An analytic model for the spatial clustering of dark matter haloes

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Motivations

- Galaxies form around haloes
  - But haloes are much easier to model
- Reinforce and illuminate simulations
- A model that agrees with observations may
  - Predict dark mass distribution from visible mass
  - Constrain cosmological parameters

Assumptions

- Fluctuations in the initial mass distribution are Gaussian
- Einstein-de Sitter universe
  - $\Omega=1$, $\Lambda=0$
  - “may be readily extended”
- Derived from Press Schechter (PS) model
- Assumes spherically symmetric halo formation

Initial conditions and considerations

Windowing function

overdensity $$\delta(x) \equiv \frac{\rho(x) - \bar{\rho}}{\bar{\rho}}$$

windowed overdensity $$\delta(x;R) = \int W(|x-y|;R)\,\delta(y)\,d^3y$$

RMS mass fluctuations $$\Delta^2(x;R) = \int P(k)\,\hat{W}^2(k;R)\,d^3y$$

probability distribution for δ $$p(\delta;R)\,d\delta = (2\pi)^{-1/2}\exp\left(\frac{\delta^2(R)}{2\Delta^2(R)}\right)\frac{d\delta}{\Delta(R)}$$

- Note: critical dependence on $\nu = \delta/\Delta$

Am I in a halo?

- From PS theory: The probability that a random mass element is part of a halo > M_1 is twice the probability that a surrounding sphere of mass M_1 in the initial conditions has lineary extrapolated overdensity > δ_c

$$F(M_1, z_1) = 2 \int_{(1+z_1)\delta_c}^{\infty} p(\delta; R_1)\, d\delta = erf\, c\left(\frac{\nu_1}{\sqrt{2}}\right)$$

leading to

$$n(M_1, z_1)\, dM_1 = -\sqrt{\frac{2}{\pi}} \frac{\bar{\rho}}{M_1} \nu_1 \frac{d \ln \Delta_1}{d \ln M_1} \exp\left(-\frac{\nu_1}{2}\right) \frac{dM_1}{M_1}$$

Halo number density

- Number of M_1 haloes at z_1 with comoving radius R_0 and overdensity δ_0

$$N(1|0)\,dM_1 = \frac{M_0}{M_1} f(1|0) \frac{d\Delta_1^2}{dM_1}\,dM_1$$

for f(1|0) the fraction of the mass in a region of initial radius R_0, δ_0, z_1 contained in dark haloes of mass M_1

This formalism due to Bond, J. R., Coles S., Efstathiou G., Kaiser N., 1991, ApJ, 379, 440

- Notation: $N(1|0) \equiv N(\Delta_1, \delta_1 \,|\, \Delta_0, \delta_0)$

Spherical Collapse Model

- Concentric shells move as unit
  - Cross only right before collapse through zero
  - Mass interior to a given shell is constant
    - $(1+\delta)R^3 = R_0^3$, with δ the present overdensity
- Gives an expression for $R(R_0, \delta_0, z)$

Bias Function

- Halo overdensity may be derived from previous expressions:
$$\delta_h(1|0) = \frac{N(1|0)}{n(M_1, z_1)V} - 1$$
  - $V = 4\pi R^3/3$, $R_0 = R(1+\delta)^{1/3}$
- Given the above dynamics we may express this in the limit where $R_0 >> R_1$ and $|\delta_0| << \delta_1$ as

$$b(M_1, z_1)\delta \equiv \delta_h(1|0) = \left(1 + \frac{\nu_1^2 - 1}{\delta_1}\right)\delta$$

with $b(M_1,z_1)$ the “bias relation”

And Finally…

- Average (Eulerian) cross-correlation between dark halo number density and mass within sphere

$$\bar{\xi}_{hm}\left(R, M_1, z_1\right) = \left\langle \delta_h\left(1|0\right)\delta \right\rangle_R = \frac{1}{n\left(M_1, z_1\right)V} \times \int_{-1}^{\infty} N\left(1|0\right) p\left(\delta; R\right) \delta \, d\delta$$

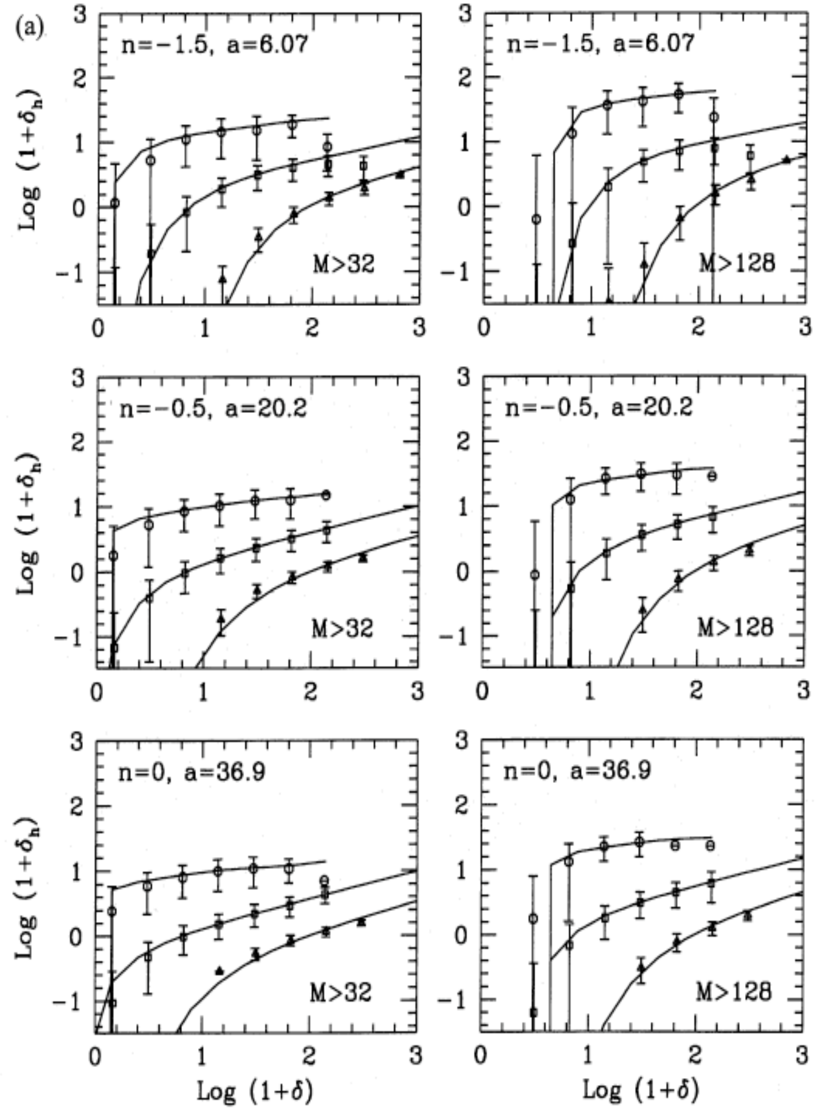
Average over spheres of radius R at z=0

N-Body Simluations

- P^3M method (See Efstathiou G., et al. 1988, MNRAS, 235, 715)
- N = one million bodies
- Periodic, cubic lattice
  - Spatial resolution L/2500 for lattice edge length L
- Time is measured by a expansion factor *a*
  - a=1 at t=0
- Normalized power spectrum $P(k) \propto k^n$
- Haloes identified via “friends-of-friends” algorithm
  - Other algorithms also tested
- Sample random spheres to gather statistics

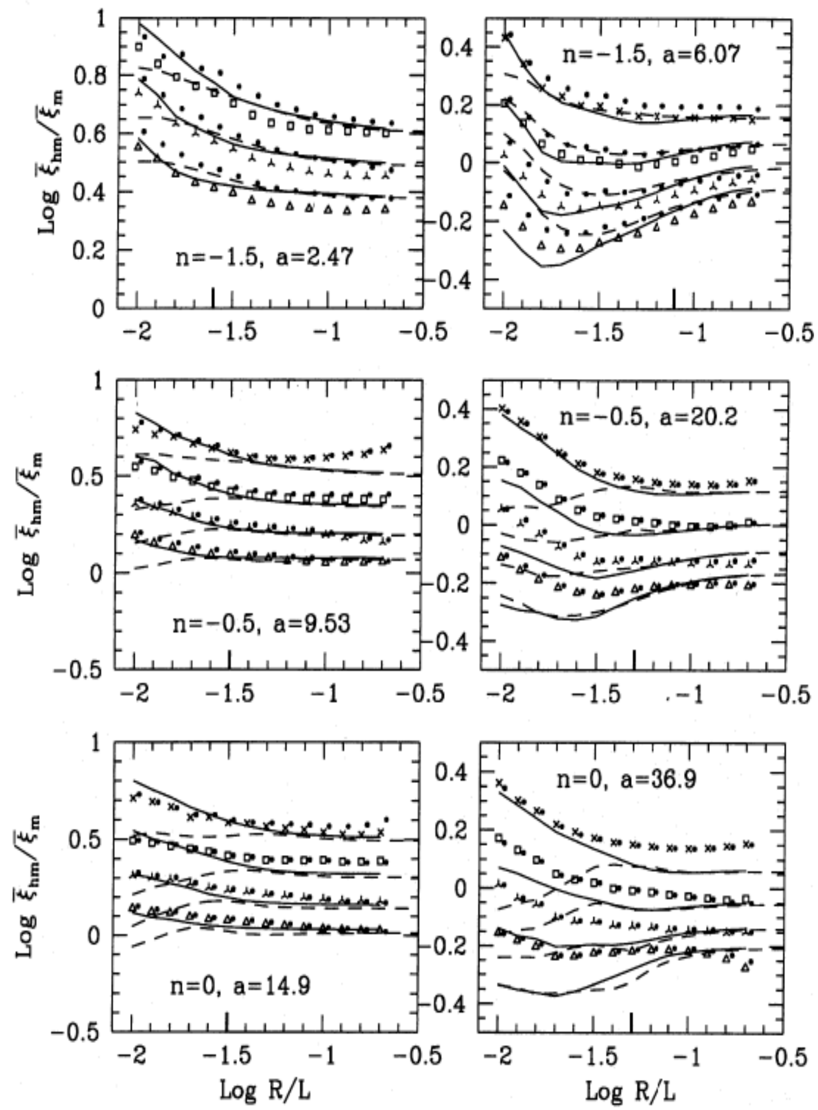
Comparison to Simulation

- Bias relation prediction
  - Overdensity of haloes δ_h versus mass overdensity δ
  - Three curves for spheres of radius R/L = 0.02, 0.05, 0.13
    - Data shifted rightward two decades for presentation
- Very good agreement



Comparison to Simulation

- Mass-Halo correlation
  - Top to bottom, M ≥ (256, 128, 64, 32)
  - Must assume a distribution for δ
    - Solid curves use the observed distribution
    - Dashed curves use lognormal approximation
- Worse agreement for ξ_m>1
  - Such cases bias haloes toward initially underdense regions
    - Suggests failure in spherical collapse model



Results

- Halo bias increases with z
  - Older low-mass haloes can become more strongly clustered than current massive haloes
    - Old clusters may be less massive but still tight
  - Clustering does not in itself predict mass
- Analytic expression for bias factor
  - Given some knowledge of P(k) for initial conditions we can use observed clustering to infer underlying dark mass

$$b\left(M_1, z_1\right)\delta = \delta_h\left(1 \mid 0\right) = \left(1 + \frac{\nu_1^2 - 1}{\delta_1}\right)\delta$$