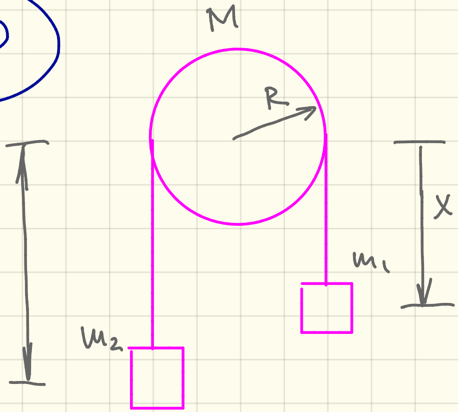
13.3



$$T = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 \dot{x}^2 + \frac{1}{2}\left(\frac{1}{2} M R^2\right)\left(\frac{\dot{x}}{R}\right)^2 =$$

$$T = \dot{x}^2 \left(\frac{1}{2} m_1 + \frac{1}{2} m_2 + \frac{1}{4} M \right)$$

$$U = g(m_2 - m_1) x$$

$$\frac{\partial \mathcal{L}}{\partial \dot{x}} = \dot{x}\left(m_1 + m_2 + \frac{1}{2} M\right) = p$$

$$\dot{x} = \frac{p}{m_1 + m_2 + M/2}$$

$$\mathcal{H} = \frac{1}{2}\left(m_1 + m_2 + \frac{M}{2}\right)\dot{x}^2 + (m_2 - m_1) g x =$$

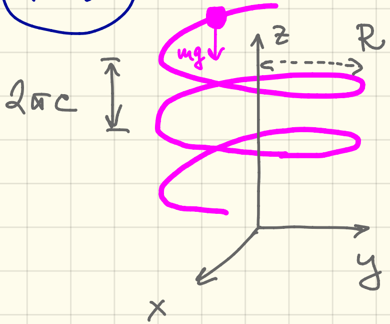
$$\mathcal{H} = \frac{1}{2} \frac{p^2}{m_1 + m_2 + M/2} + (m_2 - m_1) g x$$

$$\frac{\partial \mathcal{H}}{\partial x} = (m_2 - m_1) g = -\left(m_1 + m_2 + \frac{1}{2} M\right) \ddot{x} \quad : \quad \ddot{x} = \frac{m_1 - m_2}{m_1 + m_2 + M/2} \cdot g$$

$$\frac{\partial \mathcal{H}}{\partial p} = \frac{p}{m_1 + m_2 + M/2} = \dot{x} \quad : \quad p = \left(m_1 + m_2 + \frac{M}{2}\right)\dot{x}$$

↑ def. of generalized momentum

13.5



Helix:

$z = c\varphi$
$\rho = R$ $\rightarrow$ one degree of freedom.

Cylindrical coordinates.

Speed of the bead: $v_r = 0$, $v_\varphi = R\dot{\varphi}$,
$v_z = \dot{z} = c\dot{\varphi}$

$$T = \frac{1}{2} m \left(R^2 + c^2\right) \dot{\varphi}^2 \qquad U = mgc \cdot \varphi$$

$$p = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}} = m\left(R^2 + c^2\right)\dot{\varphi} \Rightarrow \dot{\varphi} = \frac{p}{m\left(R^2 + c^2\right)}$$

$$\mathcal{H} = \frac{p^2}{2m\left(R^2 + c^2\right)} + mgc \cdot \varphi$$

$$\frac{\partial \mathcal{H}}{\partial p} = \dot{x} = \frac{p}{m\left(R^2 + c^2\right)} \rightarrow \text{def.}$$

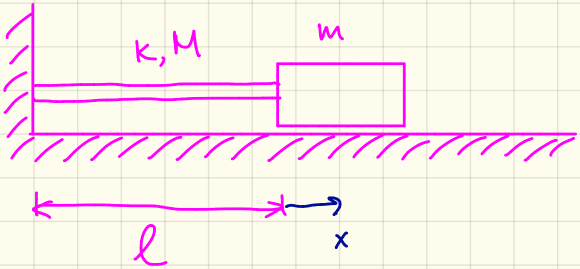
$$\frac{\partial \mathcal{H}}{\partial \varphi} = -\dot{p} = mgc$$

$$-m\left(R^2 + c^2\right)\ddot{\varphi} = mgc$$

$$\begin{cases} \ddot{\varphi} = -\dfrac{c}{R^2 + c^2}\, g \\[2ex] \ddot{z} = -\dfrac{c^2}{R^2 + c^2}\, g \end{cases}$$

if $R = 0$ φ is meaningless and $\ddot{z} = -g$ as expected

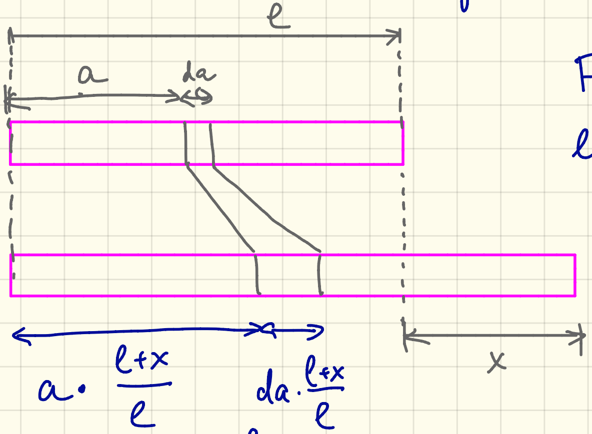
13.6



$$U = \frac{1}{2} k x^2$$

$$T = \frac{1}{2} m \dot{x}^2 + T_{spring}$$

Need to determine speed of all segments of the spring



For da which is a away from the left end of uncompressed spring:

$$y = a \cdot \frac{l+x}{l} = a\left(1 + \frac{x}{l}\right)$$

$$\dot{y} = \frac{a}{l} \dot{x}$$

$$T_{spring} = \int_0^l \frac{1}{2} \frac{M}{l} \cdot da \cdot \left(\frac{a}{l} \dot{x}\right)^2 = \frac{1}{2} \frac{M}{l^3} \cdot \dot{x}^2 \int_0^l a^2 \, da = \frac{1}{6} M \dot{x}^2$$

$$T = \frac{1}{2}\left(m + \frac{M}{3}\right)\dot{x}^2 \qquad U = \frac{1}{2}kx^2$$

$$p = \frac{\partial \mathcal{L}}{\partial \dot{x}} = \left(m + \frac{M}{3}\right)\dot{x} \quad \Rightarrow \quad \mathcal{H} = \frac{p^2}{2\left(m + \frac{M}{3}\right)} + \frac{1}{2}kx^2$$

$$\frac{\partial \mathcal{H}}{\partial x} = kx = -\dot{p} = -\left(m + \frac{M}{3}\right)\ddot{x}$$

$$\ddot{x} = -\frac{k}{m + M/3}\,x$$

$$x = A\cos(\omega t + \delta), \qquad \omega = \sqrt{\frac{k}{m + M/3}}$$

13.8

$$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \dot{y}^2 + \frac{1}{2} m \dot{z}^2 \qquad U = 0$$

$$p_x = \frac{\partial \mathcal{L}}{\partial \dot{x}} = m\dot{x}$$

$$p_y = \frac{\partial \mathcal{L}}{\partial \dot{y}} = m\dot{y} \qquad \mathcal{H} = \sum p_i \dot{q}_i - \mathcal{L} = \frac{p_x^2 + p_y^2 + p_z^2}{2m}$$

$$p_z = \frac{\partial \mathcal{L}}{\partial \dot{z}} = m\dot{z}$$

$$\frac{\partial \mathcal{H}}{\partial p_x} = \dot{x} = \frac{p_x}{m} \qquad \frac{\partial \mathcal{H}}{\partial x} = -\dot{p}_x = 0$$

$$\dot{y} = \frac{p_y}{m} \qquad \dot{p}_y = 0$$

$$\dot{z} = \frac{p_z}{m} \qquad \dot{p}_z = 0$$

$$\left.\begin{matrix} \\ \\ \end{matrix}\right\} \quad \dot{x} = \text{const}, \quad \dot{y} = \text{const}, \quad \dot{z} = \text{const}$$

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t$$

$$\vec{v}_0 = (\dot{x}, \dot{y}, \dot{z})$$

13.10

$$\vec{F} = -kx\,\hat{x} + K\,\hat{y}$$

$$U(x,y) - U(0,0) = \frac{1}{2}kx^2 - Ky$$

$$\mathcal{L} = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{y}^2 - \frac{1}{2}kx^2 + Ky$$

$$p_x = \frac{\partial \mathcal{L}}{\partial \dot{x}} = m\dot{x} \quad ; \quad p_y = \frac{\partial \mathcal{L}}{\partial \dot{y}} = m\dot{y}$$

$$\mathcal{H} = m\dot{x}^2 + m\dot{y}^2 - \frac{1}{2}m\dot{x}^2 - \frac{1}{2}m\dot{y}^2 + \frac{1}{2}kx^2 - Ky$$

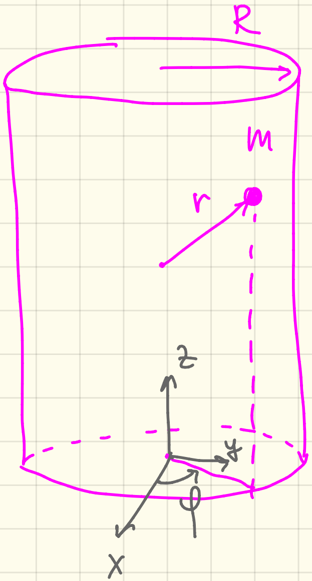
$$\mathcal{H} = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{1}{2}kx^2 - Ky$$

$$\frac{\partial \mathcal{H}}{\partial p_x} = \dot{x} = \frac{p_x}{m} \qquad \frac{\partial \mathcal{H}}{\partial x} = -\dot{p}_x = kx$$

$$\dot{y} = \frac{p_y}{m} \qquad -\dot{p}_y = -K$$

$$\left.\begin{aligned} \ddot{x} &= -\frac{k}{m}x \\ \ddot{y} &= K \end{aligned}\right\} \quad \left\{\begin{aligned} x &= A\cos\left(\sqrt{\tfrac{k}{m}}\,t + \delta\right) \\ y &= y_0 + v_0 t + \tfrac{1}{2}Kt^2 \end{aligned}\right.$$

13.13



$$U = \frac{1}{2}Kr^2 = \frac{1}{2}K\left(R^2 + z^2\right)$$

constant

$$T = \frac{1}{2}m\left((R\dot{\varphi})^2 + \dot{z}^2\right)$$

$$\mathcal{L} = \frac{1}{2}mR^2\dot{\varphi}^2 + \frac{1}{2}m\dot{z}^2 - \frac{1}{2}Kz^2$$

$$p_\varphi = mR^2\dot{\varphi} \qquad p_z = m\dot{z}$$

$$\mathcal{H} = \frac{p_\varphi^2}{2mR^2} + \frac{p_z^2}{2m} + \frac{1}{2}Kz^2$$

$$\frac{\partial \mathcal{H}}{\partial p_\varphi} = \dot{\varphi} = \frac{p_\varphi}{mR^2} \qquad \frac{\partial \mathcal{H}}{\partial \varphi} = 0 = -\dot{p}_\varphi$$

$$\dot{z} = \frac{p_z}{m} \qquad \frac{\partial \mathcal{H}}{\partial z} = Kz = -\dot{p}_z$$

$$\begin{cases} \ddot{\varphi} = 0 & \varphi = \varphi_0 + \omega_0 t \\ \ddot{z} = -\frac{K}{m}z & z = z_0\cos\left(\sqrt{\frac{K}{m}}t + \delta\right) \end{cases}$$