Preprint typeset in JHEP style - HYPER VERSION

Physics 695: Topics in Physical Mathematics, 2013

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Abstract: September 6, 2013

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1. Prerequisites

This course is primarily intended for advanced undergraduate and graduate students in physics intending to specialize in theory. There will be some bias towards particle theory, although there is much here that is useful to the nuclear and condensed matter theorist. It should also contain much of interest to the mathematics student with some interest in physics.

I will assume you have some knowledge of basic group theory, representation theory, topology and differential geometry. If there is any background I am assuming which you do not know do not hesitate to ask and we will cover it.

2. Tentative Plan

The following is an approximate plan of the topics we will cover.

The basic plan will be to cover several different topics in Physical Mathematics. There is much more material here than can be covered in one semester so some choices need to be made.

I will assume knowledge of basic manifold, cohomology and bundle theory. I am quite happy to go back and review any of this material.

The references are to various notes I have used in the past. Of course, for any topics we do agree upon I will provide more extensive sources.

2.1 Quantum symmetries and phases of gapped quantum systems

2.1.1 Quantum Symmetry

2.1.2 Group Extensions

a little crystallography

2.1.3 Wigner's theorem and quantum symmetry

2.1.4 Corepresentations

2.1.5 Real, complex, and quaternionic vector spaces and representations

2.1.6 Dyson's 3-fold way

Division algebras; Schur's lemma; Dyson's problem; Random matrices

2.1.7 Symmetries and time reversal

2.1.8 Gapped Hamiltonians and topological phases of matter

2.1.9 The 10 CT groups

2.1.10 Clifford algebras and the 10 super division algebras

2.1.11 Spinors and spin representations

2.1.12 Finite dimensional free fermions with symmetry

2.1.13 The Altland-Zirnbauer classification

2.1.14 Bott periodicity and Cartan symmetric spaces

References:

1. Twisted equivariant matter, Daniel S. Freed and Gregory W. Moore, e-Print: arXiv:1208.5055
2. http://www.physics.rutgers.edu/ gmoore/QuantumSymmetryKTheory-Part1.pdf
3. http://www.physics.rutgers.edu/ gmoore/SCGP-TWISTEDMATTER-2013D.pdf

Expected: 6-7 lectures.

2.2 K-theory in string theory and condensed matter physics

2.2.1 Bundle basics, homotopy classification, BG, Serre-Swan theorem

2.2.2 Grothendieck construction and K-theory

2.2.3 D-branes in K-theory

Just a little bit. Really belongs later on.

2.2.4 Virtual representations

2.2.5 Twisted and equivariant

2.2.6 Topological phases, again

References:

1. GMP notes
2. Freed-Moore above.
3. Husemoller et. al. Basic Bundle Theory and K-Cohomological Invariants
4. Notes from course at Aspen 2000.

Expected 3-4 lectures. Try to be brief here.

2.3 Supersymmetric Quantum Mechanics and Morse Theory

Thom class and Thom isomorphism. Poincaré dual class.

TFT integrals: Cohomological Field Theory.

Supersymmetric Quantum mechanics, Witten index, and localization. Simple examples. Hilbert space and Hodge theory. Supersymmetry and index theorems.

Review some Morse theory.

SQM and Morse theory: Morse-Smale-Witten complex

Nicely ties into the discussion of the symmetry classes via Milnor's discussion of Bott periodicity.

Comment on higher dimensions: LG models in 1+1 dimensions with (2,2) supersymmetry. Higher dimensions: Floer theory.

References:

1. Witten 82
2. Les Houches lectures
3. GMP notes

Expected 4-6 lectures.

2.4 Generalized Maxwell theory, Dirac quantization, Chern-Simons, and Self-Dual Theories

2.4.1 Generalized Maxwell Theory and (abelian) Electric-Magnetic Duality

2.4.2 Dirac quantization

2.4.3 Role of differential cohomology

2.4.4 Partition functions

2.4.5 Hilbert space and Pontryagin-Poincare Duality

2.4.6 Electric and magnetic sources

2.4.7 Self-duality

2.4.8 Chern-Simons theory

2.4.9 The abelian gauge field of M-theory

2.4.10 RR fields and differential cohomology

2.4.11 A general theory of self-dual fields

References:

1. Chapter 5, Felix Klein lectures
2. http://www.physics.rutgers.edu/ gmoore/SCGP-Minicourse.pdf

Expected 6-8 lectures

2.5 Conformal Theories

2.5.1 Conformal groups, deSitter, and anti-deSitter geometry

2.5.2 A tiny bit about conformally invariant QFT

References:

1. Group Theory Notes.

Expected 2-3 lectures

2.6 Super Lie algebras

Expected: 1 lecture

2.7 Poincaré and conformal supersymmetry

The Poincaré super algebras.

The superconformal algebras: Nahm's theorem.

Representations of the Poincaré and superconformal algebras.

Say what you can about (2,0) theories. (Felix Klein lectures: Chapters 2 and 6.)

References:

1. Group Theory notes.
2. Felix Klein lectures, chapter 3.

Expected 2-3 lectures.

2.8 Magnetic monopoles and Instantons

2.8.1 Topological sectors

Yang-Mills on a 4-manifold. Yang-Mills-Higgs on $\mathbb{R}^3$.

2.8.2 't Hooft-Polyakov monopole

2.8.3 Monopole moduli space and collective coordinates

2.8.4 ADHM construction

Expected 4-5 lectures

2.9 Donaldson theory and Seiberg-Witten invariants

2.9.1 Four-manifold topology

2.9.2 Twisting N=2 SYM and Donaldson invariants

2.9.3 Low energy Seiberg-Witten solution

2.9.4 Seiberg-Witten equations

2.9.5 Relating the Donaldson and Seiberg-Witten equations

References:

2. Lectures at http://scgp.stonybrook.edu/archives/1964
3. Lecture notes from SCGP March school lecture.

Expected: 6-8 lectures.

2.10 BPS wall-crossing and Hitchin systems

PiTP lectures.

Felix Klein lectures: Chapter 4, Chapters 11-20.

About 10 - 15 lectures

2.11 Automorphic forms in conformal field theory and string theory

Basics of modular forms.

Modular forms and functions in two dimensional CFT.

Automorphic forms and target space string dualities

Black hole counting functions

Rademacher expansion.

Elliptic genus

Borcherds products and theta lifts. K3 sigma model, mock modular forms, Mathieu Moonshine.

References:

1. Trieste lectures 2008. (Home page, talk 17)

Expected: About 6 lectures

3. Administrative

1. Notes for all the lectures will be handed out or posted on the web page for the course.

2. The grade for those taking the course for credit will be based on a short paper and possibly a presentation given at the end of the semester. I will hand out topics towards the middle of the course.

3. I will probably not hand out problem sets to be graded. However, the lecture notes contain plenty of exercises. You are encouraged to do them.

4. As a courtesy to others, PLEASE DO NOT EAT OR DRINK DURING CLASS. You may bring a water bottle.

5. Please note: Attendance at the lectures IS NOT OPTIONAL. If you are a true genius and do not need to hear the lectures to write a brilliant term paper, then you are excused. Please be aware that I will then judge your term paper by those standards.