The CGF cosmology

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The CGF cosmology is a complete fundamental theory of the Standard Model cosmological epoch — the period when the SM and General Relativity have been the complete laws of physics (supposing dark matter to be a SM effect). The CGF cosmology starts from a simple initial condition — a semi-classical state of the SM+GR completely specified by a global $\mathrm{Spin}(4) \times \mathrm{U}(1) \times \mathrm{SU}(3)$ symmetry and by a high initial energy. No physics beyond the SM is invoked. The initial condition determines the complete cosmology. The symmetric modes of the SM+GR fields are the scale factor $a(t)$ and the cosmological gauge field (CGF) which is a single global mode $b(t)$ of the $\mathrm{SU}(2)$-weak gauge field. The classical CGF oscillates anharmonically at high frequency. Macroscopically the CGF is a perfect fluid with $w \approx 0$ and only gravitational interactions, i.e. dark matter. The CGF explains dark matter as a SM effect. Ordinary matter is a sub-leading correction to the classical dark matter universe. The CGF causes an electro-weak transition in an expanding universe. Homogeneity and isotropy follow from the fundamental Spin(4) symmetry. Large initial energy implies near flatness in the present universe. The value of the initial energy does not otherwise affect local cosmology so the CGF cosmology has no adjustable parameters.

SM+GR

Suppose dark matter to be a SM effect (to be justified). Then the SM+GR are the complete laws of physics up to energy $\sim 1\,\mathrm{TeV}$.

- very well verified up to $\sim 1\,\mathrm{TeV}$.
- no experimental evidence of physics beyond the SM+GR up to $\sim 1\,\mathrm{TeV}$.

General Relativity

- Einstein's equation with nonzero cosmological constant

Standard Model

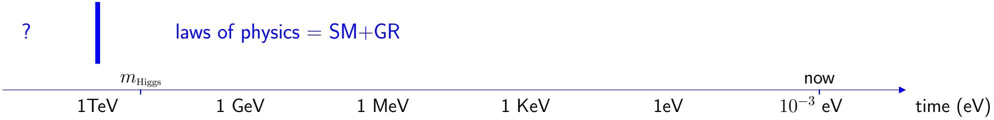
- the $\mathrm{U}(1) \times \mathrm{SU}(2) \times \mathrm{SU}(3)$ gauge fields plus the scalar Higgs field
- three generations of leptons and quarks
- with neutrino mixing

Neutrino mixing in the SM epoch is accurately described by phenomenological mixing parameters.

Neutrino mixing requires physics beyond the SM to be produced in a local quantum field theory, but that is not relevant to physics below $1\,\mathrm{TeV}$.

The Standard Model epoch

Parametrize cosmological time by characteristic energy:

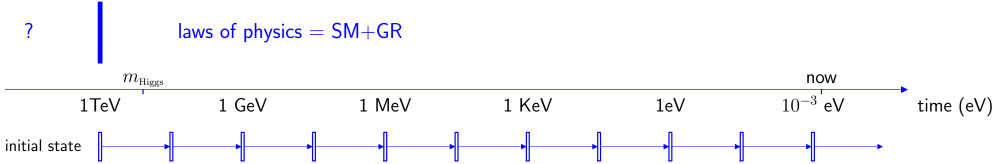


The SM epoch is the period from $\sim 1\,\mathrm{TeV}$ onward when the laws of physics were the SM+GR.

We know the laws of physics during the SM epoch.

We don't know the laws of physics before the SM epoch.

First principles cosmology of the SM epoch



?

laws of physics = SM+GR

m_{Higgs}

now

1TeV 1 GeV 1 MeV 1 KeV 1eV 10^{-3} eV time (eV)

initial state

A first principles cosmology is an initial state of the SM+GR at the beginning of the SM epoch.

The laws of physics governing the time evolution are known, so all cosmology in the SM epoch can be calculated from the initial state.

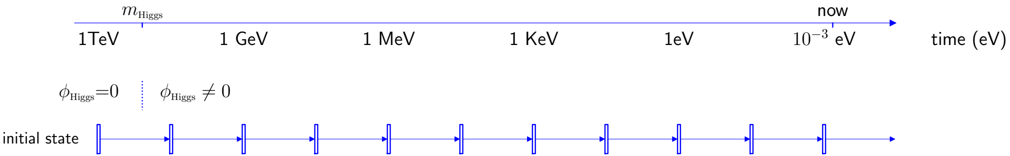
The project is to find a simple, natural initial state singled out by some basic principles.

The project is an alternative to cosmology based on speculation about physics above $1\,\mathrm{TeV}$.

There is no reason to expect that the SM universe evolved from a simple initial state.

But if it works . . .

Top-down stategy



The initial state should:

1. Start in the unbroken phase $\phi_{\text{Higgs}} = 0$.
2. Undergo an electro-weak transition to the broken phase $\phi_{\text{Higgs}} \neq 0$.
3. Expand, containing mostly dark matter, a SM effect.
4. Produce a homogeneous, isotropic, nearly flat universe.
5. Produce all the detailed structure of the actual universe.

The strategy is to get the universe correct in broad strokes 1-4 first, then check the details.

Everything can be calculated in the SM+GR starting from the initial condition.

Any failure kills the theory. The theory is maximally falsifiable.

First principles

1. The initial state is semi-classical, i.e. a classical solution of GR+SM plus small quantum fluctuations.
2. The initial state has a certain $\mathrm{Spin}(4) \times \mathrm{U}(1) \times \mathrm{SU}(3)$ global symmetry.
3. The initial energy is $> 10^{107}$ in natural units.

These three principles turn out to determine the initial state uniquely (up to the initial energy).

The present curvature of the universe varies inversely with the initial energy, so the initial energy must be large to agree with the observed upper bound on the curvature.

No other observable depends on the initial energy so there are effectively no adjustable parameters in the CGF cosmology.

If a non-zero curvature is ever measured, only a positive value will be consistent with the CGF cosmology. The value of the curvature will fix the initial energy.

Spin(4)

SO(4) is the symmetry group of the oriented unit 3-sphere in euclidean 4-space.

Spin(4) is the simply connected covering group of SO(4).

$$S^3 = \mathrm{SU}(2) \qquad \mathrm{Spin}(4) = \mathrm{SU}(2) \times \mathrm{SU}(2)$$

$$(g_L, g_R) \colon g \mapsto g_L g g_R^{-1}$$

$$\mathrm{SO}(4) = \mathrm{SU}(2) \times \mathrm{SU}(2)/\{(-1,-1)\}$$

First steps:

1. Specify how Spin(4) acts on the classical SM+GR fields.
2. Find the Spin(4)-invariant classical degrees of freedom.
3. Derive their classical equations of motion in the SM+GR.

Spin(4)-symmetric GR

Spin(4) acting as a space-time symmetry group requires space $= S^3$ and space-time $= \mathbb{R} \times S^3$.

Spin(4) acts as SO(4) on S^3.

Homogeneity and isotropy are simply symmetries of the initial state.

Deeper explanation of homogeneity and isotropy requires explaining the origin of the fundamental Spin(4) symmetry. This is a question about physics before the SM epoch, about which we are not speculating.

The only Spin(4)-symmetric degree of freedom in the space-time metric is the radius of the universe.

$$ds^2 = R(t)^2 \left(-dt^2 + \hat{g}_{ij}(\hat{x}) d\hat{x}^i d\hat{x}^j\right) \qquad \hat{g}_{ij}(\hat{x}) = \text{the metric of the unit } S^3 \subset \mathbb{R}^4$$

Spin(4)-invariant U(1) and SU(3) gauge fields

To specify how Spin(4) acts on the gauge fields and the Higgs field we to specify how Spin(4) acts on the SM gauge bundles over spacetime.

$$\begin{aligned} V_{\mathbf{1}} &= \text{the U(1) singlet at each point } x \text{ in space-time} \\ V_{\mathbf{2}} &= \text{the SU(2) doublet at each point } x \\ V_{\mathbf{3}} &= \text{the SU(3) triplet at each point } x \end{aligned}$$

The gauge fields are covariant derivatives in the gauge bundles: $D_\mu^{\mathrm{U}(1)} \quad D_\mu^{\mathrm{SU}(2)} \quad D_\mu^{\mathrm{SU}(3)}$

The U(1) and SU(3) gauge bundles are trivial product bundles.

$$V_{\mathbf{1}} = (\mathbb{R} \times S^3) \times \mathbb{C} \qquad V_{\mathbf{3}} = (\mathbb{R} \times S^3) \times \mathbb{C}^3$$

Spin(4) acts only on S^3, independent of time, not touching the singlet or the triplet vector.

There are no Spin(4)-invariant degrees of freedom in the U(1) or SU(3) gauge fields.

The Spin(4)-symmetric covariant derivatives are $D_\mu^{\mathrm{U}(1)} = \partial_\mu$ and $D_\mu^{\mathrm{SU}(3)} = \partial_\mu$.

Space-time spinors

Let S be the bundle of Dirac spinors on space-time.

At each point x there is a 4-dimensional complex vector space S_x of spinors at x.

The Dirac matrices $\gamma_\mu(x)$ at x act on the vector space of spinors S_x at x.

$$\gamma_\mu(x)\,\gamma_\nu(x) + \gamma_\nu(x)\,\gamma_\mu(x) = 2g_{\mu\nu}(x)$$

The spinor bundle splits into the two bundles of chiral spinors.

$$S = S^+ \oplus S^- \qquad \gamma_5(x) = \pm 1 \text{ on } S_x^\pm$$

Each space $S_x^\pm$ of chiral spinors at x is a 2-dimensional vector space.

Spin(4) acts on the spinors on S^3 so it acts on the chiral spinors $S_\pm$ on space-time.

SU(2) structures on chiral spinors

Space-time chiral spinors and SU(2)

The SU(2) bundle is identified with one of the chiral spinor bundles.

$$V_{\mathbf{2}} = S_{+} \text{ (or } S_{-}\text{)}$$

So now Spin(4) acts on $V_{\mathbf{2}}$.

Spin(4)-invariant Higgs and SU(2) gauge fields

The most general Spin(4)-symmetric SU(2) gauge field is

$$D_\mu^{\mathrm{SU}(3)} = \nabla_\mu + \frac{1}{4}\left[\gamma_0,\, \gamma_\mu\right] b(t)$$

where ∇_μ is the metric covariant derivative on spinors.

The single Spin(4)-invariant global degree of freedom $b(t)$ is the cosmological gauge field.

The Higgs field $\phi(x)$ lives in $V_{\mathbf{2}} = S_\pm$. The only Spin(4)-symmetric spinor is 0 so $\phi(x) = 0$.

There are no Spin(4)-invariant degrees of freedom in the Higgs field.

The SU(2) gauge bundle of the SM is identified with the spinor bundle.

This defines the action of Spin(4) on the space-time metric and on the SM fields.

The cosmology at leading order is a classical solution of the SM equations of motion invariant under this action of Spin(4).

The only nontrivial $\mathrm{Spin}(4)$-symmetric classical fields are

- the space-time metric $\quad ds^2 = R(\hat{t})^2 \left(-d\hat{t}^{\,2} + \hat{g}_{ij}(\hat{x}) d\hat{x}^i d\hat{x}^j\right)$

  $\hat{g}_{ij}(\hat{x})$ is the metric on the unit $S^3 \subset \mathbb{R}^4$.

  $\hat{t}$ is the conformal time.

- the Higgs field $\phi = 0$ (the only Spin(4)-symmetric spinor)

- the $\mathrm{SU}(2)$-weak gauge field $\quad D_i = \hat{\nabla}_i + \hat{b}(\hat{t})\hat{\gamma}_i(\hat{x})$

  $\hat{\gamma}_i(\hat{x})$ are the Dirac matrices on S^3.

  $\hat{\nabla}_i$ is the metric covariant derivative on spinors.

The $\mathrm{Spin}(4)$-symmetric degrees of freedom are

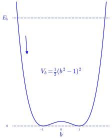
- $R(\hat{t})$ the radius of the universe

- $\hat{b}(\hat{t})$ the cosmological gauge field (CGF)

SM classical dynamics

$$\int \frac{1}{2g^2}\mathrm{tr}(-F_{\mu\nu}F^{\mu\nu})\sqrt{-g}\,d^4x \;=\; \frac{6\pi^2}{g^2}\int\left[-\frac{1}{2}\left(\frac{d\hat{b}}{d\hat{t}}\right)^2+\frac{1}{2}\left(\hat{b}^2-1\right)^2\right]d\hat{t} \qquad g^2 = 0.426$$

anharmonic
oscillator



dimensionless
conserved energy
(units $\hbar/R$)

$$\hat{E}_{\mathrm{CGF}} = \frac{1}{2}\left(\frac{d\hat{b}}{d\hat{t}}\right)^2+\frac{1}{2}\left(\hat{b}^2-1\right)^2$$

The equation of motion is solved by a Jacobi elliptic function. (This z is not the redshift.)

$$\hat{b}(\hat{t}) = \frac{k\,\mathrm{cn}(z,k)}{\epsilon} \qquad z = \frac{\hat{t}}{\epsilon} \qquad \hat{E}_{\mathrm{CGF}} = \frac{1}{8\epsilon^4} \qquad k^2 = \frac{1}{2}+\epsilon^2$$

The initial energy condition $\hat{E}_{\mathrm{CGF}} > 10^{107}$ which is $\epsilon < 10^{-27}$ comes later as a physical condition needed to produce the observed flatness of the present universe.

Local coordinates

Scale by ϵ: $\quad x^0 = z = \dfrac{\hat{t}}{\epsilon} \qquad x^i = \dfrac{\hat{x}^i}{\epsilon} \qquad a(z) = \epsilon R(\hat{t}) \qquad b(z) = \epsilon \hat{b}(\hat{t}) = k\,\mathrm{cn}(z,k)$

$$ds^2 = a(z)^2\left(-dz^2 + g_{ij}(x)dx^i dx^j\right) \qquad [\gamma_i,\,\gamma_j] = -\frac{1}{2}g_{ij}$$

x^i is coordinate, g_{ij} is the metric on the 3-sphere of radius $1/\epsilon$.

$$D_i = \frac{\partial}{\partial x_i} + \epsilon\gamma_i + b(z)\gamma_i \qquad \frac{1}{\hbar}S_{\mathrm{YM}} = \frac{6\pi^2}{g^2\epsilon^3}\int\left[-\frac{1}{2}\left(\frac{db}{dz}\right)^2 + \frac{1}{2}\left(b^2-\epsilon^2\right)^2\right]dz$$

Local physics is independent of ϵ when ϵ is very small.

Local physics does not depend on the value of $\hat{E}_{\mathrm{CGF}} \gg 1$. In effect there are no free parameters.

The CGF is semi-classical if $\epsilon \ll 1$.

CGF temperature and initial thermal state

$b(z) = k\,\mathrm{cn}(z,k)$ is doubly periodic in z.

real period $\begin{array}{l} z \sim z + 4K \\ t \sim t + 4Ka \end{array}$ imaginary period $\begin{array}{l} z \sim z + 4K'i \\ t \sim t + 4K'ai \end{array}$

K, K' are the complete elliptic integrals of the first kind.

for $k^2 = \frac{1}{2} + \epsilon^2, \quad K \approx K' \approx \frac{\Gamma(1/4)^2}{4\pi^{1/2}} = 1.854\ldots$

The imaginary time period defines a temperature. $\frac{\hbar}{k_{\mathrm{B}} T_{\mathrm{CGF}}} = 4K'a$

The initial state of the SM fluctuations is the natural thermal state whose correlation functions are periodic in imaginary time with period $4K'ai$. The CGF is a thermal bath for the fluctuations.

The electro-weak transition

$$\frac{1}{\hbar} S_{\text{Higgs}} = \int \left[\frac{1}{a^2} D_\mu \phi^\dagger D^\mu \phi + \frac{1}{2} \lambda^2 \left(\phi^\dagger \phi - \frac{v^2}{2} \right)^2 \right] a^4 \sqrt{-g}\, d^4 x$$

$$\lambda^2 = 0.258$$

$$m_{\text{Higgs}} = \hbar \lambda v = 125 \text{ GeV}$$

$$D_\mu \phi^\dagger D^\mu \phi = \partial_\mu \phi^\dagger \partial^\mu \phi + \frac{1}{2} b(z) (\partial^i \phi^\dagger \gamma_i \phi - \phi^\dagger \gamma_i \partial^i \phi) + \frac{3}{4} b(z)^2 \phi^\dagger \phi$$

$$\nabla_i = \partial_i + \epsilon \gamma_i \quad \to \quad \partial_i \qquad \text{assuming } \epsilon \ll 1$$

The energy in the CGF and the Higgs field at $\phi = 0$ drives an expanding universe. The CGF oscillates much faster than the expansion so the adiabatic approximation is accurate.

Averaging b and b^2 over the oscillation gives the effective potential for ϕ.

$$V_{\text{eff}}(\phi) = \frac{\lambda^2 v^4}{8} + \left(\frac{3}{4} \frac{\langle b^2 \rangle}{a^2} - \frac{\lambda^2 v^2}{2} \right) \phi^\dagger \phi + \frac{\lambda^2}{2} (\phi^\dagger \phi)^2$$

$$\langle b \rangle = 0 \qquad \langle b^2 \rangle = \frac{1}{4K} \int_0^{4K} k^2 \, \mathrm{cn}^2(z, k)\, dz = \frac{\pi}{4K^2}$$

The quadratic term in the potential,

$$\left(\frac{3}{4}\frac{\langle\, b^2\,\rangle}{a^2} - \frac{\lambda^2 v^2}{2}\right)\phi^\dagger\phi$$

is positive when $a(z)$ is small, so $\phi = 0$ is stable at early times.

The quadratic term turns negative when $a(z)$ reaches a_{EW}.

$$a_{\mathrm{EW}} = \left(\frac{3}{2}\frac{\langle\, b^2\,\rangle}{\lambda^2 v^2}\right)^{\frac{1}{2}} = \frac{(6\pi)^{\frac{1}{2}}}{4K\lambda v} = 0.5854\,\frac{\hbar}{m_{\mathrm{Higgs}}} = 3.08\times 10^{-27}\,\mathsf{s}$$

m_{Higgs} is the CGF energy scale.

$$\hat{a} = \frac{m_{\mathrm{Higgs}}a}{\hbar} \qquad \hat{a}_{\mathrm{EW}} = 0.5854$$

Classical solution after a_{EW}

After $a(z) = a_{\mathrm{EW}}$ the Higgs field moves away from $\phi = 0$ towards its vacuum expectation value, tracking the minimum of the effective potential.

$$(\phi^\dagger \phi)_0 = \frac{v^2}{2} - \frac{3}{4\lambda^2} \frac{\langle\, b^2 \,\rangle}{a^2}$$

The CGF continues to oscillate but now with action and dimensionless energy

$$\frac{1}{\hbar} S^{\mathrm{eff}}_{\mathrm{gauge}} = \frac{6\pi^2}{g^2 \epsilon^3} \int \left[-\frac{1}{2} \left(\frac{db}{dz} \right)^2 + \frac{1}{2} \mu^2 b^2 + \frac{1}{2} b^4 \right] dz \qquad \mu^2 = \frac{1}{2} g^2 a^2 (\phi^\dagger \phi)_0$$

$$E_{\mathrm{CGF}} = \frac{1}{2} \left(\frac{db}{dz} \right)^2 + \frac{1}{2} \mu^2 b^2 + \frac{1}{2} b^4$$

The parameters μ^2 and E_{CGF} change slowly as the universe expands.

The adiabatic approximation remains valid.

The classical solution is again a Jacobi elliptic function,

$$b(z) = \frac{k\,\mathrm{cn}(u,k)}{\alpha} \qquad dz = \alpha du \qquad \alpha^2\mu^2 = 1 - 2k^2 \qquad \alpha^4 E_{\mathrm{CGF}} = \frac{k^2(1-k^2)}{2}$$

now parametrized by slowly changing k^2 and α instead of μ^2 and E_{CGF}.

A classical identity gives

$$\alpha^2\langle\, b^2\,\rangle = \frac{1}{4K}\int_0^{4K} \alpha^2 b^2\, du = \frac{1}{4K}\int_0^{4K} k^2\,\mathrm{cn}^2(u,k)\, du = k^2 - 1 + \frac{E}{K}$$

where E is the complete elliptic integral of the second kind.

The equations for $(\phi^\dagger\phi)_0$ and μ^2 combine to give the scale $\hat{a}$.

$$\alpha^2\hat{a}^2 = \frac{3}{2}\alpha^2\langle\, b^2\,\rangle + \frac{4\lambda^2}{g^2}\alpha^2\mu^2$$

The time evolution is now completely parametrized by k^2 and α (but one too many variables).

Adiabatic invariant

The adiabatic invariant

$$\oint p\,dq$$

is a constant of the motion for an adiabatically evolving oscillator q.

The adiabatic equation for the CGF is

$$\frac{(1-k^2)K + (2k^2-1)E}{\alpha^3} = \text{constant} \quad \text{with} \;\; \alpha = 1 \;\; \text{at} \;\; k^2 = 1/2$$

This determines α in terms of k^2. The time evolution is now parametrized by k^2 alone.

As k^2 evolves from $1/2$ to 0, the scale a goes from a_{EW} to ∞ and $\phi^\dagger\phi$ goes from 0 to $v^2/2$.

CGF equation of state

The energy-momentum tensor is $\mathrm{SO}(4)$-symmetric so the CGF is a perfect fluid.

The density ρ_{CGF} and pressure p_{CGF} are obtained by substituting the classical solutions in the energy-momentum tensor. Define the dimensionless density and pressure

$$\hat{\rho}_{\mathrm{CGF}} = \frac{\rho_{\mathrm{CGF}}}{\rho_b} \qquad \hat{p}_{\mathrm{CGF}} = \frac{p_{\mathrm{CGF}}}{\rho_b} \qquad \rho_b = \frac{m_{\mathrm{Higgs}}^4}{\hbar^3} = 5.68 \times 10^{28}\,\frac{\mathrm{kg}}{\mathrm{m}^3}$$

Before a_{EW}, the gauge field is pure radiation while ϕ contributes vacuum energy.

$$\hat{\rho}_{\mathrm{CGF}}(\hat{a}) = \frac{3}{8g^2}\frac{1}{\hat{a}^4} + \frac{1}{8\lambda^2} \qquad \hat{p}_{\mathrm{CGF}}(\hat{a}) = \frac{1}{8g^2}\frac{1}{\hat{a}^4} - \frac{1}{8\lambda^2}$$

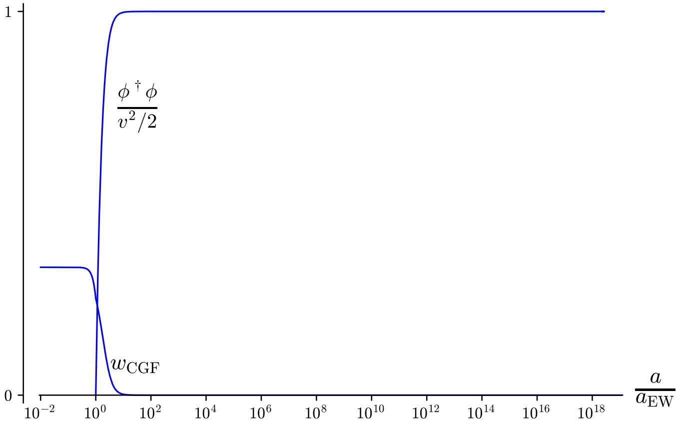
After a_{EW},

$$\hat{\rho}_{\mathrm{CGF}}(k^2) = \frac{1}{\hat{a}^4}\left(\frac{3E_{\mathrm{CGF}}}{g^2} + \frac{9\langle\, b^2 \,\rangle^2}{32\lambda^2}\right) \quad \hat{p}_{\mathrm{CGF}}(k^2) = \frac{1}{\hat{a}^4}\left(\frac{E_{\mathrm{CGF}} - \mu^2\langle\, b^2 \,\rangle}{g^2} - \frac{9\langle\, b^2 \,\rangle^2}{32\lambda^2}\right)$$

The equation of state relating $\hat{p}$ to $\hat{\rho}$ is determined implicitly.

CGF as dark matter

The equation of state parameter $w = \frac{p}{\rho}$ evolves as the universe expands.



The CGF is a nonrelativistic perfect fluid, $w_{\text{CGF}} \approx 0$, from about $10\, a_{\text{EW}}$ or $10^2\, a_{\text{EW}}$ onward.

The CGF is cold dark matter.

Present flatness

Friedmann equation $$\frac{H^2}{H_0^2} = \Omega_m + \Omega_\Lambda + \Omega_{\text{curvature}} \qquad H = \frac{1}{a}\frac{da}{dt} = \frac{1}{a^2}\frac{da}{dz}$$

$$\rho_{\mathrm{c}} = \frac{3H_0^2}{\kappa} \qquad \Omega_m = \frac{\rho_m}{\rho_{\mathrm{c}}} \qquad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{\mathrm{c}}} \qquad -\Omega_{\text{curvature}} = \frac{1}{H_0^2}\frac{1}{R^2} = \frac{\epsilon^2}{H_0^2 a^2}$$

The dark energy density is $\Omega_\Lambda = 0.685$ (assumed due to the cosmological constant).

The present curvature is small, $|\Omega_{\text{curvature}}| < 0.001$.

The only matter is the CGF: $$\rho_{\mathrm{m}} = \rho_{\mathrm{CGF}}, \qquad \Omega_m = \Omega_{\mathrm{CGF}} = \frac{\rho_{\mathrm{CGF}}}{\rho_{\mathrm{c}}}$$

$$\frac{H^2}{H_0^2} = \Omega_{\mathrm{CGF}} + 0.685$$

The present time is identified by the condition $H = H_0$ which is the equation $\Omega_{\mathrm{CGF}} = 0.315$.

Solving $\Omega_{\mathrm{CGF}} = \dfrac{\rho_{\mathrm{CGF}}}{\rho_{\mathrm{c}}} = 0.315$ gives the present values

$$k_0^2 = 7.89 \times 10^{-56} \qquad a_0 = 4.54 \times 10^{18} a_{\mathrm{EW}} \qquad -\Omega_{\text{curvature}} = \frac{\epsilon^2}{H_0^2 a_0^2} = 1.07 \times 10^{51}\, \epsilon^2$$

The present flatness $|\Omega_{\text{curvature}}| < 0.001$ is the initial energy condition

$$\epsilon^2 < 10^{-54} \qquad \hat{E}_{\mathrm{CGF}} = \frac{1}{8\epsilon^2} > 10^{107}$$

The dimensionless energy $\hat{E}_{\mathrm{CGF}}$ is the only adjustable parameter in the initial state.

If $\hat{E}_{\mathrm{CGF}} > 10^{107}$ then ϵ is so small that no local physics depends on the value of $\hat{E}_{\mathrm{CGF}}$.

In effect there are no adjustable parameters.

Dark matter stars

Having in hand the equation of state of the classical CGF allows solving the TOV stellar structure equations to find the possible stars made of the CGF.

Density fluctuations in the CGF presumably collapsed gravitationally to form self-gravitating bodies of which the simplest are spherically symmetric non-rotating stars governed by the TOV equations.

The gravitational scales are set by the CGF density scale $\rho_b = m_{\mathrm{Higgs}}^4/\hbar^3$.

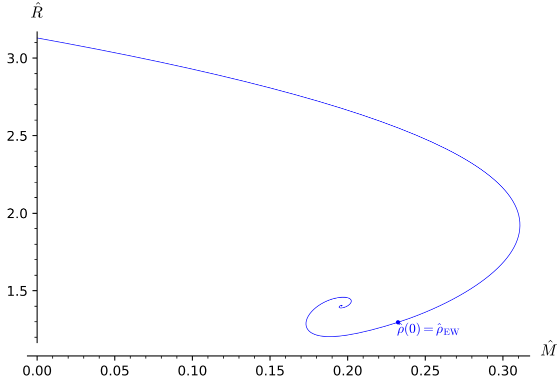
$$r_b = (4\pi G\rho_b)^{-1/2} = 4.34\,\mathrm{cm} \qquad m_b = G^{-1} r_b = 2.94\times 10^{-5}\mathrm{M}_\odot = 5.26\times 10^{42}\,\mathrm{J}$$

The dimensionless radius, mass, and binding energy are

$$\hat{R} = \frac{R}{r_b} \qquad \hat{M} = \frac{M}{m_b} \qquad \hat{\mathrm{BE}} = \frac{\mathrm{BE}}{m_b}$$

The TOV equations are solved numerically.

The next slides show plots of the mass-radius and mass-binding-energy curves.



The curve spirals inward, parametrized by increasing central density.

The dark matter universe is presumably populated with such stars.

The abundance distribution of their masses is a fluctuation calculation still to be done.

$$M = (3 \times 10^{-5} \mathrm{M}_\odot)\hat{M} \qquad R = (4\,\mathrm{cm})\hat{R}$$

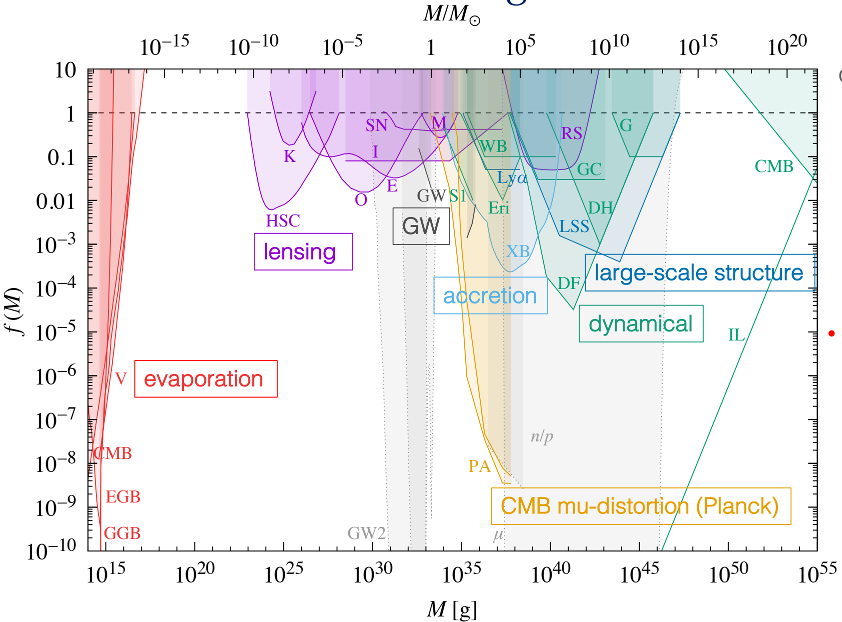
Microlensing puts an upper limit $10^{-11}\mathrm{M}_\odot$ on such compact dark matter objects as the halos.

See the next slide copied from Masahiro Takada, AstroDark-2021.

So the halos must consist mostly of stars of radius $R = 13.6\,\mathrm{cm}$ at the low mass end of the curve.

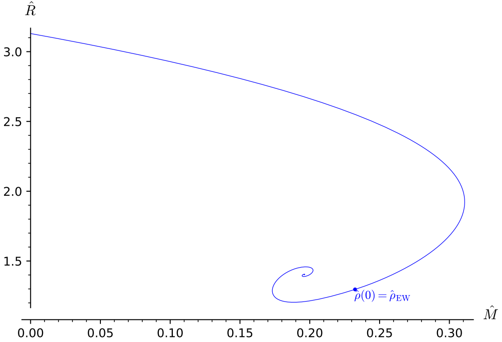
It seems challenging to detect dark matter in such a form. The rapidly oscillating CGF will likely have no significant non-gravitational interactions.

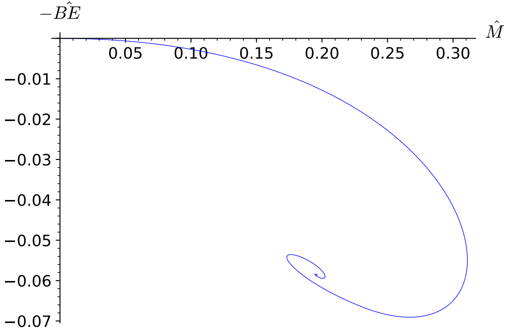
Current status: excluded regions of PBHs



Carr, Kohri, Sendouda & Yokoyama 20

- Assume that all experiments are null results (shaded regions are excluded regions of PBH assuming a monochromatic mass function in x-axis)





The binding energy curve shows the possibility of metastable dark matter stars that could undergo explosive collapse to smaller radius, emitting a burst of gravitation energy on the order of 10^{41} J in 10^{-10}s. Such bursts might be observable, perhaps taking place in the centers of ordinary stars or out in the open.

A dark matter star near the asymptotic fixed point of the spiral has high central density.

Such objects would probe energy scales physics beyond the SM.

The classical CGF cosmology has no ordinary matter, only dark matter.

The dark matter is the CGF.

The actual universe is a perturbation of this dark matter universe by the fluctuations of the SM fields.

Constructing the initial fluctuations and calculating their time evolution is a well-defined calculation within the SM and GR.

Detailed quantitative checks of the theory depend on that calculation, which remains to be done.

The SM epoch is the cosmological period starting shortly before the electro-weak transition, when the universe was at energy scale roughly $1\,\mathrm{TeV}$.

$$\rho_{\mathrm{matter}} \sim \frac{(1\,\mathrm{TeV})^4}{\hbar^3} \qquad \text{(in } c = 1 \text{ units)}$$

This is the period when the universe is governed by the SM $+$ classical GR to the best of our knowledge (taking the SM to include nonrenormalizable couplings for neutrino masses and mixings).

Hold off on assuming that dark matter requires physics beyond the SM.

The dark matter will be explained entirely within the SM.

Dark matter is a cosmological gauge field (CGF).

The initial state is a coherent state of the Higgs field and the $\mathrm{SU}(2)$ gauge field (the CGF).

The CGF is dark matter: a perfect fluid with equation of state parameter $w_{\mathrm{CGF}} \approx 0$.

In the leading order, classical approximation, the universe contains only the CGF, only dark matter, no ordinary matter. The ordinary matter is a sub-leading correction due to the fluctuations of the SM fields around the classical trajectory.

The classical CGF universe has the basic structure of the SM epoch:

- the electro-weak transition
- followed by an expanding universe
- containing only dark matter
- homogeneous and isotropic
- flat at the present time.

The classical CGF universe is the dark matter skeleton of the SM epoch, to be fleshed out by the fluctuations of the SM fields. The time evolution of the fluctuations remains to be calculated.

First principles cosmology

Laboratory HEP looks for small discrepancies from SM predictions to find new physics.

The idea is to put cosmology in the same situation. If the project works, the SM epoch will be described accurately by a systematic expansion around the classical dark matter universe. Any discrepancy will be a sign of new physics.

Maybe this is too ambitious. Maybe the SM epoch depends on so far undiscovered particles and fields. Still, until those particles or fields are actually discovered, the top-down approach seems worth trying.

Dark matter being a SM effect explains why no dark matter particles have been found. It predicts that no such particles will be found.

Dark matter being a classical effect and ordinary matter a sub-leading correction explains why most of the matter in the universe is dark matter.

It seems nontrivial that there should be a simple initial state that gives the right basic structure and a systematic scheme to correct it. It might be worthwhile to calculate the corrections to the classical trajectory to see if the details come out right.