

## 1 Introduction

I want again to thank the APS and the Donnely's for the wonderful gift of the Onsager Prize.

The work cited by the APS was done at the University of Chicago in the early 1980s in collaboration with Steve Shenker and Zongan Qiu.

I'd like to express my gratitude to Yoichiro Nambu and Leo Kadanoff, who hired Steve and me at Chicago. Their confidence in us, their generous support and the absolute autonomy they gave us made it a great situation to start out in.

I have to confess that I am not a condensed matter physicist. I feel it a very great privilege to be invited to talk here.

I think abstractly about physics. Once or twice my abstract lines of thought have possibly touched on real condensed matter systems.

Today I'll first say a few words about the result that was cited for the Onsager prize. Then I'd like to exploit this once in a lifetime opportunity to mention a slightly more recent effort, from five years ago, to describe the class of physical systems that might best be used for large-scale quantum computers.

## 2 History

First, a quick listing of the beautiful theoretical ideas whose confluence put us in a position where, by proving a theorem in abstract algebra, we could say something about the possible forms of critical behavior in 2 dimensions.

Of course, these ideas rest on a collection of beautiful experimental results, which I'm too far removed from to try to mention.

- QFT: renormalization of QED, the rg, axiomatic qft, the conformal bootstrap
- Critical phenomena: scaling, block-spinning, universality, conformal symmetry, the rg
- String theory: the Virasoro algebra
- Revival of conformal symmetry – local scale invariance – in string theory

## 3 Two Virasoro algebras

- Revival of conformal symmetry – local scale invariance – in string theory
- Axiomatic formulation of 2d conformal field theory  
conformal bootstrap

Conformal invariance introduced as a tool to permit bootstrap construction of examples of scale invariant qfts.

Two Virasoro algebras acting linearly on the space local observables.

Possible representations of Virasoro determine the possible scaling dimensions.

## 4 Unitary representations of Virasoro

For  $c < 1$ , constrains the possible scaling dimensions.

## 5 Real 2d critical phenomena?

This result looks to put severe constraints on the possible forms of 2d critical phenomena — as long as they are described by unitary conformal field theories.

Assumptions: (1) conformal symmetry, not just scale invariance; (2) unitarity.

conformal symmetry more or less follows from scale invariance, locality, translation and rotation symmetry  
translation symmetry is hard to avoid in 2d, so the essential assumptions are (1) rotation symmetry and (2) unitarity.

- Static 2d critical systems at  $T_c > 0$
- Quantum 1d critical systems at  $T_c = 0$