Steampunk cosmology

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A complete, self-contained Standard Model cosmology as theory of the physical vacuum

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Steampunk

Maybe 'Steampunk' is too frivolous.

But it's remarkably apt (at least according to my second-hand understanding of the term).

- old-fashioned styles (roughly Victorian era)
- intricate classical machines
- a few modern elements sent back in time

Cosmological Principle

Einstein (1917) started modern cosmology by proposing a cosmological symmetry principle to explain the observed homogeneity and isotropy of the universe (i.e. the fact that the large scale structure of the universe is the same at every location in space and in every direction).

Space is assumed to be a 3-sphere, say the 3-sphere S^3 of radius 1 in euclidean 4-space.

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 = 1$$

The orthogonal rotation group SO(4) of euclidean 4-space acts on the 3-sphere just like the orthogonal rotation group SO(3) of euclidean 3-space acts on the 2-sphere.

This SO(4) symmetry of space is the Cosmological Principle. The universe is assumed to be invariant under the symmetry transformations.

Cosmological Principle (2)

The unit 3-sphere S^3 gets its geometry from euclidean 4-space just like the unit 2-sphere gets its geometry from euclidean 3-space. The line element is

$$(ds)^2_{S^3} = (dx_1)^2 + (dx_2)^2 + (dx_3)^2 + (dx_4)^2$$

$ds =$ length of a tiny displacement (dx_1, dx_2, dx_3, dx_4) on the 3-sphere

The most general SO(4)-invariant geometry on the 3-sphere has to be the same everywhere and in all directions, so it has to be a 3-sphere of radius $1/\epsilon$ in euclidean 4-space for some positive number ϵ.

$$(ds)^2_{S^3_\epsilon} = \frac{1}{\epsilon^2}(ds)^2_{S^3}$$

Cosmological Principle (3)

Friedmann (1922) realized that the most general SO(4)-symmetric space-time metric is time-dependent.

$$(ds)^2 = a(t)^2 \left[-(dt)^2 + \frac{1}{\epsilon^2}(ds)^2_{S^3} \right] \qquad \text{using } c = 1 \text{ units}$$

$a(t)$ has dimension of length $\qquad t, x, \epsilon$ are dimensionless

Einstein's equation of General Relativity

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu} \qquad \kappa = 8\pi G_N$$

relates the space-time geometry on the lhs to the energy-momentum tensor $T_{\mu\nu}(t, x)$ of the matter in the universe.

Cosmological Principle (4)

The symmetric energy-momentum tensor is parametrized by two scalar functions of time,

$$T_{00} = a^2 \rho(t) \qquad T_{0i} = T_{i0} = 0 \qquad T_{ij} = a^2 \hat{g}_{ij}(x) p(t)$$

$\rho(t)$ is the energy density and $p(t)$ is the pressure of the matter in the universe.

Einstein's equation becomes the two Friedmann equations giving the time evolution of $a(t)$ in terms of $\rho(t)$ and $p(t)$.

F1 $$\left(\frac{1}{a}\frac{da}{dt}\right)^2 - \frac{1}{3}\Lambda a^2 = \frac{1}{3}\kappa a^2 \rho$$

F2 $$\frac{d}{dt}\left(\frac{1}{a}\frac{da}{dt}\right) - \frac{1}{3}\Lambda a^2 = -\frac{1}{6}\kappa a^2 \left(\rho + 3p\right)$$

Cosmological Principle (5)

The Cosmological Principle gives

- only a few invariant degrees of freedom (only $a(t)$ so far)
- obeying ordinary differential equations like a classical mechanical system

Cosmology becomes down-to-earth. The sophisticated differential geometry of General Relativity becomes a couple of ordinary differential equations.

The trouble is, we don't have a principle that determines the state of the matter in the universe, that determines the energy-momentum tensor $T_{\mu\nu}$ and thus $\rho(t)$ and $p(t)$.

Astronomers measure $\rho(t)$ and $p(t)$ and make ad hoc models of the matter to fit. The consensus Λ-CDM model starts at cosmological energy around 0.1 MeV with a soup of radiation, protons, neutrons, and hypothetical dark matter.

An extended Cosmological Principle

Proposal:

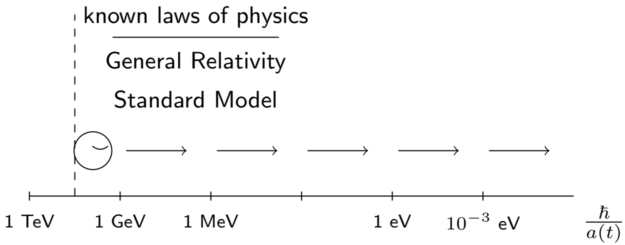
- Assume GR & the Standard Model are the complete laws of physics up to $\sim$200 GeV.
- Extend the SO(4) symmetry of space to act as a symmetry of the SM fields.
- Assume that this extended Cosmological Principle held at an *initial* time when the cosmological energy scale was $\sim$200 GeV.

Then:

- Find the invariant modes of the SM fields (a finite number < 100). These are the cosmological degrees of freedom.
- Derive their equations of motion from the SM — a classical anharmonic oscillator.
- The initial values of the invariant modes are the free parameters of the initial state.
- Derive the energy-momentum tensor $T_{\mu\nu}$ from the invariant modes.
- Solve the oscillator equations of motion and the Friedmann equations (all ode's).

An initial extended Cosmological Principle (2)

All of cosmology is to be calculated (in principle) by integrating the equations of motion of General Relativity and the Standard Model starting from the symmetric initial condition.



$$a(t) = \text{cosmological distance scale (and time scale, since } c = 1)$$

$$\frac{\hbar}{a(t)} = \text{cosmological energy scale}$$

Rationale

The laws of physics become relatively simple at energies approaching 200 GeV. The simplicity derives from symmetry principles. It does not seem unreasonable to suppose that the state of the universe was also relatively simple when the cosmological energy scale was around 200 GeV, also with a simplicity derived from symmetry.

There is no *a priori* justification for the initial symmetry principle. The justification is in the result — a cosmology that has all the basic properties needed for a fundamental theory.

- A universe that is homogeneous, nearly isotropic, nearly flat, and nearly classical.
- There is a big bang, cold dark matter, ordinary matter, baryon number asymmetry, initial thermal fluctuations, and a mechanism for the electroweak phase transition.
- There is an absolute normalization of the scale factor $a(t)$.
- There are only a finite number of free parameters to be fixed by measurements.

Moreover, the cosmology is a theory of the physical vacuum as a function of scale $a(t)$. This is the same physical vacuum that hard scattering particle collider experiments see.

Rationale (2)

The theory is testable, against both observational cosmology and high energy collider experiments, possibly even at the LHC.

I think the theory can be confirmed or dis-confirmed in the not-so-distant future. I think there is a serious chance it will turn out to be right.

There are a huge number of interesting things to think about, computational problems and conceptual questions. They are down-to-earth theoretical physics problems. No sophisticated mathematics required.

Rationale (3)

The theory does not mention any physics beyond the Standard Model. Constructing the initial condition at $\sim$200 GeV turns theoretical cosmology into a definite, concrete computational problem. It gets rid of all the uncertainty that comes from speculating about physics beyond the SM.

Explaining the initial symmetry principle and the values of the initial parameters is left for the future. Ultimately, a more fundamental theory will have to explain General Relativity and the Standard Model along with all the coupling constants, and it will also have to explain the symmetric initial condition along with its free parameters. Nobody has any reliable idea how to do this. This is a problem for the indefinite future. Quite likely it will have to wait until experiments eventually discover something beyond the Standard Model. The time scale for that is completely unknown.

Complete laws of physics?

Does it make sense to take GR&SM to be the complete laws of physics up to 200 GeV?

- High energy experiment has provided an enormous amount of data in the 50 years since the SM was finished. All of the data confirms the SM. There is no evidence of physics beyond the SM.
- Dark matter is the only anomaly. If the cosmology explains dark matter within the SM, then it is coherent to take GR&SM as the complete laws.

  There is no evidence against this. All the experimental searches for dark matter particles beyond the SM have so far been negative.
- The neutrino mixing mechanism is the only unfinished corner in the SM. I'm hoping it doesn't have a big effect on the cosmology. Maybe the cosmology can be used eventually to test neutrino mixing mechanisms. For definiteness, I'm assuming Dirac neutrinos.

Standard Model

$$S_{SM} = \int dt\, d^3x \sqrt{-g} \left(\mathcal{L}_{\text{gauge}} + \mathcal{L}_{\mathsf{quark}} + \mathcal{L}_{\mathsf{lepton}} + \mathcal{L}_{\mathsf{higgs}} \right)$$

$$\frac{1}{\hbar}\mathcal{L}_{\text{gauge}} = \text{tr}_{/V_{\text{gauge}}} \left(\frac{1}{2g^2_{\text{gauge}}} g^{\mu\mu'} g^{\nu\nu'} F_{\mu\nu} F_{\mu'\nu'} \right)$$

$$\frac{1}{\hbar}\mathcal{L}_{\mathsf{quark}} = -\frac{1}{2} \left(\overline{\boldsymbol{q}}\gamma^\mu D_\mu \boldsymbol{q} + \overline{\boldsymbol{u}}\gamma^\mu D_\mu \boldsymbol{u} + \overline{\boldsymbol{d}}\gamma^\mu D_\mu \boldsymbol{d} \right) - \overline{\boldsymbol{q}} \left(\phi Y_{\boldsymbol{u}} \boldsymbol{u} + \phi_C Y_{\boldsymbol{d}} \boldsymbol{d} \right) + \mathsf{h.c.}$$

$$\frac{1}{\hbar}\mathcal{L}_{\mathsf{lepton}} = -\frac{1}{2} \left(\overline{\boldsymbol{l}}\gamma^\mu D_\mu \boldsymbol{l} + \overline{\boldsymbol{e}}\gamma^\mu D_\mu \boldsymbol{e} + \overline{\boldsymbol{\nu}}\gamma^\mu D_\mu \boldsymbol{\nu} \right) - \overline{\boldsymbol{l}} \left(\phi Y_{\boldsymbol{e}} \boldsymbol{e} + \phi_C Y_{\boldsymbol{\nu}} \boldsymbol{\nu} \right) + \mathsf{h.c.}$$

$$\frac{1}{\hbar}\mathcal{L}_{\mathsf{higgs}} = -g^{\mu\nu} D_\mu \phi^\dagger D_\nu \phi - \frac{\lambda^2}{2} \left(\phi^\dagger \phi - \frac{v^2}{2\hbar^2} \right)^2$$

The (relative) simplicity comes from symmetry, from the $\mathrm{U}(1)\times\mathrm{SU}(2)\times\mathrm{SU}(3)$ gauge symmetry.

Standard Model (2)

The higgs field is a doublet of the SU(2) gauge group.

$$\phi(t,x) = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} (t,x)$$

In the vacuum, ϕ is at the minimum of the higgs potential.

$$V_{\text{higgs}}(\phi) = \frac{\lambda^2}{2}\left(\phi^\dagger\phi - \frac{\Lambda_{\text{higgs}}^2}{\hbar^2}\right)^2 \qquad \Lambda_{\text{higgs}} = \sqrt{\frac{v^2}{2}} = 174\,\text{GeV}$$

$$(\phi^\dagger\phi)_{\text{vac}} = \frac{\Lambda_{\text{higgs}}^2}{\hbar^2} \qquad \phi(t,x)_{\text{vac}} = \begin{pmatrix} \Lambda_{\text{higgs}}/\hbar \\ 0 \end{pmatrix}$$

Λ_{higgs} is the only dimensionful coupling constant in the SM, the only scale.

The vacuum higgs field picks out a direction in doublet space, breaking the SU(2) gauge symmetry.

The vacuum is not empty. It contains the vacuum higgs field.

Extended cosmological symmetry principle

I'll state the initial symmetry principle just to show how extremely simple it is.

- SU(2)×SU(2) is the 4-d spinor group, the covering group of SO(4), just like SU(2) is the 2-d spinor group, the covering group of SO(3)
- Identify the SU(2) doublet fields $\phi, \boldsymbol{q}, \boldsymbol{l}$ with the spinors on S^3.
- SU(2)×SU(2) acts on the spinors so now it acts on the SU(2) doublets. It leaves all the other SM fields alone.

This SU(2)×SU(2) symmetry of the SM fields combined with the $\mathrm{SO}(4)$ symmetry of space it the extended Cosmological Principle.

There are no rotationally invariant spinor fields on the 3-sphere so the SU(2)×SU(2) symmetry forces the higgs field to be 0.

The symmetry forces the SU(2) gauge symmetry to be unbroken.

This is an *initial* Cosmological Principle. $\mathrm{SU}(2)\times\mathrm{SU}(2)$ is an *initial* symmetry.

At a certain point in time, the the vacuum higgs field dynamically becomes nonzero, breaking the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ symmetry down to a subgroup.

Cosmological oscillator

First, find the invariant modes of the SM fields (depending on the symmetry group)

$$q^i(t) = \text{the invariant modes of the bosonic fields}$$
$$\psi^I(t) = \text{the invariant modes of the fermionic fields}$$

Then evaluate the SM action on the invariant modes.

$$\frac{1}{\hbar} S = \int dt \int_{S^3_\epsilon} d^3x \sqrt{\hat{g}}\ (\mathcal{L}_{\text{boson}} + \mathcal{L}_{\text{fermion}}) = \mathrm{Vol}(S^3_\epsilon) \int dt\ (\mathcal{L}_{\text{boson}} + \mathcal{L}_{\text{fermion}})$$

schematically (omitting SM coupling constants and $a(t)$ factors):

$$\mathcal{L}_{\text{boson}} = \left(\frac{dq}{dt}\right)^2 - V(q) \qquad V(q) = qq + qqq + qqqq \qquad \mathcal{L}_{\text{fermion}} = \psi^\dagger i \left(\frac{d}{dt} + q\right) \psi$$

The q^i form an anharmonic oscillator (with quartic potential because the SM is quartic).

Cosmological oscillator (2)

- The universe is a classical anharmonic oscillator (plus tiny thermal quantum fluctuations).
- The oscillator is *extremely* classical. It's a huge universe. Each invariant mode is huge.
- The fluctuations are extremely tiny ripples on the classical oscillator. (All the detailed structure of the universe is in the perturbations.)
- Having fermions $\psi^I(t)$ coupled to the bosonic oscillator $q(t)$ is a bit unusual.
- The fermion modes have the quantum numbers of quarks and leptons. The energy in the fermion modes are the ordinary baryonic matter.
- The energy in the oscillating bosonic modes is the dark matter.
- There are two time scales. The oscillator is enormously fast compared to the cosmological time scale.
- The cosmological oscillator is the physical vacuum. The vacuum higgs field is one of the bosonic oscillator modes.
- At the present time, the oscillations are so fast and small that all we see of the physical vacuum is the average vacuum higgs field.
- In high energy experiments, it should be possible to see that the physical vacuum is a more complicated classical dynamical system.

Cosmological oscillator (3)

All the nonlinearity is in the bosonic oscillator (dropping the fermions).

After solving the bosonic oscillator, it's a series of linear ordinary differential equations.

I'll ignore the fermions except at some key points.

My picture is that the fermion modes ride on the scalar oscillator.

Seeing what the bosonic oscillator does is the first task.

The cosmological oscillator is extremely classical

Later we get $\epsilon < 10^{-27}$ from the observational bound on the curvature of the universe.

So $\mathrm{Vol}(S^3_\epsilon) = 2\pi^2/\epsilon^3$ is extremely large. So the cosmological oscillator is *extremely* classical.

Also, $\epsilon < 10^{-27}$ means that space is flat for all local calculations.

The universe is a huge classical object.

The basic problem is to calculate its trajectory — a problem in classical mechanics.

All the global properties of the universe — the vacuum higgs field, the dark and ordinary energy densities, etc. — are encoded in the state of the classical cosmological oscillator.

The fluctuations around the classical solution are insignificant ripples.

It's a top-down theory. First calculate the time evolution of the classical cosmological oscillator, then calculate the time evolution of the fluctuations riding on top of it.

The fluctations condense into all the remarkable variety of phenomena we see.

Two time scales

Use our one dimensionful parameter Λ_{higgs} to make the Friedmann equations dimensionless.

$$\hat{a}(t) = \frac{\Lambda_{\text{higgs}}}{\hbar} a(t) \qquad \hat{\rho}(t) = \frac{\hbar^3}{\Lambda_{\text{higgs}}^4} \rho(t) \qquad \hat{p}(t) = \frac{\hbar^3}{\Lambda_{\text{higgs}}^4} p(t)$$

$$\hat{\kappa} = \frac{\Lambda_{\text{higgs}}^2}{\hbar} \kappa = 5.10 \times 10^{-33} \qquad \hat{\Lambda} = \frac{\hbar^2}{\Lambda_{\text{higgs}}^2} \Lambda = 10^{-88}$$

$$\left(\frac{1}{\hat{a}} \frac{d\hat{a}}{dt} \right)^2 - \frac{1}{3} \hat{\Lambda} \hat{a}^2 = \frac{1}{3} \hat{\kappa} \hat{a}^2 \hat{\rho} \qquad \frac{d}{dt} \left(\frac{1}{\hat{a}} \frac{d\hat{a}}{dt} \right) - \frac{1}{3} \hat{\Lambda} \hat{a}^2 = -\frac{1}{6} \hat{\kappa} \hat{a}^2 \left(\hat{\rho} + 3\hat{p} \right)$$

The cosmological constant term is completely negligible until late time, until large $\hat{a}(t)$.

Absorb the dimensionless gravitation constant $\hat{\kappa}$ into a definition of cosmological time.

$$t_c = \sqrt{\hat{\kappa}}\, t \qquad \sqrt{\hat{\kappa}} = 7.14 \times 10^{-17}$$

Then everything in the Friedmann equations becomes O(1).

The adiabatic oscillator

$$t_c = \sqrt{\hat{\kappa}}\, t \qquad \sqrt{\hat{\kappa}} = 7.14{\times}10^{-17}$$

Oscillator time t is 10^{16} times faster than cosmological time t_c.

In each moment of cosmological time, the oscillator has plenty of time to reach a steady state, enough time for all the time averaged observables to become constant.

The steady state evolves adiabatically in a very slowly changing geometry.

The adiabatic oscillator state is a function of the geometry (through the oscillator couplings).

The adiabatic state of the classical cosmological oscillator is the physical vacuum. The vacuum higgs field is just one mode. There is a lot more to the physical vacuum.

The basic problem is to calculate the adiabatic time evolution of the cosmological oscillator.

Invariant oscillator

Initially, the universe is invariant under the full SU(2)×SU(2) symmetry. There are only a few invariant modes of the Standard Model fields.

- one real bosonic mode $b(t)$, a mode of the SU(2) gauge field
- 12 complex fermionic modes $\psi^I(t)$, a lepton and a quark triplet for each of the 3 generations.

The dynamics is given by evaluating the Standard Model action on the invariant field modes.

$$\hat{\mathcal{L}}_{\text{boson}} = \frac{3}{2g_2^2}\left[\left(\frac{db}{dt}\right)^2 - b^4\right] - \frac{\lambda^2}{2}\frac{\Lambda_{\text{higgs}}^4 a^4}{\hbar^4} \qquad \hat{\mathcal{L}}_{\text{fermion}} = \psi^\dagger i\left(\frac{d}{dt} + \frac{3}{2}ib\right)\psi$$

g_2 = the SU(2) gauge coupling constant $\qquad \lambda$ = the higgs coupling constant

A simple anharmonic oscillator $b(t)$ coupled to the 12 anti-commuting $\psi^I(t)$.

The oscillator dynamics is independent of $a(t)$. Only the higgs energy at $\phi = 0$ depends on $a(t)$.

Absolute normalization of $a(t)$

There is a re-scaling of time that leaves the invariant theory completely unchanged.

$$(ds)^2 = a(t)^2 \left[-(dt)^2 + \frac{1}{\epsilon^2}(ds)^2_{S^3} \right]$$

$$t \to s^{-1}t \qquad a \to sa \qquad \epsilon \to s\epsilon \qquad b(t) \to sb(t) \qquad \psi(t) \to s^{3/2}\psi(t) \qquad \text{for any } s > 0$$

$a(t)$ and ϵ are not physical quantities, but only the combination $a(t)/\epsilon$, the physical radius of space. We cannot talk about $a(t)$ by itself but only about ratios like the redshift $z = a(t_0)/a(t) - 1$ (where t_0 is the present time). An absolute normalization of $a(t)$ is a basic feature that has been missing from cosmology.

The cosmological oscillator normalizes $a(t)$. Re-scale so that the oscillator constant of motion is

$$\frac{1}{2}\left(\frac{db}{dt}\right)^2 + \frac{1}{2}b^4 = \frac{1}{8} + O(\psi)$$

There is no further freedom to re-scale t. The scale factor $a(t)$ is now absolutely normalized. Now it makes sense to say things like $a(t) < \hbar/210\,\text{GeV}$.

Absolute normalization of $a(t)$ (2)

ϵ is now a well-defined physical number, a parameter of the initial state. Given that ϵ must be very small, that the universe is very close to flat, almost nothing depends on ϵ.

The remaining free parameters of the initial state are encoded in the state of the fermion modes ψ^I.

I'll make few comments and otherwise ignore the fermions, as explained earlier.

- a finite number of free parameters ($< (2^{12})^2$) but a ridiculously large number.
- The fermion modes have the quark and lepton quantum numbers, so there's plenty of room to parametrize baryon number asymmetry.
- The values of the initial parameters are not to be explained. All that matters is if there are values that give a cosmologogy that matches observation.

Stability and instability

The invariant higgs field is $\phi = 0$. The oscillator provides an effective contribution $b(t)^2\phi^2$ to the higgs potential that stabilizes $\phi = 0$, keeping the symmetry unbroken.

Consider a tiny homogeneous fluctuation of the higgs field around $\phi = 0$.

$$\phi(t,x) = \frac{\Lambda_H}{\hbar}\hat{n}\,q(t) \qquad \hat{n} = \text{ a unit doublet vector} \qquad \hat{n}^\dagger\hat{n} = 1$$

For small $q(t)$ the equation of motion is

$$\frac{d^2q}{dt^2} + \frac{3}{4}b(t)^2q - \lambda^2\hat{a}^2q = 0$$

For small $a(t)$, the solutions are oscillatory and $\phi = 0$ is stable. For large $a(t)$, the solutions are exponential and $\phi = 0$ is unstable. for small $a(t)$, then becomes unstable for large $a(t)$.

The transition is at $a_{\mathrm{EW}} = \hbar/210\,\mathrm{GeV}$. The transition from stability to instability is described analytically by some technically neat 19th century mathematics of ordinary differential equations.

Jacobi (1829), Lamé (1837), Airy (1838), Floquet (1883)

Stability and instability (2)

In the invariant phase, there are tiny fluctuations of $q(t)$.

As $a(t)$ passes through $a_{\mathrm{EW}} = \hbar/210\,\mathsf{GeV}$, the oscillator $b(t)$ squeezes the quantum fluctuations of q into a gaussian probability ensemble concentrated on a line in the classical phase space q, p.

The squeezing takes place on an intermediate time scale $t_{\mathsf{sq}} = 10^{-5}t$ so it is almost instantaneous on the cosmological time scale.

The result, just after a_{EW}, is a gaussian probability ensemble of classical trajectories. Each trajectory in the ensemble starts with tiny non-zero values of $q(t), q'(t)$. All the directions $\hat{n}$ in doublet space are equally probable. Each trajectory singles out a direction in doublet space, breaking the SU(2)×SU(2) symmetry down to just a single SU(2). The universe is still homogeneous, but no longer isotropic. This is another neat feature of the theory — an explicit, analytically calculable mechanism of spontaneous symmetry breaking.

Our universe is one of the trajectories in the ensemble.

ϵ must be nonzero for there to be fluctuations. But the exponential instability means that there is only a very weak lower bound on ϵ.

Two phases

$a(t) < a_{\mathrm{EW}}$ **initial invariant phase (small oscillator)**

The initial SU(2)×SU(2) symmetric universe was the invariant cosmological oscillator (plus tiny fluctuations).

It was expanding, driven by the oscillator energy plus the higgs potential energy at $\phi = 0$.

$a(t) > a_{\mathrm{EW}}$ **broken symmetry phase (big oscillator)**

After a_{EW}, the universe is in a probabilistic ensemble of classical trajectories. On each trajectory, the vacuum higgs field breaks the SU(2)×SU(2) symmetry down to a single SU(2). The universe is still homogeneous, but no longer isotropic. There are many more invariant field modes under the smaller symmetry group. The oscillator gains many more degrees of freedom.

The theory does not say for how long the universe was in the invariant phase before the transition.

The probabilistic character of the cosmology after a_{EW} is a striking feature.

Broken symmetry oscillator

In the broken phase, the oscillator consists of the SM field modes invariant under the single SU(2).

- 1 real higgs mode
- 36 real gauge field modes
- 12 complex lepton modes
- 36 complex quark modes

These are the individual components of the SM fields at a single point in space.

This is a much more complicated anharmonic oscillator.

It starts in the SU(2)×SU(2) invariant state perturbed by the small vacuum higgs field $q(t)\hat{n}$ just after $a_{\rm EW}$. The non-zero higgs field then starts exciting all the other modes of the oscillator.

The broken-symmetry oscillator is the steampunk contraption.

Semi-broken oscillator

The broken-symmetry oscillator is almost certainly not integrable. It will have to be simulated numerically. This looks like a big project.

I was worried about how much anisotropy was produced after the transition. Too much anisotropy would kill the theory.

To get answers with a feasible amount of effort, I ignored the fermions.

Without the fermions, the nonzero vacuum higgs field breaks the symmetry only partly, only down to SU(2)×U(1).

The semi-broken oscillator has only 4 degrees of freedom, all bosonic.

- the higgs mode $q(t)$
- 1 mode of the U(1) gauge field
- 2 modes of the SU(2) gauge field

Semi-broken oscillator (2)

Simulating the semi-broken oscillator is manageable.

It captures all the non-linearity. The corrections from putting in the fermions are given by integrating a series of linear ordinary differential equations.

We might expect that simulating the semi-broken oscillator will show the basic structure of the cosmology.

It would be nice if the expansion in the fermionic modes turns out to be perturbative, if the terms of higher degree give smaller corrections. But I don't have an argument for this.

The most general spatial metric invariant under the SU(2)×U(1) symmetry is still homogeneous because of the unbroken SU(2). But the little group at each point is only U(1). There is a distinguished direction in space. The spatial metric, up to overall scale, has the form

$$a(t)^2 \hat{g}_{ij} = a(t)^2 \begin{pmatrix} e^{-\sigma(t)} & 0 & 0 \\ 0 & e^{-\sigma(t)} & 0 \\ 0 & 0 & e^{2\sigma(t)} \end{pmatrix} \qquad \sigma(t) = \text{the anisotropy}$$

Numerical simulation compromises

The analytic treatment of the symmetry breaking produces an ensemble of oscillator trajectories after the symmetry breaking. Each trajectory in the ensemble provides an initial condition for a simulation starting just after a_{EW}.

We need to run enough simulations to sample the ensemble of trajectories accurately.

I made three compromises in order to do this with a modest amount of computation.

1. Adiabatic approximation

Literal simulation of the oscillator in cosmological time is impractical, given that the ratio of time scales is $\sqrt{\hat{\kappa}} = 7{\times}10^{-17}$.

The correct method is to simulate adiabatic evolution. For each time step in t_c, the oscillator should run in time t until it reaches a steady state, until its time-averaged observables converge.

But I had no idea how long it would take to converge. So instead I made a simple-minded approximation, simulating literally but with $\sqrt{\hat{\kappa}} = 10^{-2}$ and 10^{-3} instead of $7{\times}10^{-17}$, hoping to approach adiabaticity. It seems to, at least roughly.

Numerical simulation compromises (2)

2. Switch-over from analytic trajectory to simulated

The analytic treatment of the symmetry breaking assumes $q(t)$ very small so the linear dynamics is a good approximation. The switch-over to numerical simulation should take place when $q(t)$ is in the exponential regime but while it is still small. This leaves room for choice.

I arbitrarily set the switch-over when $q(t)$ reached 10^{-8}.

We need to investigate the beginning of the broken-symmetry phase. If we could do a perfect simulation, nothing would depend on the switch-over point. We at least need to see if the simulation results depend on the choice of switch-over point.

Numerical simulation compromises (3)

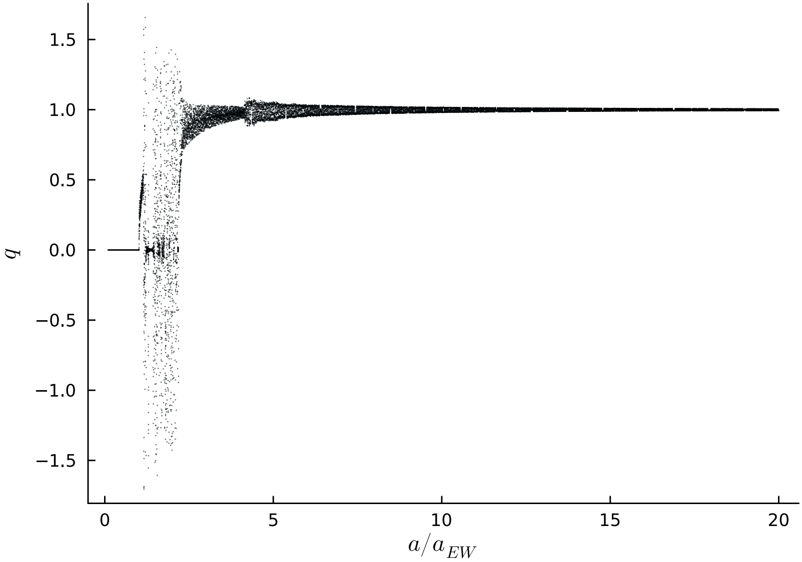
3. Dependence on ϵ

In the invariant phase, before the symmetry breaking, the gaussian width of the fluctuations in $q(t)$ is on the order of $\mathrm{Vol}((S^3_\epsilon)^{1/2} = (2\pi^2)^{1/2}\epsilon^{-3/2}$.

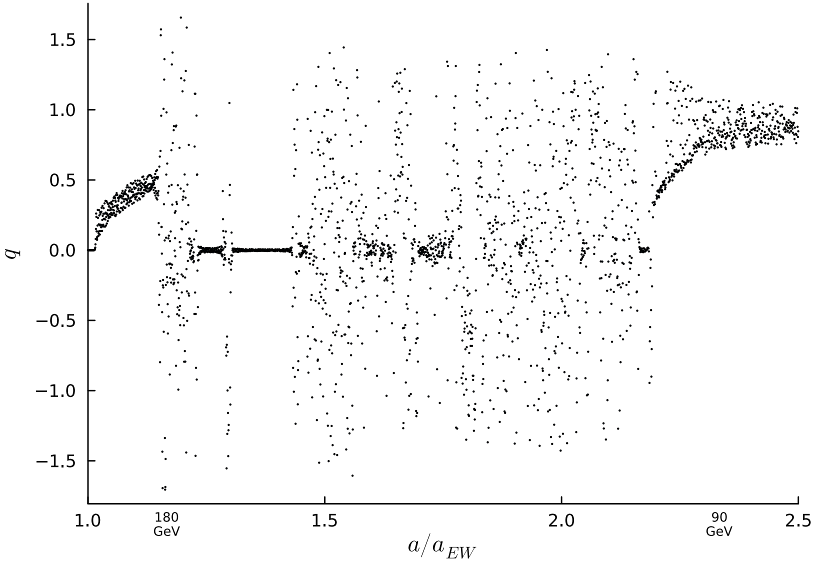
The gaussian width of the ensemble of trajectories emerging from the symmetry breaking point depends on ϵ. I made an arbitrary choice, $\epsilon = 10^{-}$.

It looks like the ensemble depends only very weakly on the value of ϵ over many orders of magnitude. But this should be investigated. If the cosmology after the transition depends significantly on ϵ, this could give a method for measuring ϵ independent of the present curvature determination.

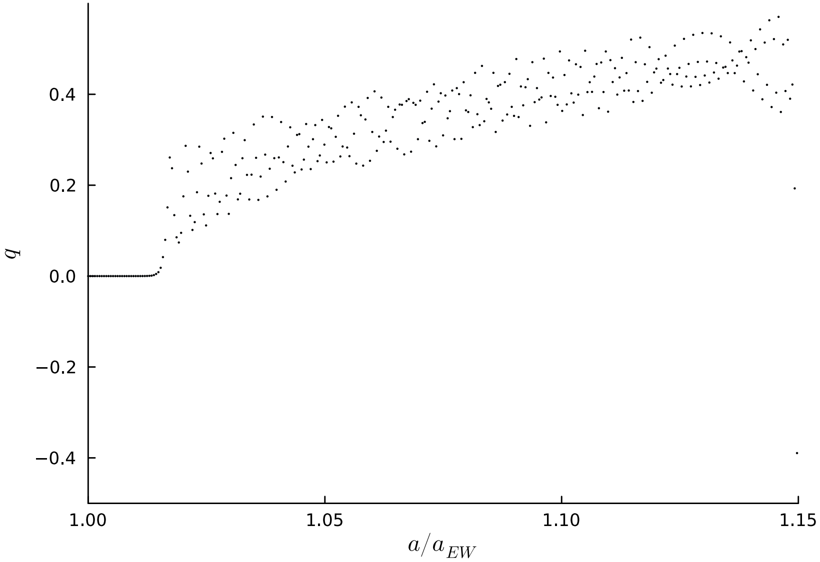
Typical trajectory after $a_{\mathrm{EW}} = \hbar/210\,\mathrm{GeV}$ (sampled)



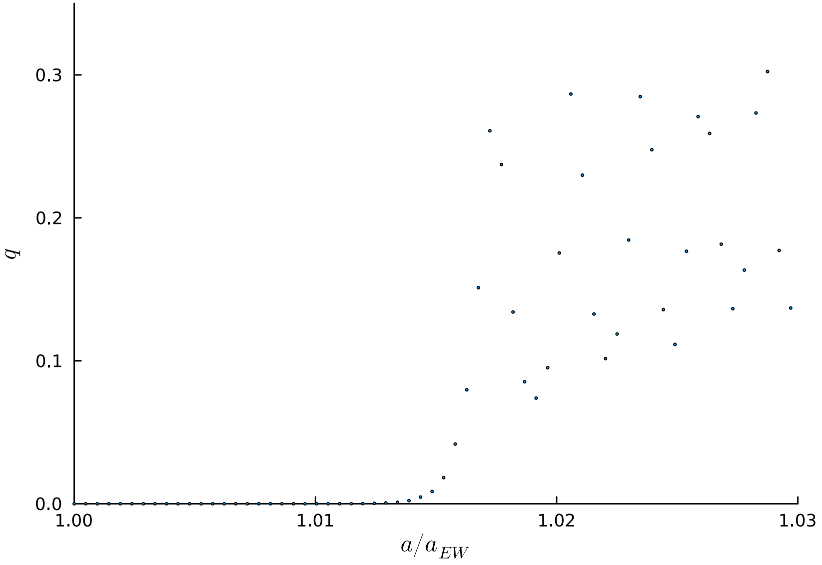
blowup



blowup (2)



blowup (3)



Electroweak transition

The electroweak transition is the period from a_{EW} until $q(t) \approx 1$, from $210\,\mathsf{GeV}$ down to $90\,\mathsf{GeV}$.

The chaos was a big surprise. I expected $q(t)$ to track the effective valley bottom of higgs potential $+\, b(t)^2 q^2$.

In the limit $q(t) \to 1$, the anharmonic oscillator becomes three weakly coupled harmonic oscillators, one for each of the W,Z, and higgs particles.

Adiabatic evolution of harmonic oscillators is a textbook problem. We get density and pressure

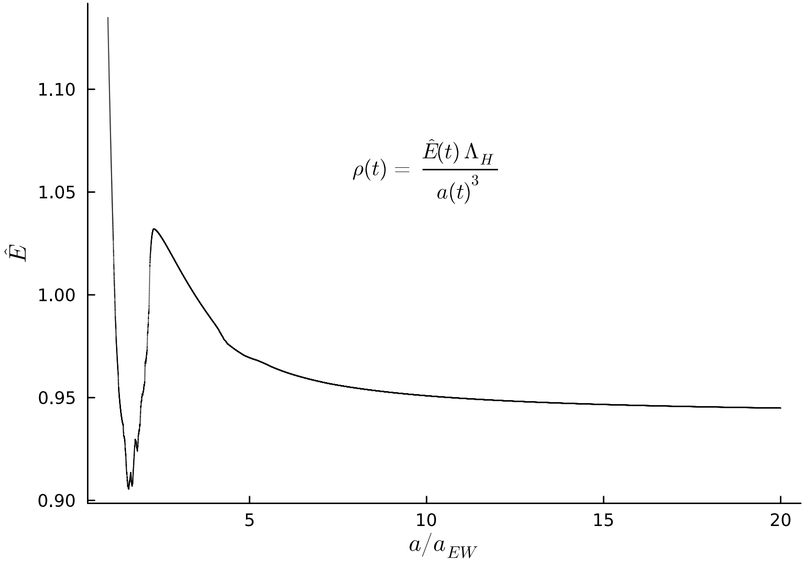
$$\rho(t) \to \frac{E}{a(t)^3} \qquad\qquad p(t) \to 0$$

where E is an energy that has to be extracted from the numerical simulation.

This is cold dark matter.

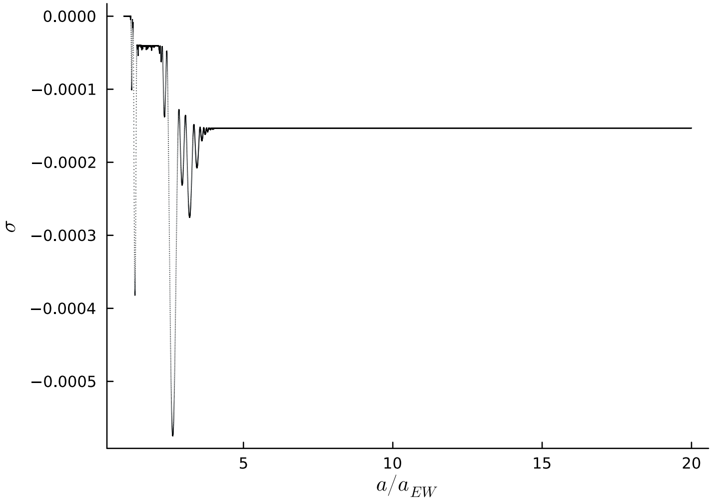
After the EW transition, the energy in the bosonic oscillator becomes cold dark matter.

$\hat{E} = E/\Lambda_{\text{higgs}}$



Anisotropy

I worried that the symmetry breaking would produce too much anisotropy, but it seems that only a little is produced. maybe an observable amount?



Butterfly effect

Just after a_{EW}, the universe is in a gaussian probability ensemble of trajectories.

The gaussian ensemble is processed by the chaotic oscillator.

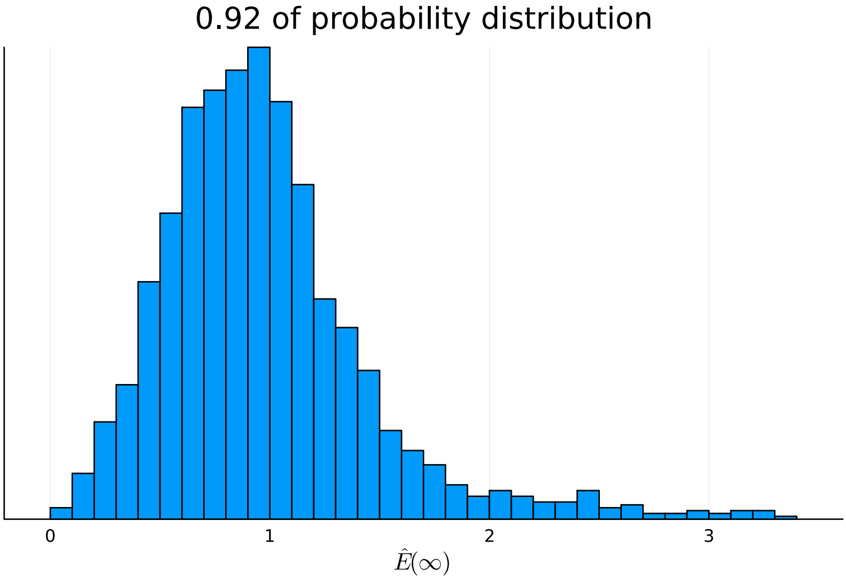
There is sensitive dependence on the initial condition.

The output is an ensemble of adiabatic oscillator states in the limit $q(t) = 1$ with the SU(2) gauge symmetry fully broken.

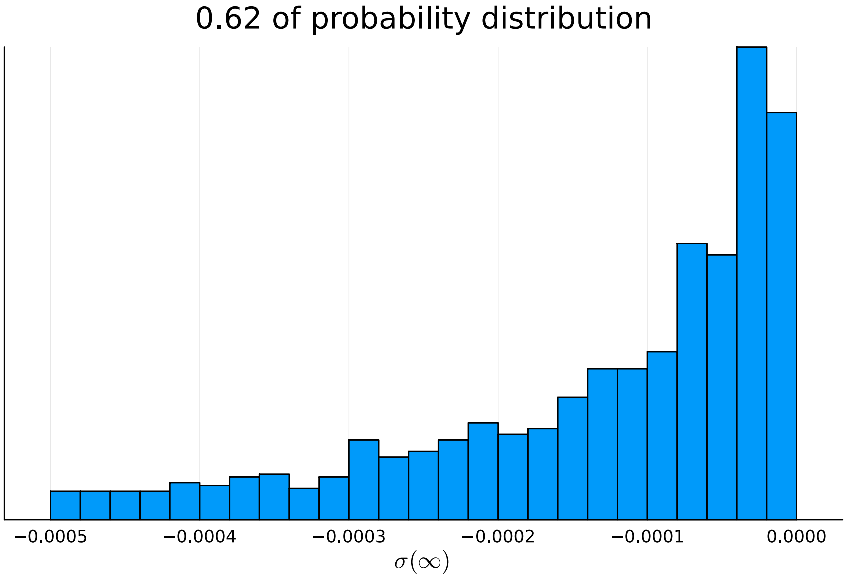
The next two slides show the ensemble probability distribution projected on two individual parameters of the adiabatic state: the dark matter energy $\hat{E}$ and the anisotropy σ.

A significant fraction of the ensemble is left out. These are outlier trajectories that stay at $q = 0$ for much longer, to much larger $a(t)$.

dark matter $\hat{E} = E/\Lambda_{\text{higgs}}$ probability distribution (1653 trajectories)



anisotropy σ probability distribution (1653 trajectories)



Physical vacuum

The adiabatic state of the cosmological oscillator is the physical vacuum.

It depends only on the energy scale $\hbar/a(t)$.

The physical vacuum is a dynamical system evolving with scale.

Hard-scattering collider experiments at energy Λ_{exp} should see the physical vacuum at scale $\hbar/a = \Lambda_{\text{exp}}$.

It is not just the SM coupling constants that depend on the energy scale (the renormalization group). The vacuum also depends on the energy scale.

To do

There is a huge amount to investigate, to think about.

1. The top priority is to confirm or dis-confirm the theory as quickly as possible by figuring out the collider experiment signatures of the chaotic vacuum at the electroweak scale. Especially: can hard-scattering events at the electroweak scale be extracted from the LHC data, to check for such signatures? This would (dis-)confirm the theory without waiting for future collider data.

2. Do systematic simulations of the adiabatic cosmological oscillator through the electroweak transition, including the fermionic modes. Confirm the chaotic phase. Analyze the resulting ensemble of adiabatic states (physical vacua).

3. Figure out if any of the properties at the end of the transition can be related to observational cosmology without integrating the oscillator equations of motion all the way to the present, for example the anisotropy or the dark matter equation of state.

soft fluctuations of the oscillator, fluid

EXTRAS

$\epsilon < 10^{-27}$

Later we will be able to argue that ϵ must be very small, $\epsilon < 10^{-27}$, in order for the theory to agree with the observed flatness of the universe.

$$|\Omega_k| = \frac{1}{H_0^2 r(t_0)^2} < 0.001 \qquad H_0 = 67.8\,(\mathrm{km/s})/\mathrm{Mpc} = \frac{1}{1.36 \times 10^{26}\,\mathrm{m}}$$

$r(t_0)$ is the present radius of the universe. H_0 is the Hubble constant. Ω_k is the cosmological curvature parameter.

$S^3 = \mathrm{SU}(2)$

The 3-sphere S^3_ϵ is identical to the group $SU(2)$

$$X = \epsilon \begin{pmatrix} x_4 + ix_3 & -x_2 + ix_1 \\ x_2 + ix_1 & x_4 - ix_3 \end{pmatrix} \qquad \begin{gathered} \det(X) = 1 = \epsilon^2 \left(x_1^2 + x_2^2 + x_3^2 + x_4^2\right) \\ X^\dagger X = 1 \end{gathered}$$

The invariant metric is

$$(ds^2)_{S^3_\epsilon} = \frac{1}{2\epsilon^2} \mathrm{tr}\left(dX^\dagger dX\right)$$

If g_L, g_R are two elements of SU(2), then they transform the 3-sphere,

$$X \mapsto g_L X g_R^{-1}$$

This transformation is a symmetry. It leaves the metric unchanged.

When $g_L = g_R = -1$, nothing happens to X. The orthogonal group SO(4) is the group $\mathrm{SU}(2){\times}\mathrm{SU}(2)$ modulo this transformation that does nothing.

$$\mathrm{SO}(4) = \mathrm{SU}(2){\times}\mathrm{SU}(2)/\mathbb{Z}_2 \qquad \mathbb{Z}_2 = \{(1,1),\ (-1,-1)\}$$

This is like SU(2) being the double covering of SO(3), the rotation group of euclidean 3-space.

$S^3 = \mathrm{SU}(2)$ (2)

$$g_L,\, g_R \in \mathrm{SU}(2) \qquad X \mapsto g_L X g_R^{-1}$$

$\mathrm{SU}(2)_L$ takes any point X to any other point X'. Likewise for $\mathrm{SU}(2)_R$. Either SU(2) enforces homogeneity.

The diagonal SU(2) $g_L, g_R = g, g$ leaves the north pole unchanged.

$$\mathbf{N} = (0,0,0,1/\epsilon) \qquad X_{\mathbf{N}} = \mathbf{1} \qquad gX_{\mathbf{N}}g^{-1} = X_{\mathbf{N}}$$

The diagonal SU(2) acts as SO(3) rotating the tangent vectors at $\mathbf{N}$, the tiny displacements dx_i.

The diagonal SU(2) enforces isotropy at $\mathbf{N}$.

The spinors at $\mathbf{N}$ are the doublet representation of this diagonal SU(2).

Integrable

The invariant oscillator is integrable, solved by elliptic functions.

The scalar solution is the Jacobi elliptic function

$$b_0(t) = k\,\mathrm{cn}(t - t_1, k) \qquad k^2 = \frac{1}{2}$$

meromorphic (analytic with poles) and doubly periodic in t

$$b_0(t) = b_0(t + 4K) = b_0(t + 4Ki) \tag{0.1}$$

The full solution $b(t)$, $\psi^I(t)$ is also given by elliptic functions.

Thermal fluctuations

The Standard Model fields are in a quantum state that is extremely close to a classical state.

There are still quantum fluctuations of the fields around the classical trajectory.

The imaginary time periodicity $t \to t + 4Ki$ of the cosmological oscillator defines a natural thermal state on the fluctuations,

$$\text{quantum state periodic in imaginary time} = \text{thermal state} \qquad \beta = \frac{\hbar}{k_B T} = 4Ka(t)$$

The cosmology is constructed from top down: first the solution of the extremely classical cosmological oscillator, then the evolution of the tiny fluctuations around the invariant classical solution. The fluctuations are tiny ripples riding on the invariant classical solution.

Calculating the time evolution of the fluctuations will be challenging because of all the condensations that take place: quarks to baryons, baryons to nuclei, nuclei and electrons to atoms, ...