First principles cosmology of the Standard Model epoch

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The project is to construct a first principles cosmology of the Standard Model epoch, the period starting shortly before the electro-weak transition. The cosmology is derived from a simple initial condition — a semi-classical quantum state of the SM precisely specified by a symmetry condition and a high energy condition. The dark matter is a classical effect in the SM, a coherent state of the $\mathrm{SU}(2)$-weak gauge field and the Higgs field. The classical universe contains only the dark matter. Ordinary matter is a sub-leading correction due to the fluctuations of the SM fields around the classical trajectory. There are no adjustable parameters consistent with the initial symmetry. No physics beyond the SM is invoked.

The Standard Model epoch

The SM epoch is the cosmological period starting shortly before the electro-weak transition, when the universe was at energy scale roughly $1\,\mathrm{TeV}$.

$$\rho_{\mathrm{matter}} \sim \frac{(1\,\mathrm{TeV})^4}{\hbar^3} \qquad \text{(in } c = 1 \text{ units)}$$

This is the period when the universe is governed by the SM + classical GR to the best of our knowledge (taking the SM to include nonrenormalizable couplings for neutrino masses and mixings).

Hold off on assuming that dark matter requires physics beyond the SM.

The dark matter will be explained entirely within the SM.

This is a top-down approach to cosmology.

It starts with a natural initial quantum state specified by a few simple conditions:

1. The universe is governed by the SM and classical GR (with Λ).
2. The universe is a 3-sphere.
3. The initial state of the SM epoch is semi-classical.
4. The initial state has a certain $\mathrm{Spin}(4)$ symmetry.
5. The initial energy is $> 10^{107}$ in natural units.

These conditions specify the initial state precisely. There are no adjustable parameters.

(Nothing observable will depend on the actual value $> 10^{107}$ of the initial energy.)

Dark matter is a cosmological gauge field (CGF).

The initial state is a coherent state of the Higgs field and the $\mathrm{SU}(2)$ gauge field (the CGF).

The CGF is dark matter: a perfect fluid with equation of state parameter $w_{\mathrm{CGF}} \approx 0$.

In the leading order, classical approximation, the universe contains only the CGF, only dark matter, no ordinary matter. The ordinary matter is a sub-leading correction due to the fluctuations of the SM fields around the classical trajectory.

The classical CGF universe has the basic structure of the SM epoch:

- the electro-weak transition
- followed by an expanding universe
- containing only dark matter
- homogeneous and isotropic
- flat at the present time.

The classical CGF universe is the dark matter skeleton of the SM epoch, to be fleshed out by the fluctuations of the SM fields. The time evolution of the fluctuations remains to be calculated.

First principles cosmology

Laboratory HEP looks for small discrepancies from SM predictions to find new physics.

The idea is to put cosmology in the same situation. If the project works, the SM epoch will be described accurately by a systematic expansion around the classical dark matter universe. Any discrepancy will be a sign of new physics.

Maybe this is too ambitious. Maybe the SM epoch depends on so far undiscovered particles and fields. Still, until those particles or fields are actually discovered, the top-down approach seems worth trying.

Dark matter being a SM effect explains why no dark matter particles have been found. It predicts that no such particles will be found.

Dark matter being a classical effect and ordinary matter a sub-leading correction explains why most of the matter in the universe is dark matter.

It seems nontrivial that there should be a simple initial state that gives the right basic structure and a systematic scheme to correct it. It might be worthwhile to calculate the corrections to the classical trajectory to see if the details come out right.

The initial state

SO(4) is the symmetry group of the 3-sphere in euclidean 4-space.

Spin(4) $=$ SU(2) $\times$ SU(2) is the simply connected covering group. Spin(4) acts on spinors on the 3-sphere.

The SU(2) gauge bundle of the SM is identified with the spinor bundle. The U(1) and SU(3) gauge bundles are trivial product bundles over the 3-sphere.

This defines the action of Spin(4) on the space-time metric and on the SM fields.

The cosmology at leading order is a classical solution of the SM equations of motion invariant under this action of Spin(4).

The only nontrivial $\mathrm{Spin}(4)$-symmetric classical fields are

- the space-time metric $ds^2 = R(\hat{t})^2 \left(-d\hat{t}^{\,2} + \hat{g}_{ij}(\hat{x})d\hat{x}^i d\hat{x}^j\right)$

  $\hat{g}_{ij}(\hat{x})$ is the metric on the unit $S^3 \subset \mathbb{R}^4$.

  $\hat{t}$ is the conformal time.

- the Higgs field $\phi = 0$ (the only Spin(4)-symmetric spinor)

- the $\mathrm{SU}(2)$-weak gauge field $D_i = \hat{\nabla}_i + \hat{b}(\hat{t})\hat{\gamma}_i(\hat{x})$

  $\hat{\gamma}_i(\hat{x})$ are the Dirac matrices on S^3.

  $\hat{\nabla}_i$ is the metric covariant derivative on spinors.

The $\mathrm{Spin}(4)$-symmetric degrees of freedom are

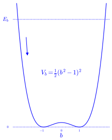
- $R(\hat{t})$ the radius of the universe

- $\hat{b}(\hat{t})$ the cosmological gauge field (CGF)

SM classical dynamics

$$\int \frac{1}{2g^2}\mathrm{tr}(-F_{\mu\nu}F^{\mu\nu})\sqrt{-g}\,d^4x \;=\; \frac{6\pi^2}{g^2}\int\left[-\frac{1}{2}\left(\frac{d\hat{b}}{d\hat{t}}\right)^2+\frac{1}{2}\left(\hat{b}^2-1\right)^2\right]d\hat{t} \qquad g^2 = 0.426$$

anharmonic oscillator



dimensionless conserved energy (units $\hbar/R$)

$$\hat{E}_{\mathrm{CGF}} = \frac{1}{2}\left(\frac{d\hat{b}}{d\hat{t}}\right)^2+\frac{1}{2}\left(\hat{b}^2-1\right)^2$$

The equation of motion is solved by a Jacobi elliptic function. (This z is not the redshift.)

$$\hat{b}(\hat{t}) = \frac{k\,\mathrm{cn}(z,k)}{\epsilon} \qquad z = \frac{\hat{t}}{\epsilon} \qquad \hat{E}_{\mathrm{CGF}} = \frac{1}{8\epsilon^4} \qquad k^2 = \frac{1}{2}+\epsilon^2$$

The initial energy condition $\hat{E}_{\mathrm{CGF}} > 10^{107}$ which is $\epsilon < 10^{-27}$ comes later as a physical condition needed to produce the observed flatness of the present universe.

Local coordinates

Scale by ϵ: $\quad x^0 = z = \frac{\hat{t}}{\epsilon} \qquad x^i = \frac{\hat{x}^i}{\epsilon} \qquad a(z) = \epsilon R(\hat{t}) \qquad b(z) = \epsilon \hat{b}(\hat{t}) = k\,\mathrm{cn}(z,k)$

$$ds^2 = a(z)^2\left(-dz^2 + g_{ij}(x)dx^i dx^j\right) \qquad [\gamma_i,\,\gamma_j] = -\frac{1}{2}g_{ij}$$

x^i is coordinate, g_{ij} is the metric on the 3-sphere of radius $1/\epsilon$.

$$D_i = \frac{\partial}{\partial x_i} + \epsilon\gamma_i + b(z)\gamma_i \qquad \frac{1}{\hbar}S_{\mathrm{YM}} = \frac{6\pi^2}{g^2\epsilon^3}\int\left[-\frac{1}{2}\left(\frac{db}{dz}\right)^2 + \frac{1}{2}\left(b^2-\epsilon^2\right)^2\right]dz$$

Local physics is independent of ϵ when ϵ is very small.

Local physics does not depend on the value of $\hat{E}_{\mathrm{CGF}} \gg 1$. In effect there are no free parameters.

The CGF is semi-classical if $\epsilon \ll 1$.

CGF temperature and initial thermal state

$b(z) = k\,\mathrm{cn}(z,k)$ is doubly periodic in z.

real period $\begin{array}{c} z \sim z + 4K \\ t \sim t + 4Ka \end{array}$ imaginary period $\begin{array}{c} z \sim z + 4K'i \\ t \sim t + 4K'ai \end{array}$

K, K' are the complete elliptic integrals of the first kind.

for $k^2 = \frac{1}{2} + \epsilon^2, \quad K \approx K' \approx \frac{\Gamma(1/4)^2}{4\pi^{1/2}} = 1.854\ldots$

The imaginary time period defines a temperature.

$$\frac{\hbar}{k_{\mathrm{B}} T_{\mathrm{CGF}}} = 4K'a$$

The initial state of the SM fluctuations is the natural thermal state whose correlation functions are periodic in imaginary time with period $4K'ai$. The CGF is a thermal bath for the fluctuations.

The electro-weak transition

$$\frac{1}{\hbar}S_{\text{Higgs}} = \int \left[\frac{1}{a^2}D_\mu\phi^\dagger D^\mu\phi + \frac{1}{2}\lambda^2\left(\phi^\dagger\phi - \frac{v^2}{2}\right)^2\right] a^4\sqrt{-g}\,d^4x$$

$$\lambda^2 = 0.258$$
$$m_{\text{Higgs}} = \hbar\lambda v = 125\ \text{GeV}$$

$$D_\mu\phi^\dagger D^\mu\phi = \partial_\mu\phi^\dagger\partial^\mu\phi + \frac{1}{2}b(z)(\partial^i\phi^\dagger\gamma_i\phi - \phi^\dagger\gamma_i\partial^i\phi) + \frac{3}{4}b(z)^2\phi^\dagger\phi$$

$$\nabla_i = \partial_i + \epsilon\gamma_i \quad\rightarrow\quad \partial_i \qquad \text{assuming } \epsilon \ll 1$$

The energy in the CGF and the Higgs field at $\phi = 0$ drives an expanding universe. The CGF oscillates much faster than the expansion so the adiabatic approximation is accurate.

Averaging b and b^2 over the oscillation gives the effective potential for ϕ.

$$V_{\text{eff}}(\phi) = \frac{\lambda^2 v^4}{8} + \left(\frac{3}{4}\frac{\langle\, b^2\,\rangle}{a^2} - \frac{\lambda^2 v^2}{2}\right)\phi^\dagger\phi + \frac{\lambda^2}{2}(\phi^\dagger\phi)^2$$

$$\langle\, b\,\rangle = 0 \qquad \langle\, b^2\,\rangle = \frac{1}{4K}\int_0^{4K} k^2\,\text{cn}^2(z,k)\,dz = \frac{\pi}{4K^2}$$

The quadratic term in the potential,

$$\left(\frac{3}{4}\frac{\langle\, b^2 \,\rangle}{a^2} - \frac{\lambda^2 v^2}{2}\right)\phi^\dagger\phi$$

is positive when $a(z)$ is small, so $\phi = 0$ is stable at early times.

The quadratic term turns negative when $a(z)$ reaches a_{EW}.

$$a_{\mathrm{EW}} = \left(\frac{3}{2}\frac{\langle\, b^2 \,\rangle}{\lambda^2 v^2}\right)^{\frac{1}{2}} = \frac{(6\pi)^{\frac{1}{2}}}{4K\lambda v} = 0.5854\,\frac{\hbar}{m_{\mathrm{Higgs}}} = 3.08\times 10^{-27}\,\mathsf{s}$$

m_{Higgs} is the CGF energy scale.

$$\hat{a} = \frac{m_{\mathrm{Higgs}}\, a}{\hbar} \qquad \hat{a}_{\mathrm{EW}} = 0.5854$$

Classical solution after a_{EW}

After $a(z) = a_{\mathrm{EW}}$ the Higgs field moves away from $\phi = 0$ towards its vacuum expectation value, tracking the minimum of the effective potential.

$$(\phi^\dagger \phi)_0 = \frac{v^2}{2} - \frac{3}{4\lambda^2} \frac{\langle\, b^2 \,\rangle}{a^2}$$

The CGF continues to oscillate but now with action and dimensionless energy

$$\frac{1}{\hbar} S^{\mathrm{eff}}_{\mathrm{gauge}} = \frac{6\pi^2}{g^2 \epsilon^3} \int \left[-\frac{1}{2} \left(\frac{db}{dz} \right)^2 + \frac{1}{2}\mu^2 b^2 + \frac{1}{2} b^4 \right] dz \qquad \mu^2 = \frac{1}{2} g^2 a^2 (\phi^\dagger \phi)_0$$

$$E_{\mathrm{CGF}} = \frac{1}{2} \left(\frac{db}{dz} \right)^2 + \frac{1}{2}\mu^2 b^2 + \frac{1}{2} b^4$$

The parameters μ^2 and E_{CGF} change slowly as the universe expands.

The adiabatic approximation remains valid.

The classical solution is again a Jacobi elliptic function,

$$b(z) = \frac{k\,\mathrm{cn}(u,k)}{\alpha} \qquad dz = \alpha du \qquad \alpha^2\mu^2 = 1 - 2k^2 \qquad \alpha^4 E_{\mathrm{CGF}} = \frac{k^2(1-k^2)}{2}$$

now parametrized by slowly changing k^2 and α instead of μ^2 and E_{CGF}.

A classical identity gives

$$\alpha^2\langle\, b^2 \,\rangle = \frac{1}{4K}\int_0^{4K} \alpha^2 b^2\, du = \frac{1}{4K}\int_0^{4K} k^2\,\mathrm{cn}^2(u,k)\, du = k^2 - 1 + \frac{E}{K}$$

where E is the complete elliptic integral of the second kind.

The equations for $(\phi^\dagger\phi)_0$ and μ^2 combine to give the scale $\hat{a}$.

$$\alpha^2\hat{a}^2 = \frac{3}{2}\alpha^2\langle\, b^2 \,\rangle + \frac{4\lambda^2}{g^2}\alpha^2\mu^2$$

The time evolution is now completely parametrized by k^2 and α (but one too many variables).

Adiabatic invariant

The adiabatic invariant

$$\oint p\,dq$$

is a constant of the motion for an adiabatically evolving oscillator q.

The adiabatic equation for the CGF is

$$\frac{(1-k^2)K+(2k^2-1)E}{\alpha^3} = \text{constant} \quad \text{with} \;\; \alpha = 1 \;\; \text{at} \;\; k^2 = 1/2$$

This determines α in terms of k^2. The time evolution is now parametrized by k^2 alone.

As k^2 evolves from $1/2$ to 0, the scale a goes from $a_{\rm EW}$ to ∞ and $\phi^\dagger\phi$ goes from 0 to $v^2/2$.

CGF equation of state

The energy-momentum tensor is $\mathrm{SO}(4)$-symmetric so the CGF is a perfect fluid.

The density ρ_{CGF} and pressure p_{CGF} are obtained by substituting the classical solutions in the energy-momentum tensor. Define the dimensionless density and pressure

$$\hat{\rho}_{\mathrm{CGF}} = \frac{\rho_{\mathrm{CGF}}}{\rho_b} \qquad \hat{p}_{\mathrm{CGF}} = \frac{p_{\mathrm{CGF}}}{\rho_b} \qquad \rho_b = \frac{m_{\mathrm{Higgs}}^4}{\hbar^3} = 5.68 \times 10^{28}\,\frac{\mathrm{kg}}{\mathrm{m}^3}$$

Before a_{EW}, the gauge field is pure radiation while ϕ contributes vacuum energy.

$$\hat{\rho}_{\mathrm{CGF}}(\hat{a}) = \frac{3}{8g^2}\frac{1}{\hat{a}^4} + \frac{1}{8\lambda^2} \qquad \hat{p}_{\mathrm{CGF}}(\hat{a}) = \frac{1}{8g^2}\frac{1}{\hat{a}^4} - \frac{1}{8\lambda^2}$$

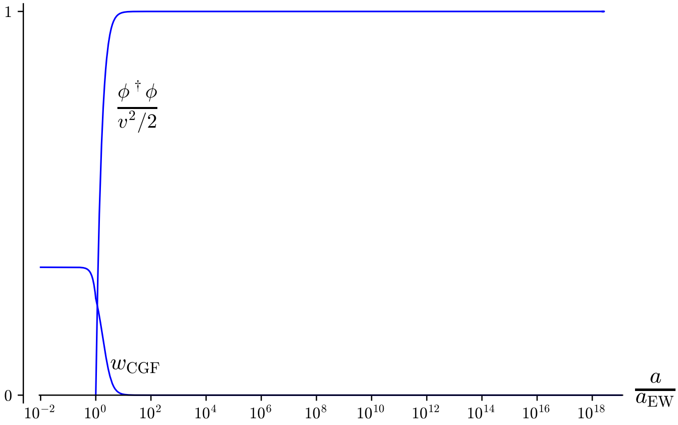
After a_{EW},

$$\hat{\rho}_{\mathrm{CGF}}(k^2) = \frac{1}{\hat{a}^4}\left(\frac{3E_{\mathrm{CGF}}}{g^2} + \frac{9\langle b^2 \rangle^2}{32\lambda^2}\right) \quad \hat{p}_{\mathrm{CGF}}(k^2) = \frac{1}{\hat{a}^4}\left(\frac{E_{\mathrm{CGF}} - \mu^2\langle b^2 \rangle}{g^2} - \frac{9\langle b^2 \rangle^2}{32\lambda^2}\right)$$

The equation of state relating $\hat{p}$ to $\hat{\rho}$ is determined implicitly.

CGF as dark matter

The equation of state parameter $w = \frac{p}{\rho}$ evolves as the universe expands.



The CGF is a nonrelativistic perfect fluid, $w_{\rm CGF} \approx 0$, from about $10\, a_{\rm EW}$ or $10^2\, a_{\rm EW}$ onward.

The CGF is cold dark matter.

Present flatness

Friedmann equation $\frac{H^2}{H_0^2} = \Omega_m + \Omega_\Lambda + \Omega_{\text{curvature}}$ $H = \frac{1}{a}\frac{da}{dt} = \frac{1}{a^2}\frac{da}{dz}$

$$\rho_c = \frac{3H_0^2}{\kappa} \qquad \Omega_m = \frac{\rho_m}{\rho_c} \qquad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} \qquad -\Omega_{\text{curvature}} = \frac{1}{H_0^2}\frac{1}{R^2} = \frac{\epsilon^2}{H_0^2 a^2}$$

The dark energy density is $\Omega_\Lambda = 0.685$ (assumed due to the cosmological constant).

The present curvature is small, $|\Omega_{\text{curvature}}| < 0.001$.

The only matter is the CGF: $\rho_m = \rho_{\text{CGF}}$, $\Omega_m = \Omega_{\text{CGF}} = \frac{\rho_{\text{CGF}}}{\rho_c}$

$$\frac{H^2}{H_0^2} = \Omega_{\text{CGF}} + 0.685$$

The present time is identified by the condition $H = H_0$ which is the equation $\Omega_{\text{CGF}} = 0.315$.

Solving $\quad \Omega_{\rm CGF} = \dfrac{\rho_{\rm CGF}}{\rho_{\rm c}} = 0.315 \quad$ gives the present values

$$k_0^2 = 7.89 \times 10^{-56} \qquad a_0 = 4.54 \times 10^{18} a_{\rm EW} \qquad -\Omega_{\text{curvature}} = \frac{\epsilon^2}{H_0^2 a_0^2} = 1.07 \times 10^{51}\,\epsilon^2$$

The present flatness $|\Omega_{\text{curvature}}| < 0.001$ is the initial energy condition

$$\epsilon^2 < 10^{-54} \qquad \hat{E}_{\rm CGF} = \frac{1}{8\epsilon^2} > 10^{107}$$

The dimensionless energy $\hat{E}_{\rm CGF}$ is the only adjustable parameter in the initial state.

If $\hat{E}_{\rm CGF} > 10^{107}$ then ϵ is so small that no local physics depends on the value of $\hat{E}_{\rm CGF}$.

In effect there are no adjustable parameters.

Having in hand the equation of state of the classical CGF allows solving the TOV stellar structure equations to find the possible stars made of the CGF.

Density fluctuations in the CGF presumably collapsed gravitationally to form self-gravitating bodies of which the simplest are spherically symmetric non-rotating stars governed by the TOV equations.

The gravitational scales are set by the CGF density scale $\rho_b = m^4_{\mathrm{Higgs}}/\hbar^3$.

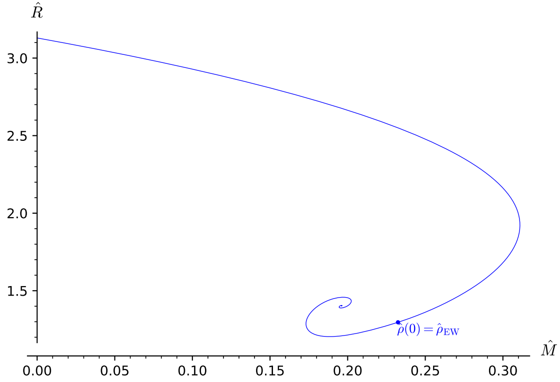
$$r_b = (4\pi G \rho_b)^{-1/2} = 4.34\,\mathrm{cm} \qquad m_b = G^{-1} r_b = 2.94 \times 10^{-5} \mathrm{M}_\odot = 5.26 \times 10^{42}\,\mathrm{J}$$

The dimensionless radius, mass, and binding energy are

$$\hat{R} = \frac{R}{r_b} \qquad \hat{M} = \frac{M}{m_b} \qquad \hat{\mathrm{BE}} = \frac{\mathrm{BE}}{m_b}$$

The TOV equations are solved numerically.

The next slides show plots of the mass-radius and mass-binding-energy curves.



The curve spirals inward, parametrized by increasing central density.

The dark matter universe is presumably populated with such stars.

The abundance distribution of their masses is a fluctuation calculation still to be done.

$$M = (3 \times 10^{-5} \mathrm{M}_{\odot})\hat{M} \qquad R = (4\,\mathrm{cm})\hat{R}$$

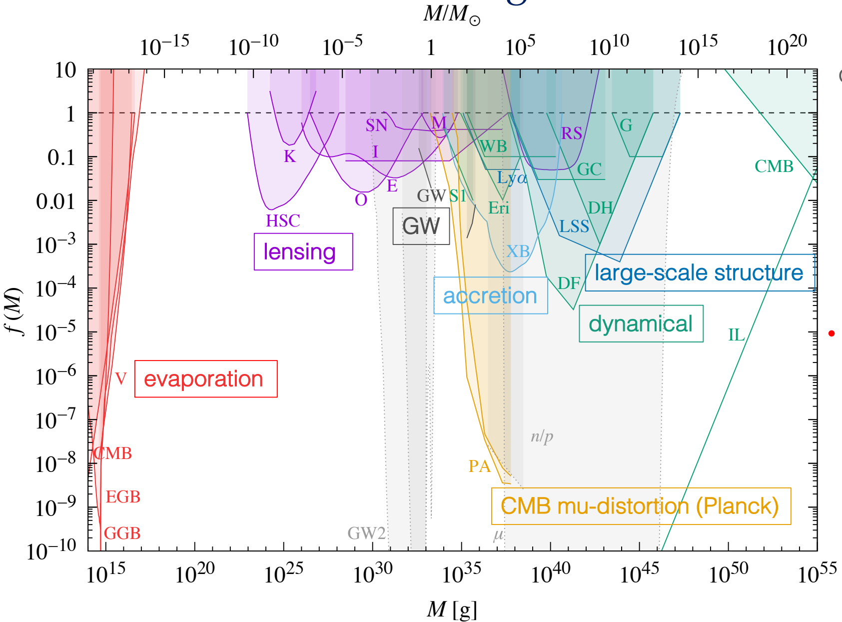
Microlensing puts an upper limit $10^{-11}\mathrm{M}_{\odot}$ on such compact dark matter objects as the halos.

See the next slide copied from Masahiro Takada, AstroDark-2021.

So the halos must consist mostly of stars of radius $R = 13.6\,\mathrm{cm}$ at the low mass end of the curve.

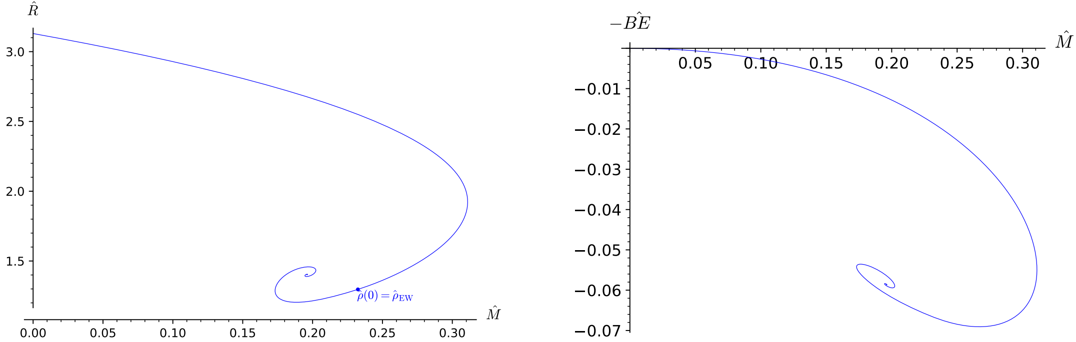
It seems challenging to detect dark matter in such a form. The rapidly oscillating CGF will likely have no significant non-gravitational interactions.

Current status: excluded regions of PBHs



Carr, Kohri, Sendouda & Yokoyama 20

- Assume that all experiments are null results (shaded regions are excluded regions of PBH assuming a monochromatic mass function in x-axis)



The binding energy curve shows the possibility of metastable dark matter stars that could undergo explosive collapse to smaller radius, emitting a burst of gravitation energy on the order of 10^{41} J in 10^{-10}s. Such bursts might be observable, perhaps taking place in the centers of ordinary stars or out in the open.

A dark matter star near the asymptotic fixed point of the spiral has high central density.

Such objects would probe energy scales physics beyond the SM.

The classical CGF cosmology has no ordinary matter, only dark matter.

The dark matter is the CGF.

The actual universe is a perturbation of this dark matter universe by the fluctuations of the SM fields.

Constructing the initial fluctuations and calculating their time evolution is a well-defined calculation within the SM and GR.

Detailed quantitative checks of the theory depend on that calculation, which remains to be done.