GRAVITATION

Name:______________________________ Section:__________

Partner:______________________________ Date:__________

PURPOSE: To explore the gravitational force and Kepler's Laws of Planetary motion.

INTRODUCTION: Newton's law of Universal Gravitation tells us that the gravitational attraction between two masses, m and M, is of magnitude

$$F = \frac{GmM}{R^2} \tag{1}$$

where $G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$ and R is their radial separation. In this experiment you will use the simulation program `Interactive Physics` to investigate a number of the consequences of this interaction. To make your calculations easier we will invent a universe where gravity is not the weakest of forces, but a very strong one; we will assume $G = 1.0\,\mathrm{N\,m^2\,kg^{-2}}$.

For this simulation you will need to know the potential energy, U, of mass m due to the gravitational attraction of M,

$$U = -\frac{GmM}{R} \tag{2}$$

and the relationship between the speed of m when in a circular orbit about M and the distance R between m and M. This is derived from Newton's second law:

$$F = \frac{GmM}{R^2} = ma = m\frac{v^2}{R}, \tag{3}$$

$$v^2 R = GM. \tag{4}$$

From this equation you can easily relate the kinetic energy to R.

SIMULATIONS: *Impact:* Open the `Interactive Physics` simulation **Escape**, which shows the two masses m and M separated by a distance R. Note the anchor on M which means it is pinned down and will not move during the simulation. Pull down the **Window** menu and click on **Properties**. Record the initial values below. To find G pull down the **World** menu and click on **Gravity**.

m: __________ x: __________ y: __________ G: __________

M: __________ x: __________ y: __________

Calculate (predict) the initial potential energy: __________. (Show your work.) At what value of R is this energy equal to zero?

Is this the potential energy of m or is it the potential energy of the entire system? Why?

Run the simulation, record the K.E. and P.E. and calculate the total energy. [Note that P.E. in the simulations may incorrectly hace units of kg^2/m instead of joules. It does correctly calculate the value.]

TABLE 1. Impact Simulation

Time (s)	K.E. (J)	P.E. (J)	Total E (J)
0			
0.032			
0.114			
0.136			

What can you conclude about the total energy during the fall? What about after the impact?

Next predict the velocity v_y, called the escape velocity, that it is necessary for m to have in order for it to move infinitely far away from M. (Show your work.) $v_y =$ ______________

Click on m and enter the escape velocity plus 1 m/s into the **Properties** box for v_y of Body[3]. Before you run the simulation reduce the size of the image. Pull down the **View** menu, then select **View Size**. Set the window width to 40 m. You may also choose the negative looking glass (negative zoom) from the tool box and click the screen several times to reduce the scale of the image. Use the scroll bars to reposition the reduced image to a convenient location. Run the simulation and record the K.E. and P.E. in the table below and calculate the total energy. What can you conclude about the total energy during the flight?

TABLE 2. Escape Velocity Simulation

Time (s)	K.E. (J)	P.E. (J)	Total E (J)
0			
0.2			
0.5			
1.0			

Circular Orbits: Now predict the velocity v_x you will need to give m in order to put it into a circular orbit around M. Click on m and enter this value for v_x into the **Properties** box for Body[3]. Reset v_y to 0. Adjust the window width to 15 m so that you will be able to see all (or most) of the orbit. Click on m, pull down the **Define** menu and click on velocity and acceleration to add these vectors to your simulation. Under **World** select **Tracking** and choose every 16 frames. Run the simulation.

⋆⋆⋆ Describe how the angle between the velocity and acceleration vectors changes with time and how this is related to the fact that both potential and kinetic energy remain constant. [Answer this in the typewritten report]

You may want to rescale your graphs to make the curves more visible. To do this, pull down the **Window** menu, choose **Appearance**, and click on each graph in turn. To rescale, type in new values for the min or max of the x (time) and y axes in the appearance box.

Kepler's First Law: All planets move in elliptical orbits with the Sun (M) at one focus. Open the simulation **Orbit**. This is almost the same simulation as **Escape** except some default settings are different. First set the initial value of v_x equal to the value you calculated for a circular orbit and run the simulation to confirm your result. Now set v_x to 25 m/s and then 15 m/s (you may want to change the number of frames tracked or time increment). Run the simulation and determine the distances of apocenter (furthest recession) R_a, pericenter (closest approach) R_p (the corresponding terms for motion around the sun are aphelion and perihelion and for motion about the earth, perigee and apogee). Note that R_a and R_p are magnitudes (i.e., always positive).

v_x: __________ R_a: __________ R_p: __________
v_x: 25 R_a: __________ R_p: __________
v_x: 15 R_a: __________ R_p: __________

$$\frac{R_p}{R_a} = \frac{1-e}{1+e}: \quad e = \_\_\_\_\_\_\_\_\_\_, \quad a = \_\_\_\_\_\_\_\_\_\_$$

The equation for an ellipse is given by $\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$. Find A and B in terms of e and a.

⋆⋆⋆ Describe how the angle between the velocity and acceleration vectors changes with time. Explain why and how it differs from the case of a circular orbit and how the change in kinetic and potential energy is related to the angle. Attach plots showing how the velocity and acceleration vectors change throughout the orbit. [Answer this in your typewritten report].

Kepler's Third Law: The square of the period of a planet is proportional to the cube of the semi-major axis of the orbit - $\frac{T^2}{a^3} = \frac{4\pi^2}{GM}$. Run the simulation for various values of v_x and measure the period T, R_a, and R_p. For each orbit calculate T^2/a^3 and compare with the predicted value. For 28 m/s you may want to increase the step size of the calculation to 0.2 s.

TABLE 3. Kepler's Third Law Simulation

v_x (m/s)	T (s)	R_a (m)	R_p (m)	a (m)	T^2/a^3 (s^2/m^3)
15					
20					
25					
28					
				Predicted Value	

Detecting Planets: The gravitational force acts equally strongly on both bodies. So in our previous simulations the motion of M is affected by the pull of m, but we didn't notice this because we anchored M to the computer screen. Open the simulation **Binary**. Notice that the anchor on M has been removed and its mass has been reduced. Calculate the velocity v_x that m needs to order to have a circular orbit, using equation (4). Put this value into the **Properties** chart for m and run the simulation. Explain why M moves to the right across the screen even through v_x was zero for it. Within roughly the past 5 years nearly 100 large planets have been discovered around nearby stars by observing the star's motion under very high resolution conditions.

Make appropriate changes in the initial parameters for m and M so that the drift in M does not appear. Do not change the masses! Print a copy of the simulation for your lab report showing that you successfully stopped the drift.

Three-Body Gravitational Problems

The solution for the two body gravitational problem has a simple analytical solution of one of the conic sections, i.e., the motion is always a circle, ellipse, parabola, or hyperbola. You might think that adding a third body would make the problem just a little more complicated. In fact, it makes it impossible to solve analytically! Even worse, the system becomes chaotic. Chaotic means that the system is so sensitive to the initial conditions that we cannot predict the long term behavior. A very small change in the starting position or velocity will make a very big change later on. The addition of a third body can also give some very interesting effects.

I. First open the file *3-body*. This has a system set up with a heavy mass (50,000 kg - the middle mass), a medium mass (500 kg, the bottom one) and a light mass (1 kg, the top one). This represents a planetary system with a star, large planet, and small planet. Run the simulation. Notice how quickly the orbit of the small planet is perturbed. Although it begins in a nearly circular orbit, it is soon forced into a very eccentric elliptical orbit. The small planet has received a boost from the larger planet. The phenomenon is used to give a space probe much larger velocities than can easily be achieved from a rocket.

II. Let's look at the boost phenomenon a little more carefully. Open file *3-body-boost*. This represents the case of a probe being launched from an orbit close to the sun and using a large planet far from the sun to get a boost. Look carefully to what happens as the probe goes by the planet. Display the acceleration and velocity of body 2 and step through the simulation. Where does the probe get its extra energy? If you change body-1's velocity from -0.5 to 0 and body-3's mass to 1 kg, you can see what the path would have been without the effect of a large planet.

Just such a boost was used by Voyager 2 (from NASA, not Star Trek). Voyager was launched in 1977 and passed Jupiter, Saturn, Uranus, and Neptune (in 1989), using each successive planet to boost itself to the next one. Such a "Grand Tour" (as it was called) is possible only if the planets are properly aligned, something which occurs only once every 175 years. Calculate how long it would take the probe to get to Neptune if it were put in an elliptical orbit with perihelion equal to the Earth's radius and aphelion equal to Neptune's radius, and compare to the 12 years it actually took.

Finally, to see an example of sensitivity to initial conditions, change the probe's initial velocity v_y from -140.57 to -141.57 (leaving v_x unchanged). Compare with the previous track to see how a very small change in the velocity changes the path completely. Watch the simulation for about 750 time units. This gives a very nice example of a triple boost. Show that after the final boost the probe will escape the system. Changing the initial velocity to -138.57 also gives an interesting path.

LAB REPORT

Please hand in the lab next week. Your report should consist of this write-up with the original data and questions answered. You should have a copy of the simulation showing the orbiting planet after you've adjusted the motion of the star to make the system stationary. Explain and show the calculations of how you determined this. In addition, give a typewritten response to the questions on pages 3 and 4 (the ones marked with $\star\star\star$). You may combine the answers into one general explanation if you feel it makes the explanation clearer.