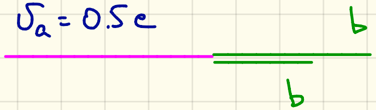
15.74

$v_a = 0.5c$



$a \to b\,b$, $m_a = 2.5\, m_b = \frac{10}{4} m_b$

a) in rest frame of a: (S^*)

$$\left(m_a c, \vec{0}\right) = \left(\frac{\varepsilon_b^*}{c}, \vec{p}_b^{\,*}\right) + \left(\frac{\varepsilon_b^*}{c}, -\vec{p}_b^{\,*}\right)$$

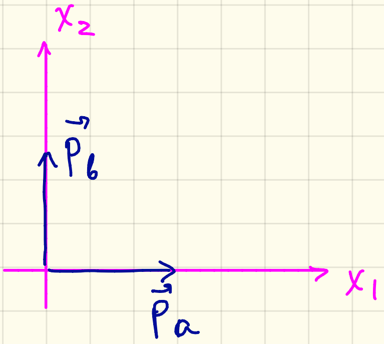
$$\varepsilon_b^* = \frac{1}{2} m_a c^2 = \gamma m_b c^2 \Rightarrow \gamma = \frac{1}{\sqrt{1-\beta^{*2}}} = \frac{m_a}{2 m_b}$$

$$1 - \beta^{*2} = \frac{4 m_b^2}{m_a^2} \qquad \beta^* = \sqrt{1 - \frac{4 m_b^2}{m_a^2}} = \sqrt{1 - \frac{4 \cdot 16}{100}} = \sqrt{0.36} = 0.6$$

b) if after decay both particles b move along x, it means that in S^* the decay is along x, too.

$$v_{b_1} = c\,\frac{0.5 + 0.6}{1 + 0.5 \cdot 0.6} = c\,\frac{1.1}{1.3} = 0.85c \qquad v_{b_2} = c\,\frac{0.5 - 0.6}{1 - 0.5 \cdot 0.6} = c\,\frac{-0.1}{0.7} = -0.14c$$

15.75



a: $m_a = 0.5 \text{ GeV}/c^2$

$\vec{p}_a = \hat{x}_1 \cdot p_a \,, \quad p_a = 2 \text{ GeV}/c$

b: $m_b = 1.0 \text{ GeV}/c^2$

$\vec{p}_b = \hat{x}_2 \, p_b \,, \quad p_b = 1.5 \text{ GeV}/c$

$$\underline{P} = \left(\frac{\mathcal{E}_a + \mathcal{E}_b}{c} \,;\, \vec{p}_a + \vec{p}_b \right)$$

$$\mathcal{E}_a = \sqrt{m_a^2 c^4 + p_a^2 c^2} = \sqrt{0.25 + 4} = \sqrt{4.25} = 2.06$$

$$\mathcal{E}_b = \sqrt{m_b^2 c^4 + p_b^2 c^2} = \sqrt{1 + 2.25} = \sqrt{3.25} = 1.80$$

$$M^2 c^4 = (\mathcal{E}_a + \mathcal{E}_b)^2 - (\vec{p}_a + \vec{p}_b)^2 c^2 = 3.86^2 - 2^2 - 1.5^2 = 14.90 - 4 - 2.25 = 8.65$$

$$\boxed{M = 2.94 \text{ GeV}/c^2}$$

$$\frac{v}{c} = \frac{\sqrt{6.25}}{3.86} = \frac{2.5}{3.86} = 0.65$$

$$\boxed{v = 0.65c}$$

15.76

$$a \xrightarrow{v_a} \qquad b \xrightarrow{v_b}$$

$$a + b \to c$$

$$\left.\begin{matrix} \\ \end{matrix}\right\} \text{given: } m_a, m_b, \quad v_a, v_b$$

$$\underline{P}_a = (\gamma_a m_a c, \gamma_a m_a v_a, 0, 0)$$

$$\underline{P}_b = (\gamma_b m_b c, \gamma_b m_b v_b, 0, 0)$$

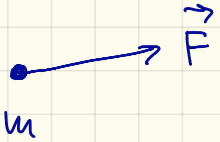
$$m^2 c^2 = \underline{P}_c^2 = (\underline{P}_a + \underline{P}_b)^2 = \underline{P}_a^2 + \underline{P}_b^2 + 2\underline{P}_a \underline{P}_b = m_a^2 c^2 + m_b^2 c^2 + 2(\gamma_a m_a c \gamma_b m_b c - \gamma_a m_a v_a \gamma_b m_a v_b)$$

$$\boxed{m^2 = m_a^2 + m_b^2 + 2 m_a m_b \gamma_a \gamma_b \left(1 - \frac{v_a v_b}{c^2}\right)}$$

$$\frac{v}{c} = \frac{pc}{E} = \frac{\gamma_a m_a v_a + \gamma_b m_b v_b}{\gamma_a m_a c + \gamma_b m_b c}$$

$$\boxed{v = \frac{\gamma_a m_a v_a + \gamma_b m_b v_b}{\gamma_a m_a + \gamma_b m_b}}$$

15.79



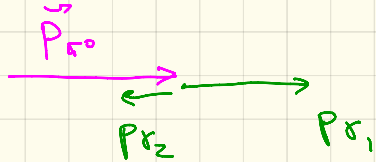
prove that $\vec F = \gamma m \vec a + (\vec F \cdot \vec v) \cdot \vec v / c^2$

$$\vec F = \frac{d\vec p}{dt} = m\frac{d}{dt}\left(\frac{\vec v}{\sqrt{1-v^2/c^2}}\right) = \gamma m \frac{d\vec v}{dt} + m\vec v \cdot \frac{d}{dt}\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right) =$$

$$= \gamma m \vec a + \frac{\vec v}{c^2}\frac{d}{dt}\left(\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}\right) = \gamma m \vec a + \frac{\vec v}{c^2}\cdot\frac{dE}{dt} =$$

$$= \gamma m \vec a + \frac{\vec v}{c^2}\cdot\left(\vec F \cdot \vec v\right) = \vec F$$

15.86



a) in π^0 rest frame: $\left(m_{\pi^0} \cdot c, 0\right) = \left(\frac{\varepsilon_\gamma^*}{c}, \vec{p}_\gamma^{\,*}\right) + \left(\frac{\varepsilon_\gamma^*}{c}, -\vec{p}_\gamma^{\,*}\right)$

$$\varepsilon_\gamma^* = \frac{1}{2} m_{\pi^0} c^2 = 77.5 \text{ MeV}$$

b) $\left(\frac{\varepsilon_{\pi^0}}{c}, p_{\pi^0}\right) = \left(\frac{\varepsilon_\gamma}{c}, p_\gamma\right) + \left(\frac{\varepsilon_\gamma}{3c}, -\frac{p_\gamma}{3}\right)$

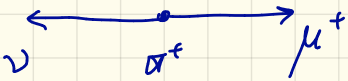
$$m_{\pi^0}^2 c^2 = 2\left(\frac{\varepsilon_\gamma}{c}\frac{\varepsilon_\gamma}{3c} + p_\gamma \frac{p_\gamma}{3}\right) = \frac{\varepsilon_\gamma^2}{c^2}\frac{4}{3}$$

So,

$$\varepsilon_\gamma = m_{\pi^0} c^2 \cdot \frac{\sqrt{3}}{2}$$

$$\varepsilon_{\pi^0} = \frac{4}{3}\varepsilon_\gamma = m_{\pi^0} c^2 \frac{2}{\sqrt{3}} \Rightarrow 1 - \frac{v^2}{c^2} = \frac{3}{4}, \quad \frac{v}{c} = \frac{1}{4}, \quad \underline{v = 0.5c}$$

15.92



$$\underline{P}_\pi = \underline{P}_\mu + \underline{P}_\nu$$

$$\underline{P}_\pi - \underline{P}_\mu = \underline{P}_\nu \Rightarrow m_\pi^2 c^2 + m_\mu^2 c^2 - 2\,\underline{P}_\pi \underline{P}_\mu = m_\nu^2 c^2 = 0$$

$$2 m_\pi c \cdot \frac{\mathcal{E}_\mu}{c} = m_\pi^2 c^2 + m_\mu^2 c^2$$

$$\mathcal{E}_\mu = \frac{m_\pi^2 + m_\mu^2}{2 m_\pi} c^2 = m_\mu c^2 \cdot \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$1 - \frac{v^2}{c^2} = \frac{4 m_\pi^2 m_\mu^2}{(m_\pi^2 + m_\mu^2)^2}$$

$$\beta = \frac{v}{c} = \frac{m_\pi^2 - m_\mu^2}{m_\pi^2 + m_\mu^2}, \quad \text{q.e.d.}$$

$$\beta_\mu = \frac{0.14^2 - 0.106^2}{0.14^2 + 0.106^2} = \frac{8.36}{30.8} = 0.27$$

$$\beta_e = \frac{0.14^2 - 0.0005^2}{0.14^2 + 0.0005^2} \approx 0.99997$$