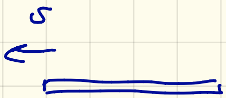
15.11



$l_0 = 1\,m$

$l = 0.8\,m$

$$\sqrt{1 - v^2/c^2} = 0.8$$

$$v^2/c^2 = 0.36$$

$$\boxed{v = 0.6\,c}$$

15.12

a) in the rest frame $\tau = \tau_0 = 1.8 \cdot 10^{-8}$ s

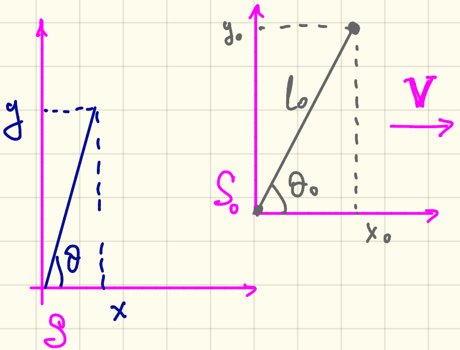
b) in rest frame of π, pipe is moving with 0.8c and has length $d' = d\sqrt{1 - v^2/c^2} = d\sqrt{1 - 0.64} = \underline{\underline{0.6d}}$

time it takes to pass $t = \dfrac{0.6d}{0.8c}$

c) $N = N_0 \cdot 2^{-t/\tau} = \underbrace{N_0 \cdot 2^{-\frac{0.6d}{0.8 \cdot c\tau}}}$ ← same as 5.8 b)

→ in 5.8 lifetime dilates by γ
here, pipe contracts by the same γ

15.13



a) in S_0: l_0, θ_0

$$y_0 = l_0 \cdot \sin\theta_0$$
$$x_0 = l_0 \cdot \cos\theta_0$$

in S:

$$y = y_0 = l_0 \sin\theta_0 = l \sin\theta$$
$$x = x_0 \sqrt{1 - v^2/c^2} = l_0 \sqrt{1 - \frac{v^2}{c^2}} \cdot \cos\theta_0 = l \cos\theta$$

$$\tan\theta = \frac{1}{\sqrt{1 - v^2/c^2}} \tan\theta_0 \qquad l = l_0 \sqrt{\sin^2\theta_0 + \left(1 - \frac{v^2}{c^2}\right)\cos^2\theta_0}$$

for $\theta_0 = 60^\circ$ and $l_0 = 1\,m$ and $V = 0.8c$: $1 - \frac{v^2}{c^2} = 0.36 = 0.6^2$

$\sin\theta_0 = \frac{\sqrt{3}}{2}$ $\cos\theta_0 = \frac{1}{2}$ $l = \sqrt{0.75 + 0.09} = \sqrt{0.84} \approx \underline{\underline{0.916\,m}}$

$\tan\theta_0 = \sqrt{3}$ $\theta = \operatorname{atan}\frac{\sqrt{3}}{0.6} \approx \underline{\underline{71^\circ}}$

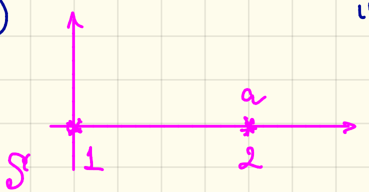
b) in S: θ , in S_0: l_0

$$\left.\begin{aligned} l_0 \sin\theta_0 &= l \sin\theta \\ l_0\sqrt{1-\frac{v^2}{c^2}}\cdot\cos\theta_0 &= l\cos\theta \end{aligned}\right\}$$ still true.

$$\tan\theta_0 = \tan\theta\sqrt{1-\frac{v^2}{c^2}} = \sqrt{3}\cdot 0.6 \quad , \quad \underline{\underline{\theta_0 \approx 46.1^\circ}}$$

$$l = l_0 \cdot \frac{\sin\theta_0}{\sin\theta} = \frac{0.72}{0.87} = \underline{\underline{0.83\ \text{cm}}}$$

15.17



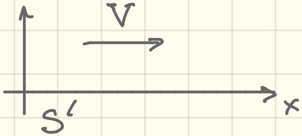
in S

$t_1 = t_2 = t_0$ ← simultaneous.

$x_1 = 0$

$x_2 = a$

a) in S'



$$x_1' = \gamma(x_1 - Vt_1) = 0$$

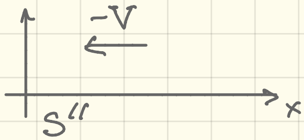
$$t_1' = \gamma\left(t_1 - x_1 V/c^2\right) = 0$$

$$x_2' = \gamma(x_2 - Vt_2) = \gamma a$$

$$t_2' = \gamma\left(t_2 - x_2 V/c^2\right) = -\gamma \frac{aV}{c^2} = -\frac{a}{c}\beta\gamma$$

t_2 is before t_1

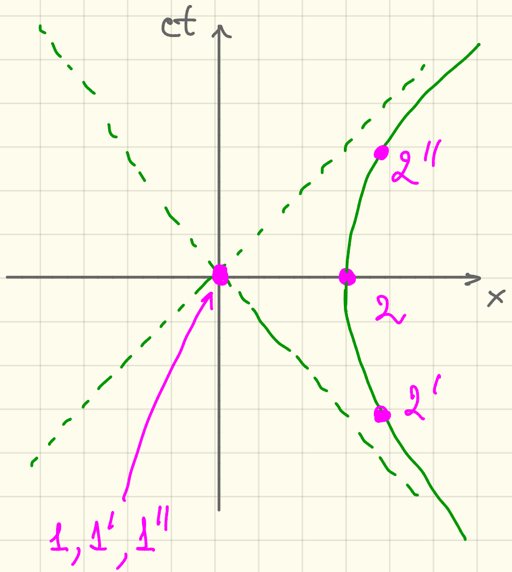
b) in S''



$x_1'' = 0 \qquad t_1'' = 0$

$x_2'' = \gamma a \qquad t_2'' = \frac{a}{c}\beta\gamma$ ← t_1 is before t_2

In both S' and S'' the distance between the events is the same.



Event 1 is at origin in all systems

Interval

$$\Delta S^2 = -a^2 \quad \leftarrow \text{space-like}$$

$$\Delta S'^2 = \Delta S''^2 = \frac{a^2}{c^2}\beta^2\gamma^2 \cdot c^2 - \gamma^2 a^2 =$$

$$= \gamma^2 a^2 \left(\frac{v^2}{c^2} - 1\right) = -a^2$$

When moving into a different system event 2 moves along a trajectory:

$c^2t^2 - x^2 = -a^2 \rightarrow$ a hyperbola with $ct = \pm x$ asimptotes.

15.23

$\vec{v}_1 = 0.9c$ $\qquad\qquad$ $\vec{v}_2 = -0.9c$

Speed of rocket 2 from 1:

$$V = \frac{0.9c + 0.9c}{1 + \frac{0.9c \cdot 0.9c}{c^2}} = \frac{2 \cdot 0.9c}{1 + 0.81} = \frac{1.8}{1.81}c \simeq 0.994c$$

15.36

$\underline{X} \cdot \underline{X}$ = scalar. if $\underline{X}$ a 4-vector

prove that $\underline{X} \cdot \underline{Y}$ = scalar if $\underline{X}$ and $\underline{Y}$ are 4-vectors

$\underline{Z} = \underline{X} + \underline{Y} \rightarrow$ 4-vector

$$\underset{\text{scalar}}{\underbrace{\underline{Z} \cdot \underline{Z}}} = \underset{\text{scalar}}{\underbrace{\underline{X} \cdot \underline{X}}} + 2\, \underline{X} \cdot \underline{Y} + \underset{\text{scalar}}{\underbrace{\underline{Y} \cdot \underline{Y}}}$$

therefore $\underline{X} \cdot \underline{Y}$ must be a scalar