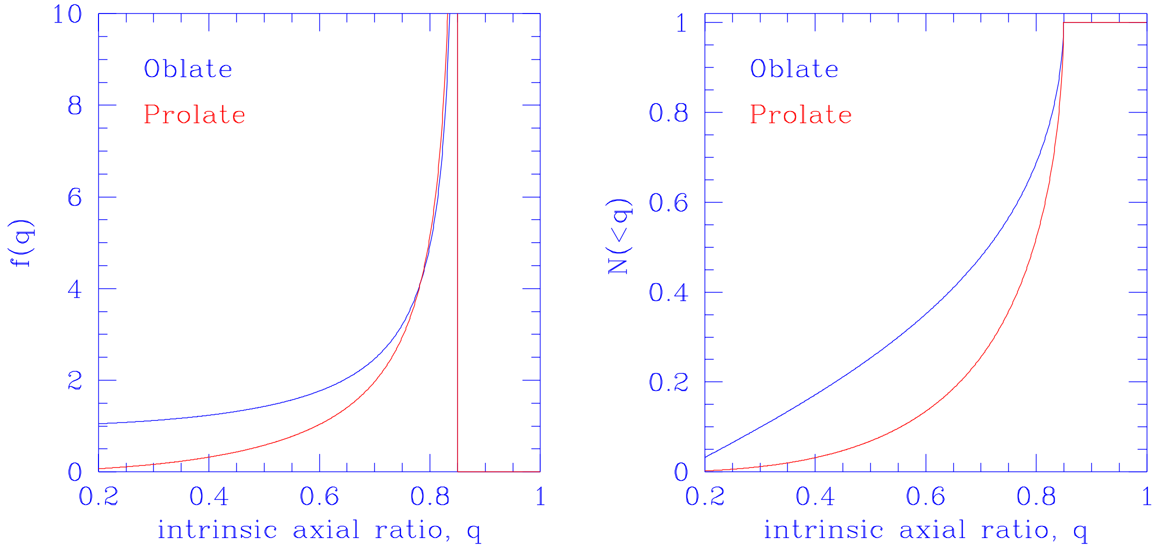
- ⋆ i: intrinsic axial ratio
- ⋆ o_0: observed axial ratio
- ⋆ $f(i, o_0)$: probability that o_0 came from an object with i ("posterior density")
- ⋆ $G_P(i)$: parent distribution of intrinsic axial ratio, i ("prior density")
- ⋆ $g(o_0, i)$: fraction of objects with i that, in random orientations produce the observed o_0 ("likelihood")

Geometry determines $g(o_0, i)$

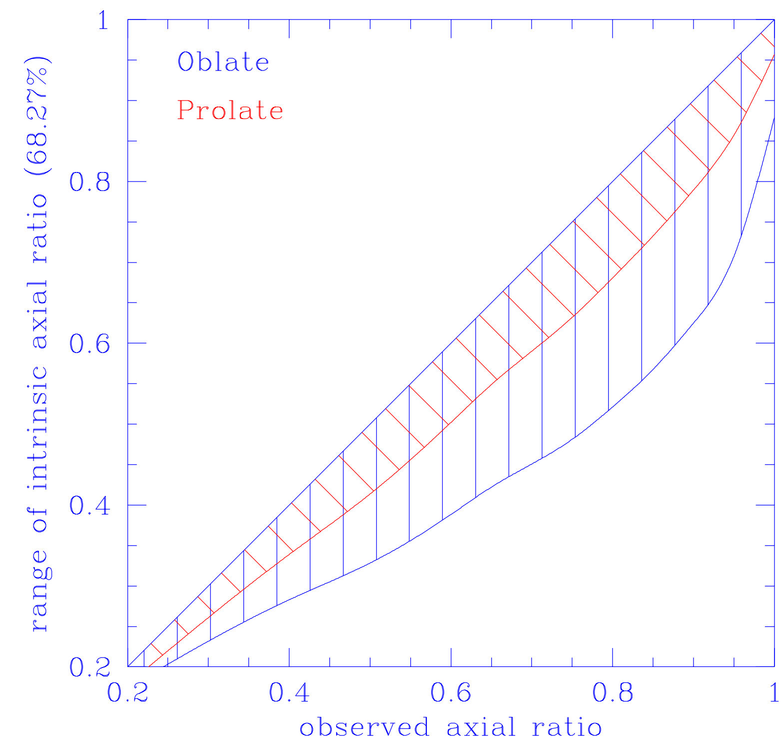
- ⋆ Oblate objects: $g(o_0, i) = \frac{o_0}{\sqrt{1-i^2}\sqrt{o_0 - i^2}}$
- ⋆ Prolate objects: $g(o_0, i) = \frac{(i/o_0)^2}{\sqrt{1-i^2}\sqrt{o_0 - i^2}}$

Initially assume uniform prior

Uniform Prior for distribution of intrinsic axial ratio



Confidence range on intrinsic axial ratio



Include Prior Information

Consider the distribution of apparent axial ratios for a sample of clusters

$$F(o) = \int f(i, o) di = \int G_P(i)\, g(o, i) di$$

- ⋆ $F(o)$: Distribution of apparent axial ratios
- ⋆ other functions: as above

Invert equation to obtain estimate of $G_P(i)$

Inverse problem with a long history in studies of elliptical galaxies (dates back to Hubble 1926)

- ⋆ "Ill-conditioned" (sensitive to small changes in the data)
- ⋆ Requires smooth estimate of $F(o)$, but data are discrete

Recent approaches

- ⋆ Assume parametric form for $F(o)$ and "fit" for parameters (Ryden 1992 ApJ, 396, 445; Lambas, Maddox, & Loveday 1992, MNRAS, 258, 404)
- ⋆ Use iterative deconvolution techniques, e.g., Lucy-Richardson algorithm (Binney & de Vaucouleurs 1981, MNRAS, 194, 679; Franx et al. 1991, ApJ, 383, 112; Fasano & Vio 1991, MNRAS, 249, 629)
- ⋆ Nonparametric (Tremblay & Merritt 1995, AJ, 110, 1039)
  - Maximum Penalized Likelihood (MPL)
  - Requires choice of smoothing length, λ

    $\lambda \rightarrow 0$: spikes at most or all data points

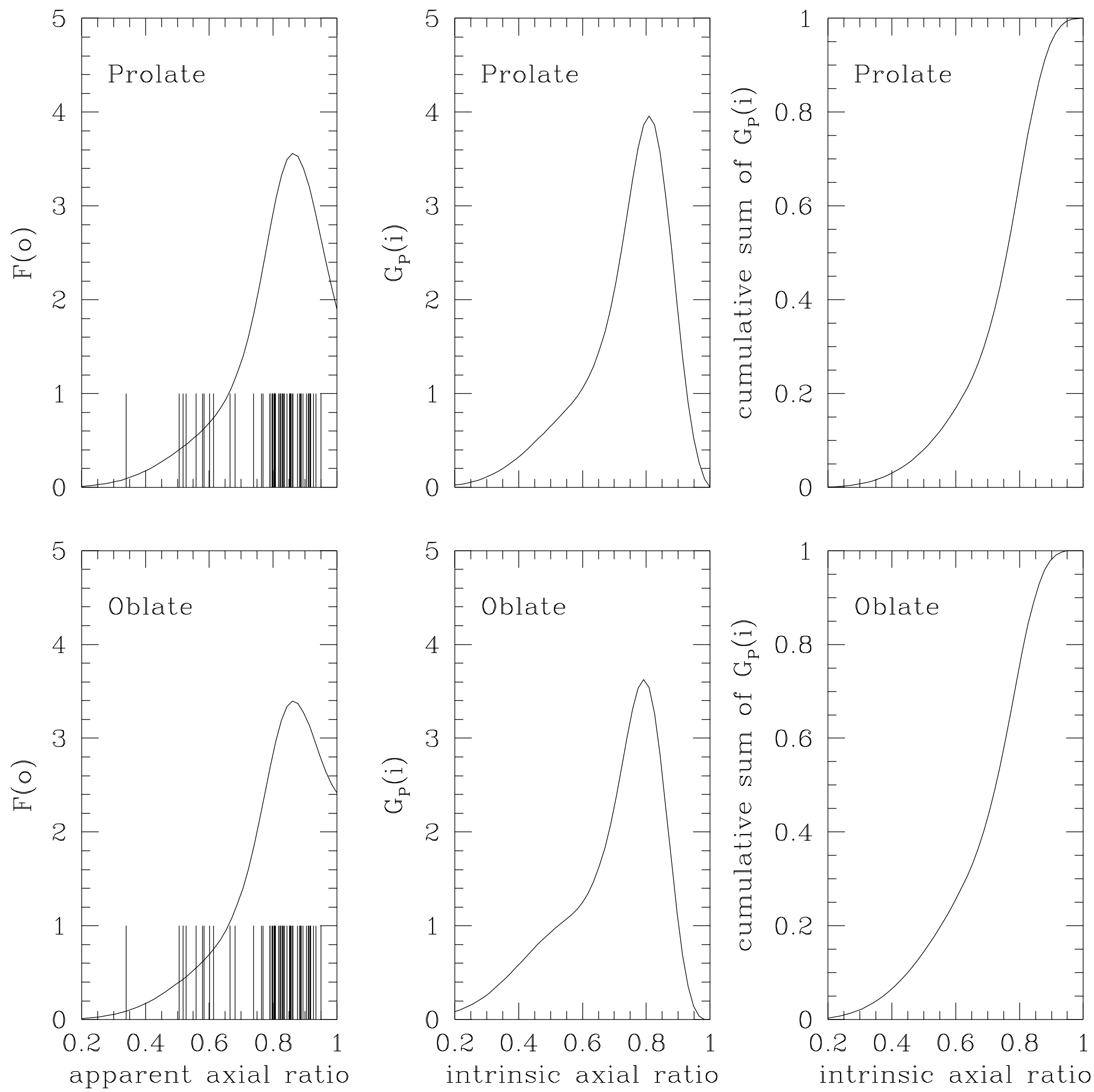
    $\lambda \rightarrow \infty$: normal density with same mean and variance as data
  - Confidence bands on estimates obtained by bootstrap resampling from the data

Approach

- ⋆ Build MPL estimate of $G_P(i)$ under oblate and prolate assumptions
- ⋆ Integrate above equation to obtain $F(o)$
- ⋆ Compare to cluster sample (Mohr et al. 1995, ApJ, 447, 8)
  - 65 galaxy clusters observed by *Einstein* IPC
  - "representative" sample
  - emission-measure weighted axial ratios

Intrinsic Distribution of Axial Ratios

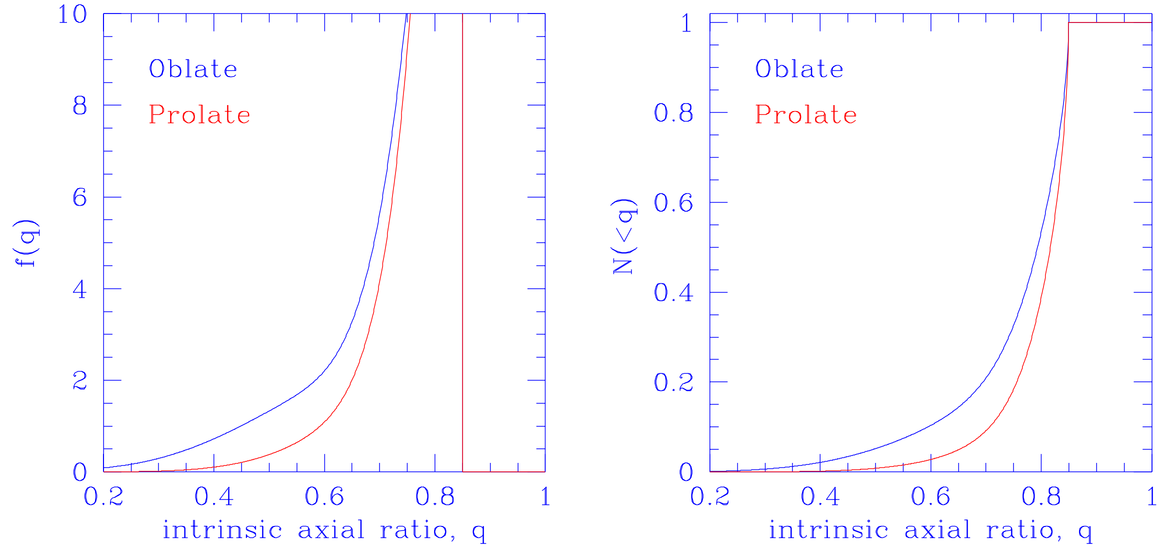
Smoothing length scale $\lambda = 10^{-7}$



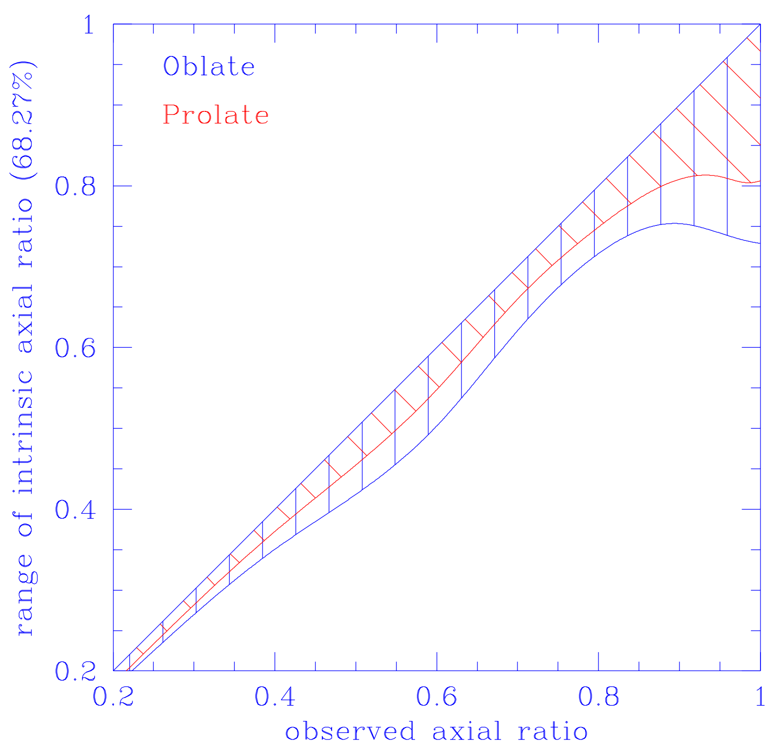
Features to note

- ⋆ Density function for apparent axial ratios (plots on left) peaks around ~0.85
- ⋆ Peak of intrinsic axial ratio density function (middle plots) shifted (by about 0.05) to lower values
- ⋆ Distributions all show a tail to lower values, indicating a (small) population of rather elliptical clusters
- ⋆ Intrinsic axial ratio density function falls off sharply for ratios greater than 0.85, indicating very few round clusters

Mohr et al. sample for Prior for distribution of intrinsic axial ratio



Confidence range on intrinsic axial ratio



Error Range on Axial Ratio and Hubble Constant for CL0016+16

Uniform prior

	68% C.L. range on intr. axial rat.	68% C.L. range on H_0 (km s^{-1} Mpc $^{-1}$)
Oblate	0.85 – 0.55	51 – 31
Prolate	0.85 – 0.73	43 – 50

Mohr et al. sample for prior

	68% C.L. range on intr. axial rat.	68% C.L. range on H_0 (km s^{-1} Mpc $^{-1}$)
Oblate	0.85 – 0.75	51 – 44
Prolate	0.85 – 0.78	43 – 47

Nota Bene: The quoted range in H_0 in the above tables comes only from geometry and inclination effects. See Hughes & Birkinshaw (1998) for the complete error budget on H_0 from CL0016+16.

Summary

- ⋆ Uncertainty in H_0 due to cluster ellipticity can now be quantified under the axisymmetric hypothesis
- ⋆ Use of prior information reduces overall uncertainty significantly
- ⋆ The range of 1-σ allowed intrinsic axial ratio is largest for apparently round clusters, a somewhat surprising result. This is a direct consequence of the deficit of clusters with intrinsic axial ratios near a value of one. Since there are few *intrinsically* round clusters, any *apparently* round ones are almost all due to inclination.

Intrinsic Distribution of Axial Ratios (Part II)

What else can we learn about the intrinsic shapes of galaxy clusters from the distribution of observed shapes?

Once again, consider the distribution of apparent axial ratios for a sample of clusters

$$F(o) = \int G_P(i)\, g(o,i) di$$

Approach

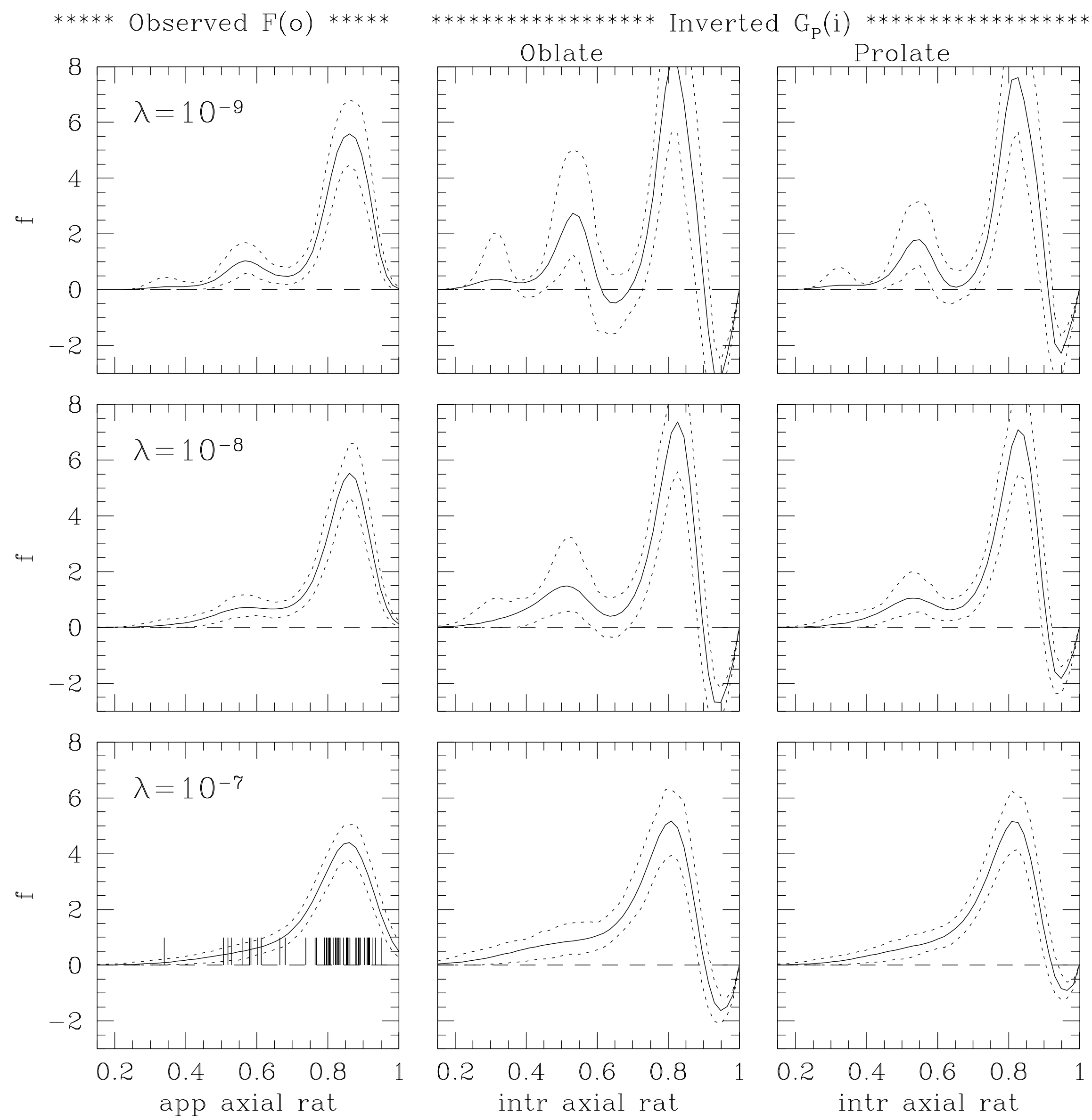
- ⋆ Build MPL estimate of $F(o)$ from cluster sample
- ⋆ Invert above equation, under prolate and oblate assumptions, to obtain estimates of $G_P(i)$.

Difference with before:

- ⋆ Before: $G_P(i) \geq 0$ which guarantees that $F(o) \geq 0$
- ⋆ Now: $F(o) \geq 0$ which does *not* guarantee that $G_P(i) \geq 0$

Effects of varying smoothing length scales:

- ⋆ λ increases from top to bottom
- ⋆ Dashed lines are 90% confidence bands
- ⋆ Data from Mohr et al. (1995)



Results

- ⋆ All $G_P(i)$ estimates for both prolate and oblate assumptions dip below zero near $i = 0$
- ⋆ Highly significant: even the 99% confidence band excludes a positive (or zero) density function near $i = 0$

Axisymmetric hypothesis must be rejected for this sample

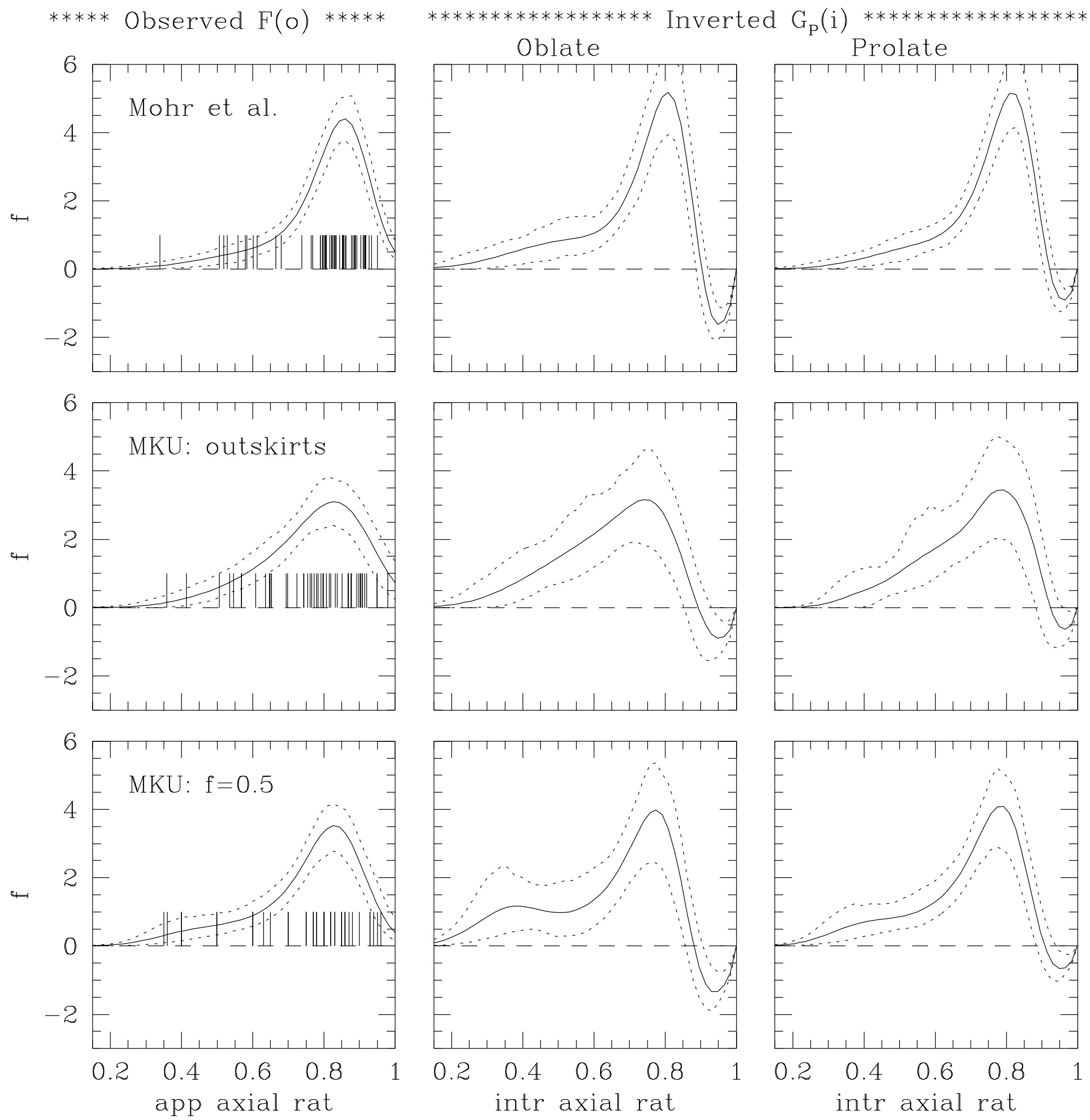
Other cluster samples

McMillan, Kowalski, & Ulmer 1989, ApJS, 70, 723, (hereafter MKU)

- ⋆ 49 Abell clusters observed by the *Einstein* IPC
- ⋆ 32 clusters overlap with Mohr et al. (1995) sample
- ⋆ Axial ratios determined independently using different techniques
- ⋆ Published plots of axial ratio as function of brightness level
- ⋆ Consider 2 samples:
  - Cluster outskirts (i.e., faint brightness levels) (values given in MKU's Table 2)
  - Axial ratios determined at the half brightness level, $f = 0.5$ (values estimated from plots in MKU)

Compare different cluster samples

- ⋆ Smoothing $\lambda = 10^{-7}$ for all; dashed lines are 90% confidence bands
- ⋆ Three different cluster samples as indicated



Results

- ⋆ All $G_P(i)$ estimates dip below zero near $i = 0$
- ⋆ Highly significant: >99% confidence for top and bottom panels; 90%-99% confidence for middle panels (cluster outskirts)

Axisymmetric hypothesis must be rejected for X-ray clusters

- ⋆ X-ray clusters are complex, either triaxial or multiple component
- ⋆ In agreement with numerical simulations (Splinter et al. 1996, astro-ph/9607144; P. Thomas, this conference).