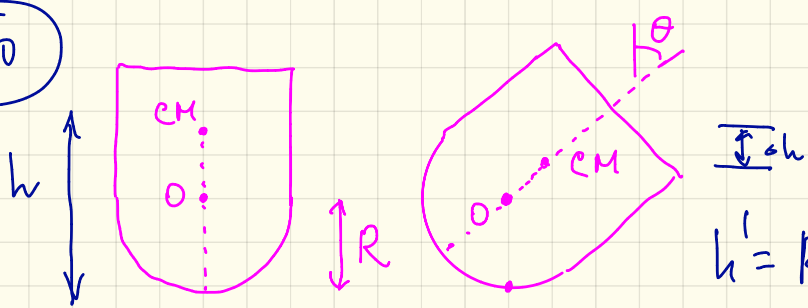
4.30



$$h' = R + (h - R)\cdot\cos\theta =$$

$$h' = h\cdot\cos\theta + R(1-\cos\theta)$$

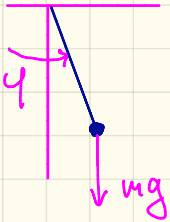
$$\Delta U = mg\,\Delta h = mg\left(h\cos\theta + R(1-\cos\theta) - h\right) =$$

$$= mg\left(-h(1-\cos\theta) + R(1-\cos\theta)\right) = mg(R-h)(1-\cos\theta)$$

if $\Delta U > 0$ its equilibrium.

$$\boxed{\text{so } R > h}$$

$\boxed{4.34}$



a) Equilibrium is @ $\varphi = 0$

$$U(\varphi) = mg(l - l\cos\varphi) = mgl(1-\cos\varphi)$$

Kinetic energy: $K = \frac{1}{2}mv^2 = \frac{1}{2}m\cdot(\dot\varphi l)^2$

$$E_{total} = \frac{1}{2}ml^2\dot\varphi^2 + mgl(1-\cos\varphi)$$

b) $$\frac{dE_{total}}{dt} = ml^2\dot\varphi\ddot\varphi + mgl\sin\varphi\cdot\dot\varphi = 0$$

$$\underbrace{ml^2}_{\text{moment of inertia w.r.t. hinge}}\ddot\varphi = \underbrace{-mgl\sin\varphi}_{\text{torque w.r.t. hinge}}$$

$$I\ddot\varphi = \tau$$

c) for small φ: $\ddot\varphi = -\frac{g}{l}\varphi$, $\varphi = A\cos\left(\sqrt{\frac{g}{l}}t + \varphi_0\right)$

$$\boxed{\text{period is } 2\pi\sqrt{\frac{l}{g}}}$$

4.41

$$U = kr^n \qquad \vec{F} = -\vec{\nabla} U = -\frac{\partial U}{\partial r} - \cancel{\frac{1}{r}\frac{\partial U}{\partial \theta}} - \cancel{\frac{1}{r\sin\varphi}\frac{\partial U}{\partial \varphi}}$$

$$-\frac{mv^2}{r} = -kn \cdot r^{n-1}$$

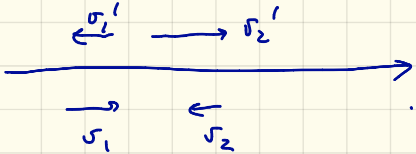
$$\Downarrow$$

$$\frac{1}{2}mv^2 = \frac{1}{2}knr^n = \frac{n}{2}U$$

$$\boxed{T = \frac{n}{2}U}$$

4.47

$$v_2 - v_1 \stackrel{?}{=} -(v_2' - v_1')$$



$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$m_1 (v_1' - v_1) = m_2 (v_2 - v_2')$$ ← momentum

$$m_1 v_1^2 + m_2 v_2^2 = m_1 v_1'^2 + m_2 v_2'^2$$

$$m_1 (v_1'^2 - v_1^2) = m_2 (v_2^2 - v_2'^2)$$

$$m_1 (v_1' - v_1)(v_1' + v_1) = m_2 (v_2 - v_2')(v_2 + v_2')$$ ← energy

$$v_1' + v_1 = v_2 + v_2' \Rightarrow v_2 - v_1 = v_1' - v_2' = -(v_2' - v_1')$$

4.48

$$\underset{m_1, \vec{v}_1}{\circ} \longrightarrow \qquad \underset{m_2}{\circ} \qquad\qquad \underset{m_1+m_2}{\bullet} \xrightarrow{v}$$

$$v = \frac{m_1}{m_1+m_2} v_1 \qquad\qquad E_{in} = \frac{1}{2} m_1 v_1^2$$

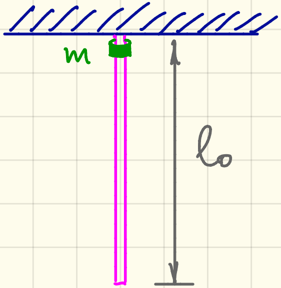
$$E_f = \frac{1}{2}(m_1+m_2) v^2 = \frac{1}{2}(m_1+m_2)\frac{m_1^2}{(m_1+m_2)^2} v_1^2 = \frac{1}{2}\frac{m_1^2}{m_1+m_2} v_1^2$$

$$f = 1 - \frac{E_f}{E_{ini}} = 1 - \frac{\frac{1}{2} m_1 v_1^2 \cdot m_1/(m_1+m_2)}{\frac{1}{2} m_1 v_1^2} = 1 - \frac{m_1}{m_1+m_2} = \frac{m_2}{m_1+m_2}$$

for $m_2 \ll m_1$: $f \simeq 0$ $(v \simeq v_1)$ $\Rightarrow$ m_1 continues unimpeded (car in the rain)

for $m_1 \ll m_2$ $f \simeq 1$ $(v \simeq 0)$ $\Rightarrow$ m_1 stops (putty ball thrown against a wall)

Additional



washer m is dropped from rest

friction between washer & rubber cord is constant F (does not depend on speed)

Hook's constant for the cord is k

a) Δl just before washer slips off the cord

b) v of the washer at that time:

c) what energy Q will dissipate into heat by that time?

a)

$k\Delta l = F$, $\boxed{\Delta l = \frac{F}{k}}$

b)

$E_{in} = 0$

$$E_f = \frac{1}{2}mv^2 - mg(l_0+\Delta l) + F(l_0+\Delta l) = 0 \Rightarrow$$

$$\frac{1}{2}mv^2 = (l_0+\Delta l)\,m\left(g - \frac{F}{m}\right) \Rightarrow \boxed{v = \sqrt{2\left(g-\frac{F}{m}\right)\left(l_0+\frac{F}{k}\right)}}$$

c) the same force F will stretch the cord:

Work done by friction: $F(l_0 + \Delta l) = F\left(l_0 + \frac{F}{k}\right)$

Goes into stretching cord and heat:

$$\frac{1}{2} k \Delta l^2 + Q = F\left(l_0 + \frac{F}{k}\right)$$

$$Q = F l_0 + \frac{F^2}{k} - \frac{1}{2} k \cdot \frac{F^2}{k^2} = F l_0 - \frac{1}{2} \frac{F^2}{k}$$