Where does quantum field theory come from?

Daniel Friedan

Rutgers University and the University of Iceland

Vadim Knizhnik Memorial Conference, IHÉS, October 31, 2013

I will describe (sketch) a mechanism that produces a quantum field theory in spacetime (a measure on euclidean spacetime fields) and, at the same time, produces the corresponding quantum string background.

It seems to me that this is the correct construction of the quantum string background (a purely formal question). I don't know if it is the correct construction of quantum field theory. That is, I don't know if the mechanism actually operates in the real world. To find out, I am trying to derive predictions of exotic low-energy phenomena that might be observable.

I am in the process of investigating a possibility that this mechanism produces exotic non-canonical low-energy degrees of freedom whenever there is $SU(2)$ gauge invariance. These non-canonical degrees of freedom are associated with the nontrivial homotopy group $\quad \pi_1(SU(2) \text{ gauge fields on } \mathbb{R}^4) = \mathbb{Z}_2$.

This project is (very) speculative fundamental physics — obviously an extreme longshot.

The 2d general nonlinear model (1979)

A renormalizable 2d qft with infinitely many coupling constants

$$\int \mathcal{D}X \; e^{-A(X)} \qquad A(X) = \int d^2z \; g_{\mu\nu}(X) \partial X^\mu \bar{\partial} X^\nu$$

$$X(z,\bar{z}) \in M, \;\; \text{a manifold}$$

$$\{\text{2d coupling constants}\} \;=\; g_{\mu\nu}(X), \;\; \text{a riemannian metric on } M$$

RG equation $\qquad \Lambda \dfrac{\partial}{\partial \Lambda} g_{\mu\nu}(X) = -R_{\mu\nu}(X) + O(R^2)$

Acting at small 2d distances, the 2d RG drives the metric $g_{\mu\nu}$ to a fixed point, $R_{\mu\nu} = 0$.

The 2d RG is a mechanism that produces classical spacetime field theory (taking M to be spacetime).

This was enormously exciting (at least to me). It seemed to be possibly a clue towards explaining where the laws of physics come from.

Immediate questions:

1. $R_{\mu\nu} = 0$ is not quite Einstein's equation. How to produce a realistic field theory? (with gauge, fermion, scalar fields)

2. The fixed points $R_{\mu\nu} = 0$ have unstable directions (relevant couplings). How to get stability?

3. Where does *quantum* field theory come from?

Questions 1 and 2 were answered when the nonlinear model was interpreted as the string background.

String backgrounds

The worldsurface of a string moving in a background spacetime is described by a 2d nonlinear model.

$\beta = 0$ is a consistency condition (to get the Virasoro algebra, to get a unitary string S-matrix)

$\beta = 0$ becomes a classical field equation derived from a spacetime action when all possible renormalizable couplings (esp the dilaton field) are included.

The fermionic string, with 2d N=1 susy, has more renormalizable 2d couplings: spacetime gauge fields, fermion fields, scalar fields

Stability is given by the GSO projection (absence of relevant couplings = absence of tachyons) and by the remarkable cancellation of higher loop terms in β when there is $N = 2$ susy (spacetime supersymmetry).

Questions

- Should the string background be given by a *classical* field?

  Shouldn't it be given by a quantum mechanical state?

  What is the quantum string background?

- Is there a mechanism — a generalization of the 2d RG — that produces a quantum field theory in spacetime and a quantum string background?

It seems to me that we need a mechanism that produces quantum field theory. There are too many quantum field theories. A mechanism actually producing quantum field theory might bring explanatory power.

Eventually, a proposal.

The λ-model (2002)

Write λ^i for the 2d coupling constants,

$$\{\lambda^i\} = \{\text{the modes of the spacetime fields } g_{\mu\nu},\ A_\mu,\ \psi^\alpha,\ \phi,\ \ldots\}$$

Let $\mathcal{M}$ be the manifold of spacetime fields, so the λ^i are coordinates on $\mathcal{M}$.

Equivalently, $\mathcal{M}$ is the manifold of 2d qfts parametrized by the coupling constants λ^i.

The λ^i couple to the approximately marginal 2d fields $\phi_i(z, \bar{z})$ (e.g., the vertex operators $\partial X^\mu \bar{\partial} X^\nu e^{ipX}$).

The λ^i have scaling dimensions of the form $-p(i)^2$, where $p(i)$ is the spacetime momentum of the mode λ^i.

The modes λ^i with $p(i)$ small (in dimensionless units) are only slightly irrelevant.

Allow the λ^i to vary on the surface, becoming local sources $\lambda^i(z, \bar{z})$.

Set the $\lambda^i(z, \bar{z})$ fluctuating at 2d distance scales $< \Lambda^{-1}$,

$$\int \mathcal{D}\lambda \ e^{-\int d^2z \ \frac{1}{g^2} G_{ij}(\lambda) \partial\lambda^i \bar{\partial}\lambda^j} \ e^{\int d^2z \ \lambda^i(z,\bar{z})\phi_i(z,\bar{z})}$$

$$\begin{aligned} G_{ij}(\lambda) &= \text{the natural metric on the manifold of spacetime fields} \\ &\quad\ \text{(the natural metric on the manifold of 2d qfts)} \\ g &= \text{the spacetime coupling constant (the string cc)} \end{aligned}$$

Quantum string background

$$\int \mathcal{D}\lambda \; e^{-\int d^2z \, \frac{1}{g^2} G_{ij}(\lambda)\partial\lambda^i \bar{\partial}\lambda^j} \;\; e^{\int d^2z \, \lambda^i(z,\bar{z})\phi_i(z,\bar{z})}$$

Integrating out fluctuations of the $\lambda^i(z, \bar{z})$ at 2d distances $< \Lambda^{-1}$ produces insertions of the $\phi_i(z, \bar{z})$ in the surface.

The λ-model is designed so that these insertions are the same as the insertions produced by the froth of small handles at 2d distances $< \Lambda^{-1}$ in the worldsurface.

The λ-model acts at 2d distances $< \Lambda^{-1}$ to produce an effective 2d qft whose correlation functions are defined at distances $> \Lambda^{-1}$. This is the quantum string background.

The λ-model calculates (nonperturbatively) the effects of the froth of tiny handles. It substitutes for the froth of small handles.

The crucial design principle is 2d locality. The effective worldsurface is local at 2d distances $> \Lambda^{-1}$, so the string S-matrix will have all the good properties implied by 2d locality.

Production of QFT

Under the 2d RG, the small distance fluctuations of the $\lambda^i(z,\bar{z})$ are integrated out while at the same time the $\lambda^i(z,\bar{z})$ are being driven along $\beta^i(\lambda)$.

At each z, what evolves is a measure on $\mathcal{M}$, a measure on the the manifold of spacetime fields.

The 2d RG, acting over 2d distances up to Λ^{-1}, drives this measure to a limiting equilibrium measure.

Production of QFT

Easier to study the evolution of functions $f(\lambda)$ on $\mathcal{M}$,

$$\Lambda \frac{\partial}{\partial \Lambda} f(\lambda(z, \bar{z})) = \beta^i \partial_i f - \nabla_i (g^2 G^{ij} \partial_j f)$$

the first term coming from the evolution of λ^i under $\beta^i(\lambda)$ and the second term from integrating out the small distance fluctuations. The measure then evolves as the dual

$$\Lambda \frac{\partial}{\partial \Lambda} \rho = \nabla_i (\beta^i + g^2 G^{ij} \partial_j) \rho$$

If β^i were 0, the equilibrium measure would be the metric volume element $\mathcal{D}\lambda$. For $\beta^i = G^{ij} \partial_j S$, the equilibrium measure is

$$\mathcal{D}\lambda \, e^{-\frac{1}{g^2} S(\lambda)} .$$

In this way the λ-model produces a quantum field theory, a measure on the spacetime fields.

Radial quantization

The production of quantum field theory can be seen equivalently in the radial quantization of the λ-model.

Write $\mathcal{M}$ for the manifold of spacetime fields — the target manifold of the λ-model. The λ^i are local coordinates on $\mathcal{M}$.

Under 2d dilation, the loops in $\mathcal{M}$ evolve quantum mechanically.

The fluctuations of the non-zero modes of the loops $\lambda(s)$ are severely suppressed. The ground state wave function is concentrated on the constant loops.

The evolution of the zero-modes is just stochastic quantization of the quantum field theory with classical action $S(\lambda)$.

Small 2d distance = long distance in spacetime

Suppose the 2d qft is renormalized at 2d distance μ^{-1}.

The λ^i are slightly irrelevant, with dimensions $-p(i)^2$.

At 2d distance $\Lambda^{-1} \ll \mu^{-1}$, the effects of λ^i are suppressed by

$$(\Lambda\mu^{-1})^{-p(i)^2} = e^{-L^2 p(i)^2}, \qquad L^2 = \ln(\Lambda\mu^{-1})\,.$$

So λ^i with $p(i)^2 >> L^{-2}$ are effectively irrelevant. They can be disregarded. Their fluctuations would have no effect.

L^{-1} is an effective UV cutoff in the λ-model.

Each 2d distance scale Λ^{-1} corresponds to a spacetime distance scale L.

Small distance 2d physics is long distance spacetime physics.

QFT/S-matrix complimentarity

The effective worldsurface is cutoff at 2d distance Λ^{-1}. The string propagator is

$$\frac{1-(\Lambda\mu^{-1})^{-p^2}}{p^2}$$

cutoff in the spacetime infrared at spacetime momentum L^{-1}.

The string S-matrix in the effective background describes scattering at spacetime distances $< L$.

The λ-model produces a qft describing spacetime physics at distances $> L$ and a string S-matrix describing physics at distances $< L$.

The choice of Λ and thus of L is arbitrary, as long as $L \gg 1$.

As L varies, the λ-model ensures consistency between qft and S-matrix.

This is a realistic version of the string background. The string background is the quantum mechanical apparatus where scattering takes place.

This seems to me a practical form for a physical theory. It avoids idealism — both the idealism of S-matrix theory and the idealism of fundamental quantum field theory.

The λ-model has an intriguing property: it acts from the smallest 2d distances upwards, so it builds the spacetime qft from the longest spacetime distances downwards.

There are many internal, formal questions to investigate. But I want to know whether or not the theory describes the real world.

Just because I find the theory philosophically satisfactory does not mean that it is true. I want low-energy predictions that can be checked against experiments.

There doesn't seem much possibility of deriving the standard model, or of verifying string theory. So take another tack. Assume that the λ-model produces something like the standard model. Would it necessarily produce something else at low energy, something non-canonical and therefore exotic, that might be possible to observe?

Question

- The λ-model is a 2d qft. Are there nonperturbative 2d effects that would show up in the spacetime qft produced by the λ-model?

$\mathcal{M}$ is the manifold of classical spacetime fields on $\mathbb{R}^4$.

Semi-classical 2d effects are

- winding modes coming from $\pi_1(\mathcal{M})$
- 2d instantons coming from $\pi_2(\mathcal{M})$

$\pi_1(SU(2)$ gauge fields$) = \mathbb{Z}_2$, $\quad \pi_2 = \mathbb{Z}_2$

$\pi_1(SU(3)$ gauge fields$) = 0$, $\quad \pi_2 = \mathbb{Z}$

Questions

1. Are there non-canonical low energy degrees of freedom in spacetime associated with the nontrivial winding mode in the manifold of $SU(2)$ gauge fields?
2. Are there non-canonical low energy couplings associated with the nontrivial 2-spheres in the manifolds of $SU(2)$ and $SU(3)$ gauge fields?

Minimal representatives (2010)

From computer investigations of the Yang-Mills flow, minimal representatives of the nontrivial homotopy groups were found:

	Minimal representatives
$\pi_1(SU(2)$ gauge fields)	a loop of instanton pairs
$\pi_2(SU(2)$ gauge fields)	a 2-sphere of instanton quartets
$\pi_2(SU(3)$ gauge fields)	a 2-sphere of instanton pairs

The minimal loop has zero length (to be explained). The minimal 2-spheres have zero area. So there are chances of low energy phenomena.

Most interesting is the possibility that the $SU(2)$ winding mode will show up as non-canonical low-energy degrees of freedom in $SU(2)$ gauge field theory.

Twisted pairs

The minimal nontrivial loop of $SU(2)$ gauge fields is a loop of twisted pairs.

A twisted pair is an instanton in $\mathbb{R}^4$ with a tiny anti-instanton glued in (or, alternatively, an anti-instanton with a tiny instanton glued in).

A_+	an $SU(2)$ instanton of size ρ_+ centered at x_+
$g_- A_- g_-^{-1}$	an anti-instanton of size $\rho_- \approx 0$, centered at x_-, twisted by $g_- \in SU(2)/\pm\mathbf{1}$

The nontrivial element of $\pi_1(SU(2)$ gauge fields) is represented by a nontrivial loop in this $SU(2)/\pm\mathbf{1}$.

The metric on twisted pairs

Instead of the parameters ρ_-, g_- write

$$v_- = \rho_-(g_-\hat{1}) \in \mathbb{C}^2 \qquad \text{where } \hat{1} = (1,0) \in \mathbb{C}^2$$

The metric is smooth on this $\mathbb{C}^2$. At the origin,

$$(ds)^2 = \frac{1}{g^2}|dv_-|^2.$$

The space of twisted pairs (for fixed x_+, ρ_+, x_-) is the orbifold

$$\pm v_- \in \mathbb{C}^2/\mathbb{Z}_2\,.$$

The minimal nontrivial loop is the zero-length loop in $SU(2)/\pm\mathbf{1}$ at $\rho_- = 0$ (at the blow-up of the origin).

The λ-model winding mode is the twist field of this $\mathbb{C}^2/\mathbb{Z}_2$ orbifold. (x_+, ρ_+, x_-) are collective coordinates for the twist field.

If massless fermion fields are coupled to the $SU(2)$ gauge fields, there will be chiral zero modes in the instanton and in the anti-instanton. The zero mode in the tiny anti-instanton is odd under the $\mathbb{Z}_2$ (this is the $SU(2)$ global anomaly made explicit) so the orbifolding projects it away. The instanton zero modes survive, becoming fermionic collective coordinates for the twist field. So there are possibilities of interesting quantum numbers in the twist field.

The RG (YM) flow

For $\rho_- \approx 0$,

$$S_{YM} = 2 + \frac{4|v_+|^2|v_-|^2 - 16(v_+ \cdot v_-)^2}{|x_+ - x_-|^4}$$

where

$$v_- = \rho_-(g_-\hat{1}), \qquad v_+ = \rho_+(g_+\hat{1}) \qquad \hat{1} = (1,0) \in \mathbb{C}^2$$

($g_+ = \mathbf{1}$ is global gauge fixing.)

Writing $v_- = (\chi_0 + i\chi_1,\ \chi_2 + i\chi_3)$, $v_+ = (\rho_+, 0)$,

$$S_{YM} = 2 + \frac{4\rho_+^2}{|x_+ - x_-|^4}\left(-3\chi_0^2 + \chi_1^2 + \chi_2^2 + \chi_3^2\right)$$

so there is one unstable direction (flowing to the flat connection).

The flow becomes interesting when $\rho_+ \approx 0$ as well as $\rho_- \approx 0$.
The unstable direction then becomes only marginally unstable.

To do

The problem now is to understand the contribution of the twist fields in the λ-model functional integral.

I suppose that we can express those contributions by introducing new λ-fields λ_{tw}^{α} which couple to the twist field, where the index α stands for the collective coordinates of the twist field.

The dynamics of the new λ-fields is derived from the scaling behavior of products of twist fields and ordinary fields.

The first question:

- Is the classical 2d scale invariance of the twist field ruined by perturbative quantum corrections? Are cancellations needed? spacetime susy?

It seems that the unstable trajectory from the twisted pair to the flat connection should be the key ingredient in the operator product of two twists.