

# Where does quantum field theory come from?

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## Projects

- Geometry of the rg of 2-d quantum field theory  
(with Anatoly Konechny)
  - 2004 Gradient formula for the beta-function of 2-d boundary qft  
(implying the  $g$ -theorem)
  - 2010 Gradient formula for the beta-function of 2-d qft
  - 2011 Curvature formula for the space of 2-d cfts (in preparation)
- Large-scale quantum computation in critical quantum circuits  
(in abeyance)
  - 2005 Entropy flow in near-critical quantum circuits
- Where does quantum field theory come from?  
[1979, 1985, 2003, 2010]

## Renormalization of the 2-d nonlinear model

The fields are maps  $X^\mu(z, \bar{z})$  from 2-d to the target manifold.

$$Z = \int \mathcal{D}X e^{-A[X]} \quad A[X] = \int d^2z g_{\mu\nu}(X) \partial X^\mu \bar{\partial} X^\nu$$

the 2-d couplings given by a metric  $g_{\mu\nu}(X)$  on the target manifold.

The renormalization group

$$\Lambda \frac{\partial}{\partial \Lambda} g_{\mu\nu} = R_{\mu\nu} + O(R^2)$$

acts at short distance in 2-d, driving to a fixed point,  $R_{\mu\nu} = 0$ .

The 2-d renormalization group *produces* classical field theory.

What could produce quantum field theory?

Interested because it might produce something observably different from canonical quantization, . . . .

## $\beta = 0$ in string theory

The string worldsurface is described by a 2-d qft.

$$\begin{aligned} &\{\text{2-d couplings of the string worldsurface}\} \\ &= \{\text{background spacetime fields } g_{\mu\nu}, A_\mu, \psi^\alpha, \phi, \dots\} \end{aligned}$$

$\beta = 0$  as consistency condition on the string worldsurface

possibility of a realistic field theory produced by the 2-d rg

Is there a *quantum* string background?

## The $\lambda$ -model

The 2-d coupling constants:

$$\{\lambda^i\} = \{\text{modes of the spacetime fields } g_{\mu\nu}, A_\mu, \psi^\alpha, \phi, \dots\}$$

are allowed to vary in 2-d, becoming local sources  $\lambda^i(z, \bar{z})$ :

$$Z[\lambda] = \langle e^{-\int d^2z \lambda^i(z, \bar{z}) \phi_i(z, \bar{z})} \rangle$$

and then are set fluctuating:

$$\int \mathcal{D}\lambda e^{-\int d^2z g^{-2} G_{ij}(\lambda) \partial\lambda^i \bar{\partial}\lambda^j} Z[\lambda], \quad \mathcal{D}\lambda = \prod_{z, \bar{z}} d\rho(\lambda(z, \bar{z}))$$

where  $G_{ij}(\lambda)$  is the natural metric on the spacetime fields,  $g$  is the coupling constant, and  $d\rho(\lambda)$  is a measure on the spacetime fields – the *a priori* measure.

## The rg of the $\lambda$ -model

Under the 2-d rg, the *a priori* measure  $d\rho(\lambda)$  diffuses, because of the fluctuations of the  $\lambda^i(z, \bar{z})$ .

But the  $\lambda^i(z, \bar{z})$  are not exactly dimensionless fields, since

$$\Lambda \frac{\partial}{\partial \Lambda} \lambda^i = \beta^i(\lambda)$$

so the diffusion is driven by  $\beta^i(\lambda)$ , driving towards  $\beta^i = 0$ .

The equilibrium measure is a functional integral on the spacetime fields – a quantum field theory – with equation of motion  $\beta = 0$ .

Like stochastic quantization. Except the process is not ordinary diffusion, because the fluctuations are in 2-d, not 1-d.

The  $\lambda$ -model operates at short distance in 2-d: a quantum 2-d rg.

It produces an effective worldsurface (quantum string background).

In spacetime, a nice reconciliation:

at large distance: a qft (the *a priori* measure)

at small distance: a string S-matrix (the effective worldsurface)

The  $\lambda$ -model is *derived* from string theory, expressing the effect of tiny handles in the worldsurface.

In principle, the target manifold of the  $\lambda$ -model is the space of all string backgrounds, and the fluctuations of the  $\lambda$ -model should produce *the* string background.

Figuring this out seems impractically ambitious for someone who would like to find out – in his own lifetime – if the theory describes the real world.

So pivot. Simply *assume* that we somehow arrive at a 4-d spacetime with the fields and classical action of the standard model, so the  $\lambda$ -model now operates with this  $\{\lambda^i\}$  and  $\beta^i(\lambda)$ .

The  $\lambda$ -model produces a functional integral on the fields of the standard model that might differ from canonical quantization because of non-perturbative 2-d effects.

Ask: does the  $\lambda$ -model predict anything different from canonical quantization, something that might be seen in experiment?

## Semi-classical 2-d effects

$\pi_1(\text{target manifold}) \implies$  winding fields

$\pi_2(\text{target manifold}) \implies$  2-d instantons

$\pi_1(SU(2) \text{ gauge fields in } \mathbb{R}^4) = \mathbb{Z}_2$

$\pi_2(SU(2) \text{ gauge fields in } \mathbb{R}^4) = \mathbb{Z}_2$

$\pi_2(SU(3) \text{ gauge fields in } \mathbb{R}^4) = \mathbb{Z}$

The minimal non-trivial loops of  $SU(2)$  gauge fields:

For  $x_{1,2} \in \mathbb{R}^4$ , put a zero-size instanton at  $x_1$  and a zero-size anti-instanton at  $x_2$ . The space of relative internal orientations is  $SU(2)/\{\pm 1\}$ .

The non-trivial loops in this  $SU(2)/\{\pm 1\}$  are the minimal non-trivial loops of  $SU(2)$  gauge fields.

The minimal nontrivial 2-spheres of gauge fields are made similarly.

These non-trivial loops have zero-length, so they should give rise to new massless fields (a multiplet of fields, because of the fermion zero-modes in the instanton cores).

Problems:

- Will 2-d quantum corrections give large masses to these fields? (do the fermionic zero-modes help? need spacetime susy?)
- The new fields will be bi-local,  $\Phi(x_1, x_2)$  – how could the Wick rotation be unitary?
- How to calculate the couplings to ordinary fields?
- Investigate non-canonical spacetime couplings coming from the 2-d instantons – the minimal non-trivial 2-spheres of gauge fields.