BALLISTIC PENDULUM

INTRODUCTION: In this experiment you will use the principles of conservation of momentum and energy to determine the speed of a horizontally projected ball and use this speed to predict the distance the ball will travel before striking the ground, along with an estimate of the error in your prediction. You will then measure the distance the ball actually travels, determine the error in your measurement, and compare the predicted and experimental ranges to see if they agree. Finally you use a computer simulation to study the effect of air resistance on the trajectory of a horizontally projected ball

EXPERIMENT: *Measuring the Projectile Speed* — Consider a steel ball of mass m that is fired from a spring-loaded gun into a catcher-swing of mass M. The ball has an initial velocity of v. The catcher is initially at rest and is free to swing like a pendulum. After capturing the ball, the catcher + ball have a velocity V. At the moment the ball is captured there is no net external force acting on the catcher + ball system. Thus, its linear momentum is constant, and from the law of conservation of momentum:

$$\begin{aligned} \text{initial momentum} &= \text{final momentum} \\ mv &= (m+M)V \\ \text{or} \qquad v &= \frac{(m+M)}{m}V. \end{aligned} \tag{1}$$

The ball's velocity v can be computed if we can measure V, which should be easier since it is much slower ($M \gg m$). Note that kinetic energy is not conserved in this collision because of the dissipating force of friction which locks the ball into the catcher.

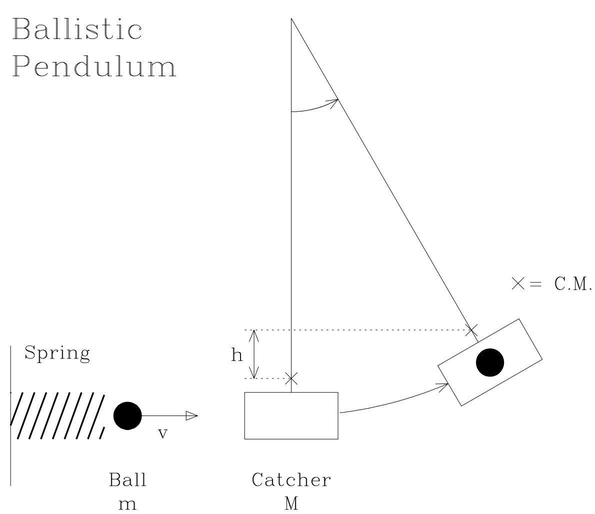

FIGURE 1. Ballistic Pendulum

After the ball is caught in the catcher and the swing-arm starts to move, momentum is no longer conserved because there is a net external force (the force of the swing-arm and gravity are no longer parallel). However, once the catcher-swing starts in motion, conservation of mechanical (kinetic plus potential) energy applies because the force of the swing-arm is always perpendicular to the motion of the catcher. As shown in Figure 1, the catcher with the ball continues to swing upwards until it stops with its center of mass at a vertical distance, h, above the starting level. The kinetic energy of the catcher plus ball, $K = 1/2(m+M)V^2$, has become gravitational potential energy, $(m+M)gh$.

From conservation of energy:

$$\frac{1}{2}(m+M)V^2 = (m+M)gh$$

$$V = \sqrt{2gh} \tag{2}$$

$$v = \frac{(m+M)}{m}\sqrt{2gh} \tag{3}$$

where eqn. (3) comes from combining eqns. (1) and (2).

A ratchet arrangement keeps the pendulum from falling back after it reaches its highest point and h is easily measured. [We assume that no energy is lost by drag of the ratchet latch as the catcher swings upward.] Thus the ball's initial velocity can be computed from easily measured quantities.

Calculation of the Range of the Projectile: We will now move the ballistic pendulum out of the way and fire the ball horizontally from an initial elevation y and see whether the ball hits the floor at the predicted horizontal distance x. See Figure 2.

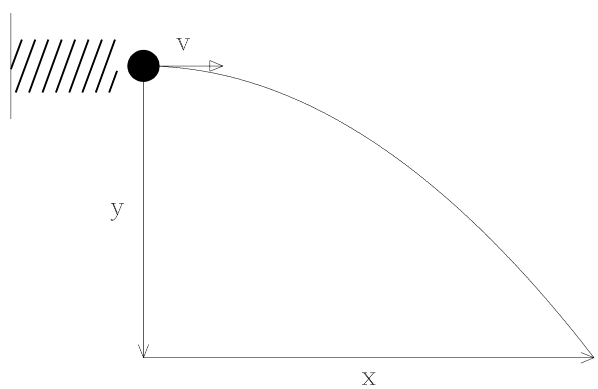

FIGURE 2. Range of Projectile

The horizontal component of the velocity is constant and equal to the initial speed v of the bullet so that

$$x = vt \tag{4}$$

where t is the time of flight of the bullet. Along the vertical, the bullet falls from rest with constant downward acceleration g so that

$$y = \frac{1}{2}\,gt^2. \tag{5}$$

Eliminating t between eqns. (4) and (5) gives

$$x = v\,\sqrt{2y/g} \tag{6}$$

Finally, combining eqns. (3) and (6) gives the location x at which the ball should hit the floor as predicted from a single pendulum measurement,

$$x = \frac{(m+M)}{m}\,\sqrt{2gh}\,\sqrt{2y/g}$$

which reduces to
$$x = \frac{(m+M)}{m}\,2\,\sqrt{hy} \tag{7}$$

Note that g cancels out. [Here is an important lesson: An analysis should becarried to the end using only symbols and mathematics. Don't use numerical quantities until you have to!] In order to make a prediction of the range x, you will measure h a number of times and use the average value, $\overline{h}$, in eqn. (7). You will then measure x a number of times and compare the average, $\overline{x}$, with the value of x from eqn. (7).

UNCERTAINTIES: *You should read and understand sections 2.8 and 2.9 in Baird.* In using eqn. (7) to predict where the projectile hits the floor we must take into account the errors in our measurements of the quantities we plug into the equation — m, M, h, and y. We will ignore the errors in measuring m and M since they are small. To get Δy, the error in y, you will simply estimate how accurate our measurement is — probably within the order ofa mm. To get Δh, the error in h, you will repeat the measurement a number of times and calculate the standard deviation, σ_h. Then $\Delta h = \sigma_h$. But we must now combine Δh and Δy to determine Δx. To do this we use the relation, that if x is a function f of y and h,

$$x = f(y,h)$$

Then

$$\Delta x = \frac{\partial f}{\partial y}\Delta y + \frac{\partial f}{\partial h}\Delta h,$$

where $\frac{\partial f}{\partial y}$ is the partial derivative of f with respect to y. All that is meant by the term partial derivative with respect to y is that you ignore that h is a variable and just take the regular derivative $\frac{df}{dy}$ (and similarly for $\frac{\partial f}{\partial h}$). Then from eqn. (7) we see

$$f(y,h) = \frac{(m+M)}{m}\,2\,\sqrt{hy}$$

and

$$\Delta x = \frac{(m+M)}{m}\left[\sqrt{\frac{h}{y}}\,\Delta y + \sqrt{\frac{y}{h}}\,\Delta h\right].$$

Dividing both sides of this equation by eqn. (7), we find the simple result:

$$\frac{\Delta x}{x} = \frac{\Delta y}{2y} + \frac{\Delta h}{2h}. \tag{8}$$

Thus, we predict the projectile will land at range values of $x + \Delta x$ and $x - \Delta x$, where x is given by eqn. (7) and Δx is given by eqn. (8).

SIMULATION: For the last part of this laboratory you will use a commercial software package, Interactive Physics, to study the trajectory of a horizontally projected ball with and without air friction. One question you will be examining, among others, is "What conservation laws apply during the flight of the ball and how does this change when you include air resistance?". The tutorial (see below) tells you how to use Interactive Physics to produce the simulation

PROCEDURE:

Part A When cocking the spring plunger, push straight back so as not to bend the rod. Careful: there are several catches on the plunger: always use the same catch. Shoot the ball into the catcher and measure the vertical rise h (measure to mark on catcher arm showing center of mass of combined system of catcher + ball). Repeat at least five times. The numbers on the ratchet are only for reference. Calculate $\overline{h}$, the average of your measurements, and the standard deviation, σ_h.

Measure the mass, m, of the ball with a balance to within 0.1 gm. The mass (M) of the catcher, is given to you on the body of apparatus. Aim the mechanism toward the wall which should have a protective sheet to absorb the impact of the bouncing ball. **BE CAREFUL OF YOURSELF AND OTHERS; DON'T PLAY GAMES WITH THE APPARATUS! DO NOT COCK THE GUN UNTIL YOU ARE READY TO FIRE!** Measure with the meter stick the vertical distance y through which the ball will fall while in flight.

Part B Tape a large sheet of paper to the floor to where a test shot landed. In some cases a piece of carbon paper can be placed on the sheet to give a better imprint; often this is unnecessary and will slow you down. You will compare the predicted (theoretical) values with your what you measure in the experiment.

SAFETY WARNING: Treat the spring-loaded gun with great caution. Load or cock the mechanism only just prior to firing. No one should ever be in front of a loaded gun. Warn others when you are ready to make a shot. Watch your fingers!

Tutorial for Interactive Physics Simulation: Horizontally Projected Ball Falling Due to Gravity

For this part of this laboratory you will use a commercial software package, `Interactive Physics`, to study the trajectory of a horizontally projected ball with and without air friction. With `Interactive Physics` it is very easy to develop your own simulation. The tutorial below will show you how to do this. If you have trouble or run out of time, you can alternatively use the simulation already written and stored on your computer with filename: Horizontal ball. Note, however, you will have to modify this simulation to include air resistance.

How to create the `Interactive Physics` simulation:

1. Double click on the `Interactive Physics` (IP) icon, or if IP is already open select *New* under the *File* menu.
2. The first step will be to **add a ball** (**circle**) to your simulation. To do this select the circle from the toolbar on the left-hand side of IP. Move your mouse to the upper left hand side of the page and click on the mouse and holding the button down move the pointer (this is called click and drag) until a circle about 1 cm in diameter appears.
3. The next step will be to **locate the circle at y = 0** so that initial gravitational potential energy is zero. Click again anywhere inside the circle and four small squares will appear surrounding the sphere. This means you have selected the circle and any subsequent action you take will refer to the circle. If you click and drag on the circle you can move it where ever you want on the page. Alternatively, click twice on the circle and a box will pop up that allows to change any of the properties of the sphere. Type 0.0 into the box for the y-coordinate.
4. We now want to **set the initial velocity** to about 5 m/s horizontal. You can again double click on the circle and type 5.0 into the box for Vx. Alternatively click and drag on the center of the circle. A vector labeled V will appear, which you should drag until it is about 3 cm long and horizontal. The other properties are not important, so just leave them at the default values.
5. You can **test your simulation** at any point by clicking on the *run* button, which then changes over to a *stop* sign to allow you to stop the simulation. You will need to click on *reset* and on the circle again before you make any further changes to the simulation.
6. Next we want to *define* which **vectors** we want to show up on the simulation. To do this select the circle, pull down the *Define* menu, choose *Vectors*, and one at a time check *Velocity*, *Acceleration*, and *Gravitational force*. If you now run the simulation these vectors will appear. If the length of any of the vectors is too long or too short, you can change it (without affected the trajectory) by again pulling down the *Define* menu, picking *Vector length*, and moving the slide in the appropriate direction.
7. As things are set up now it's difficult to follow your trajectory. To **"plot" your trajectory**, pull down the *World* menu and choose *Every 16 frames* under

Tracking. Reset, then run your simulation again. Stop the simulation when the circle moves off screen. Notice the slide bar at the bottom of the screen. Use it to run back through your simulation. Notice that you can advance one frame at a time.

8. The next step is to **measure some of the properties of the ball** as it moves along the trajectory. Click on the circle, then pull down the *Measure* menu and select *momentum*, *translational kinetic energy*, and *gravitational potential energy.* This will produce boxes on your screen. Click on each box in turn and select graph. Click and drag on the lower right hand corner of each box to increase the size of the graph. Now repeat these steps to produce three more boxes, but leave them in digital form. Move the boxes around on the screen so they don't interfere with the trajectory (i.e., cover it up).

9. Run your simulation and answer the questions on the data sheet. Caution! Do not run the simulation for more than two (graph) seconds or the graphs will rescale to an inconvenient scale, which will take a little work to undo.

10. Finally **add air resistance** by pulling down the *World* menu and choosing *Air Resistance*. Click on *Standard* and set the value of k to 0.25. This setting assumes the air resistance is proportional to the speed of the object. Add the **vector Air Force** to your simulation.

11. Set the ball mass, radius, height, and initial velocity to the same values you found using the ballistic pendulum. Determine how far the ball goes with and without air resistance. Run your simulation and answer the questions on the data sheet. Explore the effect of changing k. You may want to try *High* setting which assumes the air resistance is proportional to the speed squared.

Simulation of a Horizontally Projected Ball Falling Due to Gravity

All responses should be typed and in complete sentences.

A. Trajectory without air friction

1. Describe how the acceleration changes during the fall and explain why.

2. Which of the quantities that you measure are conserved during the fall? Explain.

B. Trajectory with air friction

1. Attach to your report, a plot of the trajectory of the ball with moderate air resistance.

2. Describe how the acceleration changes during the fall and explain why. [Hint: You may want to add the vector Total Force to your simulation.]

3. Which of the quantities that you measure are conserved during the fall? Explain.

4. What is meant by the term "terminal velocity"? Describe how each of the five quantities you measured vary when the ball is moving with terminal velocity. [You will want to use a larger value for k or the *High* air friction setting in order to reach terminal velocity within 2 seconds.]

5. How much does the distance traveled differ for the cases with and without air resistance? Discuss whether or not air resistance was a significant source of systematic uncertainty in your determination of the distance traveled.