SIMPLE HARMONIC MOTION

PURPOSE: To study the relationships of displacement, velocity, acceleration, kinetic energy, potential energy, amplitude and frequency in simple harmonic motion, and to explore the effects of non-linearity on these relationships.

APPARATUS: Computer, universal lab interface, spring, weights, `Interactive Physics` software

INTRODUCTION: There are many cases in nature where an object oscillates. An oscillation is a swinging or vibrating motion. As the object moves away from its **rest position**, there is a force that opposes the displacement, forcing the object back. This force, called the **restoring force**, tries to return (restore) the object to the rest position. However, the object overshoots, and the force then acts in the opposite direction pushing the object back. If there is no loss of energy, the oscillation continues indefinitely.
When the restoring force is opposite and directly proportional to the displacement, the oscillating object will exhibit Simple Harmonic Motion, SHM. The object is called a Simple Harmonic Oscillator, SHO.

Oscillating Spring: As an example of a SHO, let's consider a mass, m, attached to a spring which is fixed at one end and free to slide on a frictionless surface. Mathematically, the restoring force exerted by the spring on the mass is given by Hooke's law,

$$F = -kx \tag{1}$$

The constant k, called the **spring constant**, characterizes the stiffness of the spring. A large k means a large force is needed to stretch (or compress) the spring a small distance.
The rest position is at $x = 0$ where the force is zero. The force on the mass reaches a maximum at $x = \pm A$, where the mass momentarily stops and then moves back toward its rest position. A is called the **amplitude** of the oscillation.
By conservation of energy, the total mechanical energy of a SHO is constant,

$$\text{K.E.} + \text{P.E.} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant} = C.$$

When the spring is stretched to its maximum, $x = A$, and the velocity, v, is zero, so the total mechanical energy is

$$C = \frac{1}{2}kA^2$$

For any other position of the spring, x and v are related through

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2. \tag{2}$$

Then, solving this equation for v, the velocity as a function of the displacement, x, is

$$v = \pm A\sqrt{\frac{k}{m}\left(1 - \frac{x^2}{A^2}\right)} \tag{3}$$

The SHO moves back and forth, so its velocity will be either in the plus or minus direction, but the velocity's magnitude depends only on the magnitude of x. Next, we want to derive an expression for how the displacement varies with time, t. Substituting Eqn. (1) and the definition of acceleration, $a = d^2x/dt^2$, into Newton's second law, $F = ma$, we get:

$$m\frac{d^2x}{dt^2} = -kx \tag{4}$$

This is a differential equation that you may not have yet learned how to solve.Fortunately, any way you get a solution is fine. We'll "guess" that the solution is

$$x = A\cos\left(\frac{2\pi t}{T}\right) \tag{5}$$

where T is the **period** of the oscillation, the time it takes for the mass to complete one cycle of oscillation. We check that our "guess" is right by substituting Eqn. (5) into Eqn. (4), and find that it a solution, provided the period is related to m and k by the relation:

$$T = 2\pi\sqrt{\frac{m}{k}} \tag{6}$$

The **frequency** (usually expressed in Hertz) of the SHO's oscillations is

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{7}$$

Substituting Eqn. (6) into Eqn. (5) we get,

$$x(t) = A\cos\left(\frac{2\pi t}{T}\right) = A\cos\left(\sqrt{\frac{k}{m}}t\right) \tag{8}$$

Note that this solution assumes that at $t = 0$, $x = A$; the SHO starts at the positive amplitude. The velocity as a function of time is found by taking a derivative with respect to time, $v = dx/dt$

$$v(t) = A\sqrt{\frac{k}{m}}\sin\left(\sqrt{\frac{k}{m}}t\right) = A\sqrt{\frac{k}{m}}\cos\left(\sqrt{\frac{k}{m}}t - \frac{\pi}{2}\right), \tag{9}$$

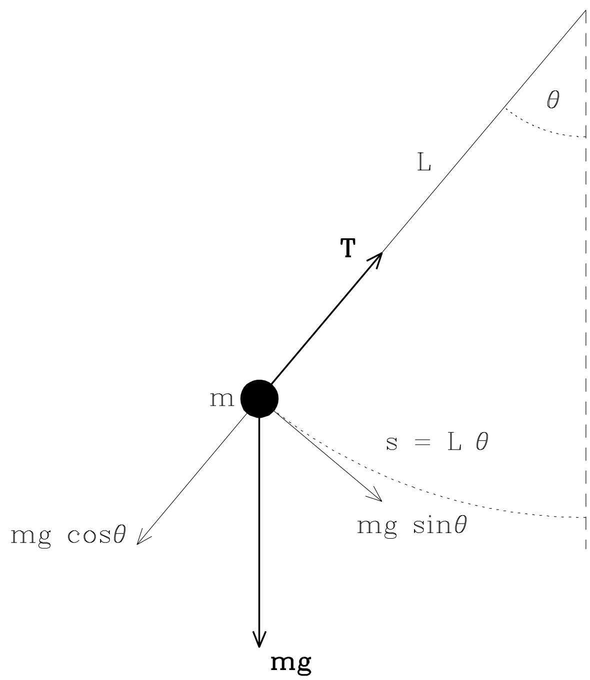

FIGURE 1. Simple Pendulum

where we have used the identity: $\sin\theta = \cos(\theta - \pi/2)$. The acceleration, dv/dt, is

$$a(t) = -A\frac{k}{m}\cos\left(\sqrt{\frac{k}{m}}t\right) = A\frac{k}{m}\cos\left(\sqrt{\frac{k}{m}}t - \pi\right), \tag{10}$$

where we have used the identity: $\cos\theta = -\cos(\theta - \pi)$.

Simple pendulum: As a second example of an oscillating system, let us study the simple pendulum – a mass attached to light string and swinging from side to side, as shown in the accompanying figure 1. The only two forces acting on the mass are the tension T in the string and the weight mg. The tension is partly balanced by the component $mg\cos\theta$ of the weight. The sum of these forces produces the inward (centripetal) acceleration as the ball moves in a circle. The other component of the weight is $F = -mg\sin\theta$, where the minus sign indicates that F is acting opposite to the displacement, $s = L\theta$, where s is the arc length. This force accelerates the mass along the circumference of the circle toward $\theta = 0$ and produces SHM. If we assume that θ is small, then $\sin\theta \approx \theta$ and we can write the restoring force as:

$$F = -mg\theta = -\left(\frac{mg}{L}\right)s. \tag{11}$$

Comparing this equation to Eqn. (1) shows that we again have Hooke's law with the displacement now being the arc length s instead of x and with $k = mg/L$. Then we can immediately see, using Eqn. (6), that the period of the (simple) pendulum is

$$T = 2\pi\sqrt{\frac{L}{g}} \tag{12}$$

In this lab you will use *Interactive Physics* to simulate the simple pendulum to verify this relationship and then explore how the period changes for large oscillations when θ is no longer small and the restoring force is no longer simply proportional to s. You may also want to explore the behavior of the physical pendulum, where the oscillating object is not a point mass but an extended body. In that case the period becomes:

$$T = 2\pi\sqrt{\frac{I}{mgh}} \tag{13}$$

where I is the moment of inertia of the body about the pivot point and h is the distance from the pivot to its center of mass.

PROCEDURE: The ULI (Universal Lab Interface) should be connected to a sonic ranger that you will place on the floor looking up at a mass hanger. The mass hanger consists of two parts whose total mass must be included in your calculations. Don't forget this. Additional masses are placed on top of the hanger's tilted top side. The mass hanger is attached to a spring that is suspended from a stand. The stand must be securely clamped or screwed into the table top.

First, determine the spring constant, k, using Eqn. (1) and $F = mg$. Do this by measuring the CHANGE in the mass hanger's height for the CHANGE in the added weight and determine k as outlined on the data sheet. Use masses of 50, 100, 150, 200, and 250 g. For this part of the lab, you do not need to include the weight of the holder or spring. Use *LoggerPro* to measure the distance. Set the data rate to 40 Hz.

Now suspend a total mass of 200 g (including hanger) from the spring. Pull the hanger down by a small amount so that it oscillates with a full swing (i.e., twice the amplitude) of only 5 to 8 cm. That's enough to get good data. Make sure it is oscillating vertically and not rocking or swinging sideways.

WARNING: DO NOT DROP WEIGHTS ON THE SONIC RANGER BY CARELESSLY YANKING DOWN AND OVERDRIVING THE SPRING. THIS WILL MAKE THE WEIGHTS FALL OFF THE HANGER.

Collect data for about 10 seconds. `LoggerPro` should give you a graph of x versus time. If the x- and/or y-axis scales are too small to clearly see the SHM, you will want to change the axes setting on the graph to magnify it. Be sure that you can clearly see several whole waves and use this to determine the period of oscillation. You may want to use `LoggerPro`'s **Analyze** feature. *Attach a copy of the graph to your report form.*

Next you will examine the relationship between the displacement, velocity, and acceleration. Under **Display** select **Two Graphs** and set up separate displacement and velocity windows. The scale of the velocity graph may be too large to be useful,

so rescale it in the same way as you did for the displacement graph. Also the horizontal x axis may too large. Change its span from 0 to 3 s for BOTH graphs.
Next, under **Analyze** select **DATA A** and you will get a screen with a large vertical bar that enables you to analyze the graphs. Notice that below the graphs there is a tabular readout of Time, Distance (Displacement), Velocity, and Acceleration. Ignore Force because its meaning is useless; we are not using a force probe. Note these values on the graphs as you move the cursor. You will use this table to analyze the sinusoidal graphs. After you have finished studying the displacement, velocity, and acceleration relationships, you will examine the dependence of the period on the amplitude and mass.
When you have finished the experiment go to **File** and select **Quit**. Select **Don't Save** to get rid of your data so that the disk doesn't get cluttered with old data.