Hydrodynamic Simulations of Galaxy Formation

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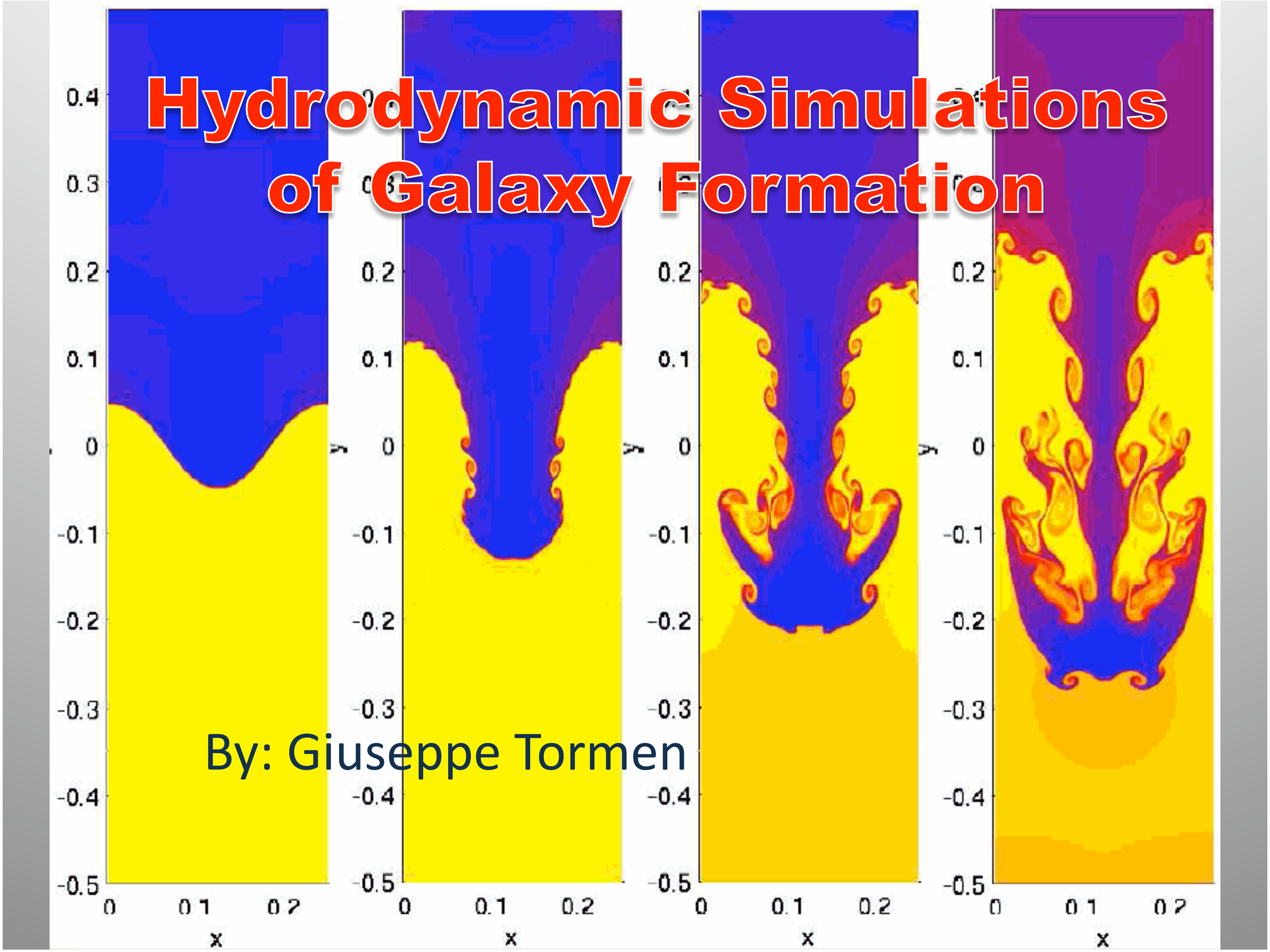


Heating Processes

- Adiabatic Compression

$$dQ = dU + pdV = 0 \Longrightarrow dU = -pdV$$

- Viscous Heating
  - Due to internal friction of the gas.
- Photoionization

$$\gamma + H \rightarrow e^- + H^+$$

$$J(\nu) \propto (\nu/\nu_L)^{-\alpha}$$

Cooling Processes

- Adiabatic Expansion: Opposite of adiabatic Compression- Heat is Converted to work
- Compton Cooling

$$\text{Compton effect:} \quad \gamma + e^- \rightarrow \gamma + e^-$$

- Radiative Cooling

$$dE/dt \equiv \Lambda(\rho, T) = n_e n_i f(T)$$

F(T) is The Cooling Function, whose processes are…

- Collisional ionization

$$e^- + H \rightarrow H^+ + 2e^-$$

- Collisional Excitation and Line Cooling

$$e^- + H \rightarrow e^- + H^* \rightarrow e^- + H + \gamma.$$

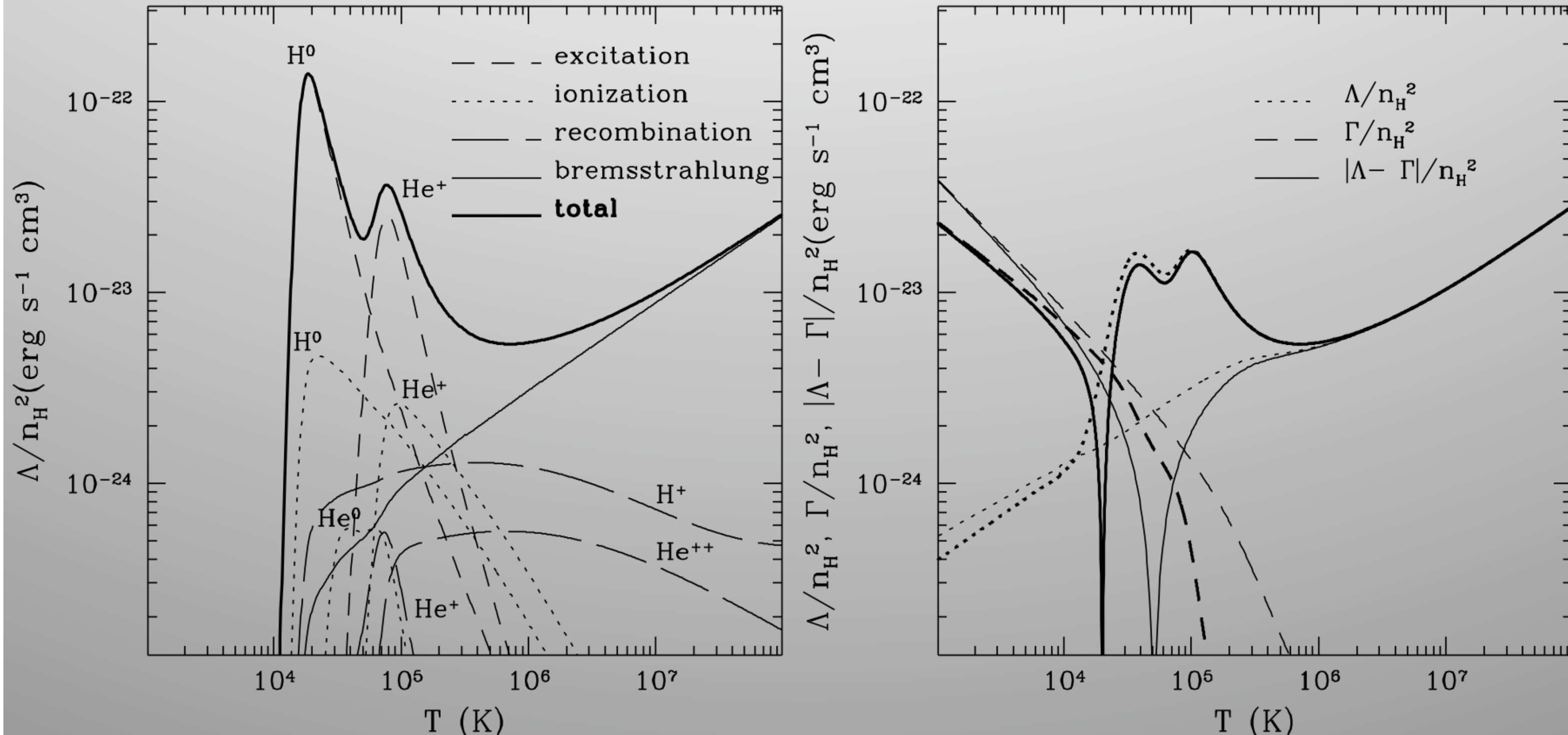
- Recombination

$$e^- + H^+ \rightarrow H + \gamma.$$

- Bremsstrahlung

$$H^+ \rightarrow e^- + H^+ + \gamma$$

Cooling and Heating Functions



Other Processes

- Thermal Conduction: Direct heat transfer across a continuous thermal gradient
- Radiative Transfer: The gas absorbs photons which are thermalized through multiple scatterings and are emitted as black body radiation
- Star Formation: An additional constraint on the heating and metal enrichment of the gas. Currently, star formation is poorly understood

Fluid Equations

- Adiabatic State Equation is used $ds/dt = 0$

$p = p(\rho, T)$. These equations may be written as

$$\frac{d\rho}{dt} = -\rho \vec{\nabla} \cdot \vec{v}, \quad (1)$$

$$\rho \frac{d\vec{v}}{dt} = -\vec{\nabla} p + \text{viscosity terms} - \rho \vec{\nabla} \Phi, \quad (2)$$

$$\rho \frac{d\epsilon}{dt} = -p \vec{\nabla} \cdot \vec{v} + \text{viscosity terms} + \vec{\nabla} \cdot \left(\kappa \vec{\nabla} \mathrm{T} \right) + (\mathrm{Q} - \Lambda), \quad (3)$$

$$p = p(\rho, T). \quad (4)$$

Eulerian Methods

- Densities are put on a grid.

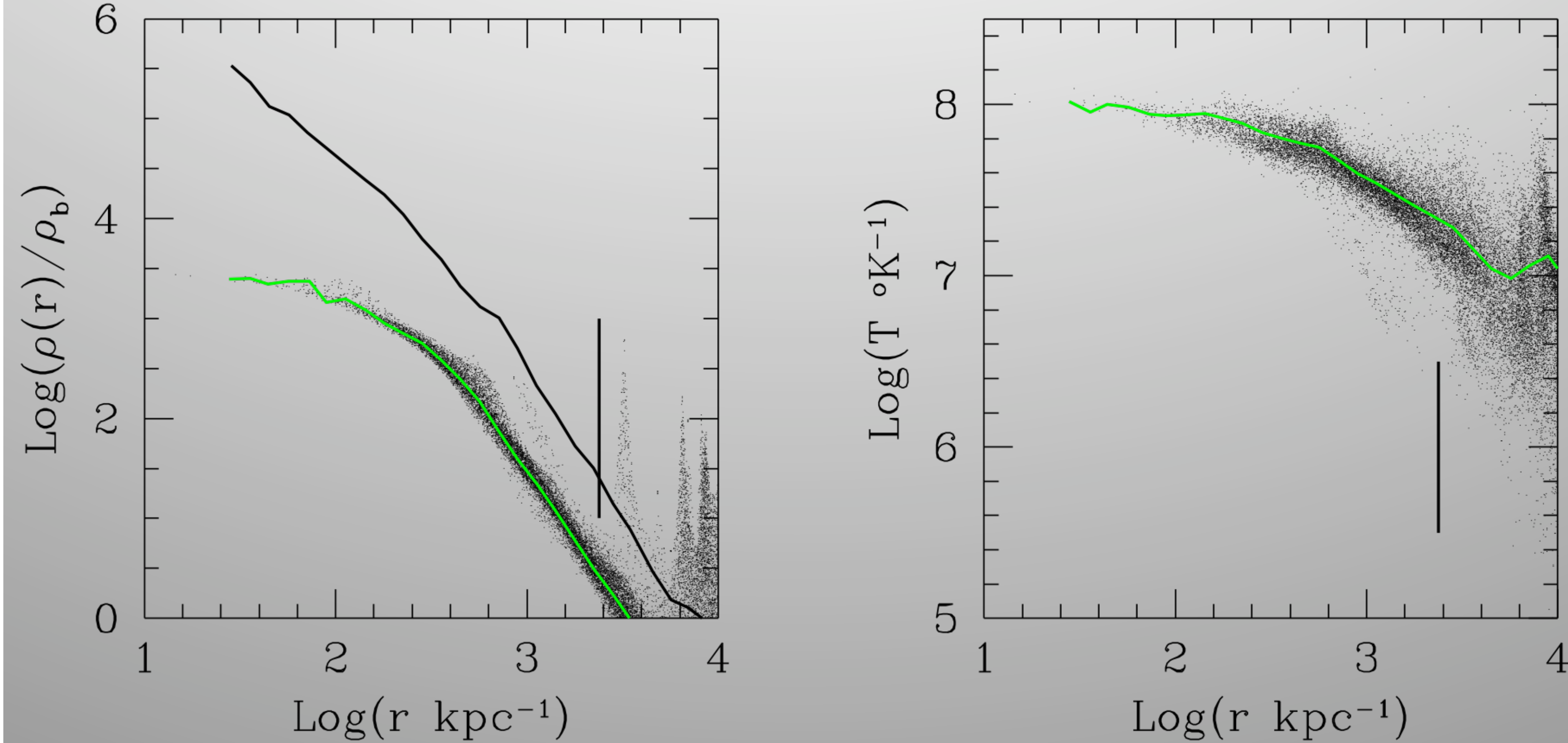
- The Scalar Field is Solved Via Taylor Expansion

- Discontinuities are Modeled Well

Lagrangian Method: SPH

- Smooth Particle Hydrodynamics
- N-Body Simulation with discretized particles
- Solutions are solved with smooth estimates at particles' positions

Simulation of the Formation of a Galaxy Cluster



Conclusions

Lagrangian-SPH

- Pro-Models Cosmological Size Collapse into galaxies better then Eulerian methods and traces out the mass to higher resolution.
- Con-Is not adept to modeling shocks and other contact discontinuities.

Eulerian

- Pro-Models shocks well through the shock tube model for neighboring interactions.
- Con-Computationally limited and does not to well in collapsing cosmological scale gas into galaxies.

Eulerian Methods

- Conservation Equation $$\frac{\partial f}{\partial t} = -\frac{\partial F}{\partial x};$$

- Taylor Expand f(x,t) in time:

$$f(x, t + dt) = f(x, t) + \frac{\partial f}{\partial t} dt + \frac{1}{2}\frac{\partial^2 f}{\partial t^2} dt^2 + O(dt^3)$$

- Inserting the first into the second equation:

$$f(x, t + dt) = f(x, t) - \frac{\partial F}{\partial x} dt + \frac{1}{2}\frac{\partial}{\partial x}\left[\frac{\partial F}{\partial x}\frac{\partial F}{\partial f}\right] dt^2 + O(dt^3).$$

Lagrangian Methods: SPH

- A field is used which is smoothed by using the local averages of the particles Properties

$$\langle f(\vec{r})\rangle = \int d^3u f(\vec{u}) W(\vec{r}-\vec{u};h)$$

- W is the smoothing kernel which is strongly peaked near zero with a 3-D dirac delta function.

$$\lim_{h\to 0}\langle f(\vec{r})\rangle = \int d^3u f(\vec{u})\delta_D(\vec{r}-\vec{u}) = f(\vec{r})$$

In Practice…

- If you evalulate discretely on the particles' positions, f(r) becomes

$$\langle f(\vec{r})\rangle = \sum_{j=1}^{N} \frac{m_j}{\rho_j} f(\vec{r}_j) W(|\vec{r}-\vec{r}_j|;h)$$

- Where the particles Number Density is Defined as follows… $\langle n(\vec{r}_i)\rangle = \rho(\vec{r}_i)/m_i$
- When f is the density equals this reduces to…

$$\langle \rho(\vec{r})\rangle = \sum_{j=1}^{N} m_j W(|\vec{r}-\vec{r}_j|;h)$$

In this Context the Fluid Equations for a perfect Adiabatic Gas, become…

$$\frac{d\vec{v}_i}{dt} = -\sum_{j=1}^{N} m_j \left[\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2}\right] \vec{\nabla}_i W(|\vec{r}_i - \vec{r}_j|; h)$$

$$\frac{d\epsilon_i}{dt} = \frac{p_i}{\rho_i^2} \sum_{j=1}^{N} m_j (\vec{v}_i - \vec{v}_j) \cdot \vec{\nabla}_i W(|\vec{r}_i - \vec{r}_j|; h)$$