Homework 4, Quantum Mechanics 501, Rutgers

October 18, 2016

1) Consider a harmonic oscillator which is in an initial state $a|n\rangle + b|n+1\rangle$ at $t = 0$, where a, b are real numbers with $a^2 + b^2 = 1$. Calculate the expectation values of $X(t)$ and $P(t)$ as a function of time. Compare your results to the classical motion $x(t)$ of a harmonic oscillator with the same physical parameters (ω, m) and the same (average) energy $E \approx (n+1)\hbar\omega$.

2) A particle of mass m is in a one-dimensional potential of form $V(x) = 1/2m\omega^2x^2 + mgx$ with some real number g. (Think of this as an oscillator potential plus a constant force mg in $-x$ direction acting on the particle).

   Without doing much heavy math, can you write down the lowest energy eigenstate of this potential? (Think about the classical analog a weight hanging on a vertical spring. How does gravity affect the equations and solution for the harmonic spring potential energy?)

   What is the probability that a particle starting out in the ground state of the harmonic oscillator potential only (first part of $V(x)$) ends up in the new ground state once the force is "switched on"?

3) Find the eigenvalues and eigenstates of the one-dimensional Hamiltonian with potential

$$V(x) = \begin{cases} \frac{1}{2}m\omega^2x^2 & x < 0 \\ \infty & x \geq 0 \end{cases} \tag{1}$$

   Nearly no math is needed, only some clever argument.

4) The wavefunction of a particle of mass m is in a 1D potential $V(x)$ is

$$\psi(x) = \begin{cases} Axe^{-ax} & x \geq 0 \\ 0 & x < 0 \end{cases} \tag{2}$$

   a) Assuming the particle is in an eigenstate of the Hamiltonian, find the potential $V(x)$ and the total energy E for this state.
   b) Find the potential energy expectation value $\langle V\rangle$ for this state
   c) Find the expectation value of the kinetic energy for this state.

5 The eigenstates, which are accesible to a single electron, have energies ε_0, ε_1 and ε_2 and their states are $|0\rangle$, $|1\rangle$ and $|2\rangle$. When two electrons are introduced in such system, what are possible wave-functions of the system of two electrons, if we neglect interaction between the two electrons?

a) How many possible states can you write down, which have correct statistics? Write them down.

b) What are the energies of these states?

c) Is the state $|0\rangle \otimes |1\rangle$ a valid wave function of such system? Why (not)?