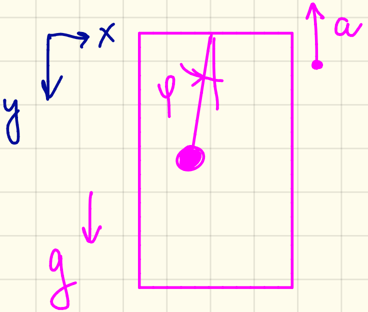
7.22



$$x = l \cdot \sin\varphi$$

$$y = l \cdot \cos\varphi - \frac{1}{2} a t^2$$

$$\dot{x} = l \cdot \cos\varphi \cdot \dot{\varphi}$$

$$\dot{y} = -l \sin\varphi \cdot \dot{\varphi} - at$$

$$U = -mgy = \underline{-mg\left(l\cos\varphi - \frac{1}{2}at^2\right)}$$

$$\dot{x}^2 = l^2 \cos^2\varphi \cdot \dot{\varphi}^2$$

$$\dot{y}^2 = l^2 \sin^2\varphi\, \dot{\varphi}^2 + a^2t^2 + 2lat\sin\varphi \cdot \dot{\varphi}$$

$$\dot{x}^2 + \dot{y}^2 = l^2\dot{\varphi}^2 + a^2t^2 + 2lat\sin\varphi \cdot \dot{\varphi}$$

$$T = \frac{1}{2} m \left(l^2\dot{\varphi}^2 + a^2t^2 + 2lat\sin\varphi\, \dot{\varphi}\right)$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\varphi}} = ml^2\dot{\varphi} + mlat\sin\varphi$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\varphi}}\right) = ml^2\ddot{\varphi} + mla\sin\varphi + {} + mlat\cos\varphi \cdot \dot{\varphi}$$

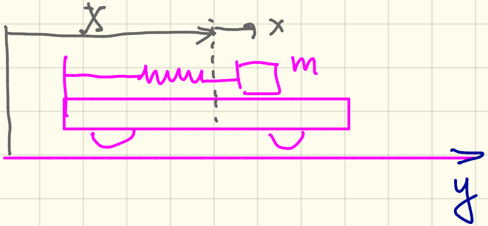
$$\frac{\partial \mathcal{L}}{\partial \varphi} = -mgl\sin\varphi + mlat\dot{\varphi}\cos\varphi$$

$$ml^2\ddot{\varphi} + mla\sin\varphi + \cancel{mlat\cos\varphi\,\dot{\varphi}} = -mgl\sin\varphi + \cancel{mlat\dot{\varphi}\cos\varphi}$$

$$ml^2\ddot{\varphi} = -ml(g+a)\sin\varphi$$

$$\ddot{\varphi} = -\frac{g+a}{l}\sin\varphi$$

7.23



in lab frame (coordinate y)

$$y = X + x = x + A\cos\omega t$$

$$\dot{y} = \dot{X} + \dot{x} = \dot{x} - A\omega\sin\omega t$$

$$U = \frac{1}{2}kx^2$$

$$T = \frac{1}{2}my^2 = \frac{1}{2}m(\dot{x} - A\omega\sin\omega t)^2 = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}A^2\omega^2 m\sin^2\omega t - mA\omega\sin\omega t\cdot\dot{x}$$

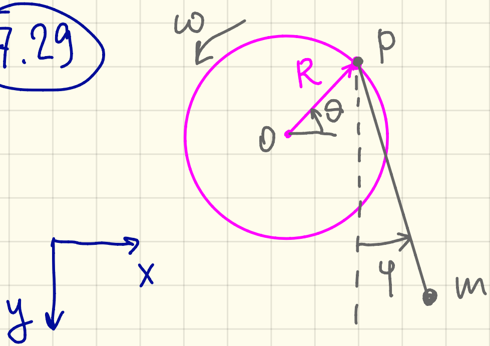
$$\frac{\partial\mathcal{L}}{\partial x} = -kx \qquad \frac{\partial\mathcal{L}}{\partial\dot{x}} = m\dot{x} - mA\omega\cdot\sin\omega t$$

$$-kx = m\ddot{x} - mA\omega^2\cos\omega t$$

$$\ddot{x} + \frac{k}{m}x = mA^2\omega^2\cdot\cos\omega t$$

$$\sqrt{\frac{k}{m}} = \omega_0 \qquad mA^2\omega^2 = \beta \quad \Rightarrow \boxed{\ddot{x} + \omega_0^2 x = B\cdot\cos\omega t}$$

7.29



$$x = R\cdot\cos\theta + l\cdot\sin\varphi \qquad \theta = \omega\cdot t$$

$$y = -R\sin\theta - l\cos\varphi$$

$$\dot{x} = -R\omega\sin\omega t + l\cos\varphi\cdot\dot{\varphi}$$

$$\dot{y} = -R\omega\cos\omega t + l\sin\varphi\,\dot{\varphi}$$

$$\dot{x}^2 = R^2\omega^2\sin^2\omega t + l^2\cos^2\varphi\cdot\dot{\varphi}^2 - 2lR\omega\sin\omega t\cdot\cos\varphi\cdot\dot{\varphi}$$

$$\dot{y}^2 = R^2\omega^2\cos^2\omega t + l^2\sin^2\varphi\cdot\dot{\varphi}^2 - 2lR\omega\cos\omega t\cdot\sin\varphi\cdot\dot{\varphi}$$

$$\dot{x}^2+\dot{y}^2 = R^2\omega^2 + l^2\dot{\varphi}^2 - 2lR\omega\cdot\sin(\varphi+\omega t)\dot{\varphi}$$

$$\mathcal{L} = \frac{1}{2}m\left(R^2\omega^2 + l^2\dot{\varphi}^2 - 2lR\omega\sin(\varphi+\omega t)\cdot\dot{\varphi}\right) + mgR\sin\omega t + mgl\cos\varphi$$

$$\frac{\partial\mathcal{L}}{\partial\varphi} = -mlR\omega\cos(\varphi+\omega t)\cdot\dot{\varphi} - mgl\sin\varphi$$

$$\frac{\partial\mathcal{L}}{\partial\dot{\varphi}} = ml^2\dot{\varphi} - mlR\omega\sin(\varphi+\omega t)$$

$$\frac{\partial \mathcal{L}}{\partial \varphi} = m l R \omega \cos(\varphi+\omega t) - m g l \sin\varphi$$

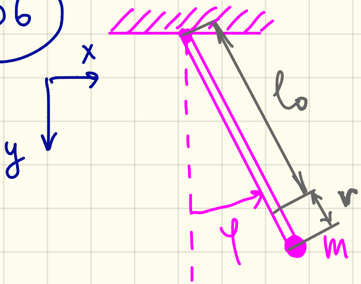
$$\frac{\partial \mathcal{L}}{\partial \dot\varphi} = m l^2 \dot\varphi - m l R \omega \sin(\varphi+\omega t)$$

$$-\cancel{m l R \omega \dot\varphi \cos(\varphi+\omega t)} - m g l \sin\varphi =$$

$$= m l^2 \ddot\varphi - m l R \omega \cos(\varphi+\omega t)(\omega+\cancel{\dot\varphi})$$

$$\boxed{\ddot\varphi = -\frac{l}{g}\sin\varphi \underbrace{- \frac{R}{l}\omega^2 \cdot \cos(\varphi+\omega t)}_{\text{disappears if } \omega = 0}}$$

$\boxed{7.36}$



$$x = (l_0+r)\sin\varphi \qquad \dot{x} = \dot{r}\sin\varphi + (l_0+r)\cos\varphi\cdot\dot{\varphi}$$

$$y = (l_0+r)\cos\varphi \qquad \dot{y} = \dot{r}\cos\varphi - (l_0+r)\sin\varphi\cdot\dot{\varphi}$$

$$\dot{x}^2 = \dot{r}^2\sin^2\varphi + (l_0+r)^2\cos^2\varphi\,\dot{\varphi}^2 + 2(l_0+r)\dot{r}\dot{\varphi}\sin\varphi\cos\varphi$$

$$\dot{y}^2 = \dot{r}^2\cos^2\varphi + (l_0+r)^2\sin^2\varphi\,\dot{\varphi}^2 - 2(l_0+r)\dot{r}\dot{\varphi}\sin\varphi\cos\varphi$$

$$\dot{x}^2+\dot{y}^2 = \dot{r}^2 + (l_0+r)^2\dot{\varphi}^2$$

$$\boxed{\mathcal{L} = \frac{1}{2}m\dot{r}^2 + \frac{m}{2}(l_0+r)^2\dot{\varphi}^2 - \frac{1}{2}kr^2 + mg(l_0+r)\cos\varphi}$$

two Lagrange eqn's:

$\textcircled{r}$ $$\frac{\partial\mathcal{L}}{\partial r} = m(l_0+r)\dot{\varphi}^2 - kr + mg\cos\varphi$$

$$\frac{\partial\mathcal{L}}{\partial\dot{r}} = m\dot{r}$$

$\textcircled{\varphi}$ $$\frac{\partial\mathcal{L}}{\partial\varphi} = -mg(l_0+r)\sin\varphi$$

$$\frac{\partial\mathcal{L}}{\partial\dot{\varphi}} = m(l_0+r)^2\dot{\varphi}$$

$$\boxed{\ddot{r} = -\frac{k}{m}r + (l_0+r)\dot{\varphi}^2 + g\cos\varphi}$$

$$\boxed{\ddot{\varphi} = -\frac{g}{l_0+r}\cdot\sin\varphi}$$

7.36 cont

For small oscillations, $r, \varphi, \dot{r}, \dot{\varphi} << 1$

$$\ddot{r} = -\frac{k}{m} r + \cancel{(l_0+r)\dot{\varphi}^2} + g\cancel{\cos\varphi}$$

$$\ddot{r} = -\frac{k}{m} r + g$$

$$\boxed{r = \frac{mg}{k} + A\cos\left(\sqrt{\frac{k}{m}}\,t + \delta\right)}$$

particular solution

general solution

$$\ddot{\varphi} = -\frac{g}{l_0+r}\sin\varphi = -\frac{g}{l_0}\left(1-\cancel{\frac{r}{l_0}}\right)\cdot\varphi$$

$$\ddot{\varphi} = -\frac{g}{l_0}\varphi$$

$$\boxed{\varphi = A\cos\left(\sqrt{\frac{g}{l_0}}\,t + \delta\right)}$$

Small oscillations occur independently in r and φ