16.3

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \qquad u = f(x-ct)$$

$$\frac{\partial u}{\partial t} = -c f' \qquad \frac{\partial^2 u}{\partial t^2} = c^2 f''$$

$$\frac{\partial u}{\partial x} = f' \qquad \frac{\partial^2 u}{\partial x^2} = f''$$

$$c^2 f'' = c^2 f'' \quad , \text{ q.e.d.}$$

16.9 $\quad \dfrac{\partial^2 u}{\partial t^2} = c^2 \dfrac{\partial u}{\partial x^2}$

$u(0,t) = u(L,t) = 0.$

$u(x,t) = X(x) \cdot T(t)$

$X \cdot T'' = c^2 X'' T$

$\dfrac{T''}{T} = c^2 \dfrac{X''}{X}$

$\downarrow$ function of t $\qquad$ $\downarrow$ function of x

$\Downarrow$

constant!

$X'' = \dfrac{K}{c^2} X$

if $K > 0$:

$X = A \cdot e^{\frac{\sqrt{K}}{c} \cdot x} + B \cdot e^{-\frac{\sqrt{K}}{c} \cdot x}$

Check boundary conditions:

$X(0) = 0 = A + B \Rightarrow A = -B$

$X(L) = 0 = A \cdot \left(e^{\sqrt{K}/c \cdot L} + e^{-\sqrt{K}/c \cdot L}\right)$

$\hookrightarrow$ no solution for A!

K, therefore is negative.

So: $K < 0$, let's denote $K = -\omega^2$

$$X'' = -\frac{\omega^2}{c^2} X$$

$$X = A \cdot \cos\frac{\omega x}{c} + B \sin\frac{\omega x}{c}$$

Check boundary conditions:

$$X(0) = 0 \Rightarrow A = 0$$

$$X(L) = 0 \Rightarrow B \sin\frac{\omega L}{c} = 0$$

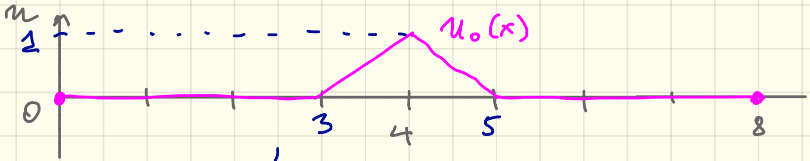
$$\omega = \frac{\pi}{L} \cdot n \leftarrow \text{allowed frequencies.}$$

$$T'' = -\omega^2 T \qquad T = A \cdot \cos(\omega t + \delta)$$

$$y(x,t) = A \sin\left(\frac{\omega}{c} x\right) \cos(\omega t + \delta),$$

where $\omega = \dfrac{\pi \cdot n}{L}$

(16.10)



$$B_n = \frac{2}{L}\int_0^L u_0(x)\sin\frac{n\pi x}{L}\,dx \qquad (L = 8)$$

u_0 is symmetric under reflection about $x=4$

For even $n=2m$, $\sin\frac{2m\pi x}{8}$ is changes sign if reflected about $x=4$.

Therefore $B_{2m} = 0$

if $n = 2m+1$

$$B_n = \frac{2}{8}\int_3^5 u_0(x)\sin\frac{n\pi x}{8} = \frac{4}{8}\int_4^5 u_0(x)\sin\frac{n\pi x}{8}\,dx =$$

$$y = x - 4 \qquad u_0(y) = \begin{cases} 0, & y < -1 \\ 1+y, & -1 < y < 0 \\ 1-y, & 0 < y < 1 \\ 0, & y > 1 \end{cases}$$

$$B_n = \frac{1}{2} \int_0^1 u_0(y) \sin \frac{n\pi(y+4)}{8}\, dy = \ldots$$

odd n means

$n = 2m+1$

$$\frac{n\pi(y+4)}{8} = \frac{n\pi y}{8} + \frac{1}{2} n\pi = \frac{n\pi y}{8} + m\pi + \frac{\pi}{2}$$

$$B_n = \frac{1}{2} \int_0^1 (1-y) \cos \frac{n\pi y}{8} \cdot dy = (-1)^m \frac{32}{n^2\pi^2}\left[1 - \cos\frac{n\pi}{8}\right]$$