15.35

q - 4 vector

if one component is 0 in all frames: all are zero.

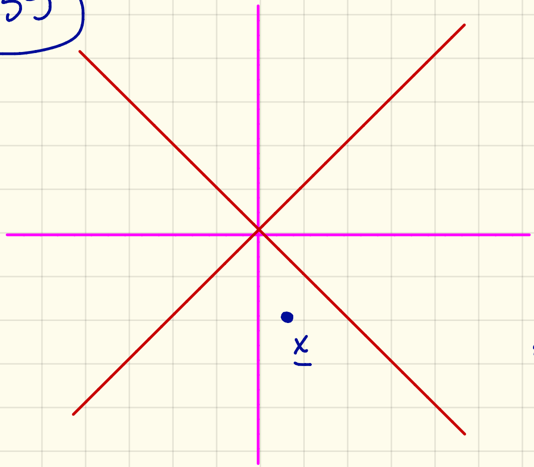
Suppose $q^0 = 0$

in a different frame moving along x $\quad q'^0 = \gamma(q^0 + \beta q^1) \Rightarrow q^1 = 0$

— " — y $\quad q''^0 = \gamma(q^0 + \beta q^2) \Rightarrow q^2 = 0$

etc.

15.35



in system S:

$$x^0 < 0$$

$$x^i x_i = \underline{x}^2 > 0$$

Prove that $x'^0 < 0$ and $\underline{x}^2 > 0$ in every inertial system.

$\underline{x}^2$ is a scalar $\Rightarrow$ same in every frame.

in S $\quad (x^0)^2 > (x^1)^2 + (x^2)^2 + (x^3)^2$

in S' moving along x^1: $\quad x'^0 = \gamma(x^0 + \beta x^1) < 0,$

since $|x^1| < \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2} < |x^0|$

and $|\beta| < 1$

Same for all other possible inertial systems

15.43

a) in S $v/c < 1$: displacement $\underline{d} = (c\,dt, \vec{v}\,dt) \to$ 4-vector

$\underline{d}^2 = c^2 dt^2 - v^2 dt^2$ if $\frac{v}{c} < 1$, $d^2 > 0$ in every system

b) in S $v = c$ Pretty much the same argument as a)

$\underline{d} = (c\,dt, \vec{v}\,dt)$ $\underline{d}^2 = c^2 dt^2 - v^2 dt^2 = c^2 (dt^2 - dt^2) = 0$

$\underline{d} = 0$ in every system

15.48

$$\omega = \frac{\omega_0}{\gamma(1-\beta\cos\theta)} \quad , \quad \beta = 0.2 \Rightarrow \gamma = \frac{1}{\sqrt{1-0.2^2}} \approx$$

$$\approx 1 + \frac{1}{2}\cdot 0.04 = 1.02$$

a) $\cos\theta = 0$:

$$\omega = \frac{\omega_0}{1.02} = 0.98\,\omega_0$$

b) $\cos\theta = 1$

$$\omega = \frac{\omega_0}{1.02\cdot 0.8} = 1.23\,\omega_0$$

15.52 $\underline{P} = \left(\frac{\mathcal{E}}{c}, \vec{p}\right) \to$ 4-vector

Suppose $\vec{p}$ conserves in every inertial frame.
Let's look at a (time-like) interval

$\underline{\Delta P} = \underline{P}_2 - \underline{P}_1 \leftarrow$ i.e. change in 4-momentum between two events.

$\underline{\Delta P} = \left(\frac{\mathcal{E}_2 - \mathcal{E}_1}{c}, \vec{0}\right)$ momentum conserves!

in another system S' (boost along x)

$\Delta P' = \left(\frac{\mathcal{E}_2' - \mathcal{E}_1'}{c}, \vec{0}\right)$ momentum conserves in all inertial systems

Lorentz transform: $(\Delta p')_x = 0 = \gamma\left(\Delta p_x + \beta \frac{\mathcal{E}_2 - \mathcal{E}_1}{c}\right) = \gamma\beta \frac{\mathcal{E}_2 - \mathcal{E}_1}{c}$

therefore $\mathcal{E}_2 = \mathcal{E}_1$ in <u>every inertial system</u>

15.53 Two particles a & b with m_a and m_b

in rest frame of a:

$$\underline{p}_a = (m_a c, \vec{0}) \qquad \underline{p}_b = \left(\frac{E_b}{c}, \vec{p}_a\right)$$

$$\underline{p}_a \cdot \underline{p}_b = m_a c \frac{E_b}{c} - \vec{0} \cdot \vec{p}_a = m_a E_b$$

in rest frame of b:

$$\underline{p}_a = \left(\frac{E_a}{c}, \vec{p}_a\right) \qquad \underline{p}_b = (m_b \cdot c, \vec{0}), \qquad \underline{p}_a \cdot \underline{p}_b = m_b \cdot E_a$$

if speed of a in rest frame of b is v_{rel}, then

$$E_a = \gamma m_a c^2, \text{ so}$$

$$p_a \cdot p_b = m_a \cdot m_b \cdot c^2 \cdot \frac{1}{\sqrt{1 - \frac{v_{rel}^2}{c^2}}}$$

15.56

a) $M_i c^2 + T_i = M_f c^2 + T_f \Rightarrow \Delta M c^2 = -\Delta T = -5\,eV$

$$\Delta M = -5.4 \cdot 10^{-9}\,u$$

b)

$$M_i = M(2H_2 + 1O_2) = 36\,u$$

$$\frac{\Delta M}{M} = -\frac{5.4 \cdot 10^{-9}}{36} \simeq -1.5 \cdot 10^{-10}\,u$$

c) whatever initial mass, the fractional change is the same, so if $M_i = 10\,g$ then

$$\Delta M = -1.5 \cdot 10^{-9}\ \text{gram}$$

↳ tiny!