The shape of a more fundamental theory?

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Abstract

A minimal practical formal structure is suggested for a more fundamental theory than the Standard Model + General Relativity. A mechanism that produces such a structure is reviewed. The proposed mechanism has possibilities of producing non-canonical phenomena in SU(2) and SU(3) gauge theories which might give conditional predictions to be checked.

Contents

1 Formal fundamental physics

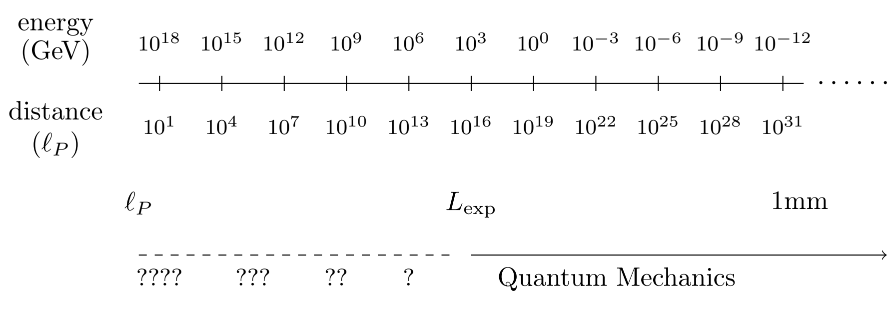
For 45 years, our most fundamental theory has been the Standard Model combined with General Relativity. The combined theory describes almost all known physics at distances larger than about $(10^3\text{GeV})^{-1}$. Only dark matter, neutrino mixing, and some CP violation are left out.

SM+GR is used as an *effective* quantum field theory with short distance cutoff on the order of $(10^3\text{GeV})^{-1}$. There is no problem with treating GR as an effective QFT because quantum corrections in GR are negligible at distances much larger than the Planck distance ℓ_P, so GR as an effective QFT with short distance cutoff $(10^3\text{GeV})^{-1} = 10^{16}\ell_P$ is indistinguishable from the classical field theory.

The project of formal fundamental physics is to hypothesize a "more fundamental" formal machinery that can "produce" SM+GR, and then predict consequences beyond SM+GR that can be checked against experiment.

In the 45 years since the SM was finished, no attempt at formal fundamental physics has worked. One possible reaction is to give up on the project, perhaps hoping that experiment will eventually provide more guidance. Another possible reaction is to examine the assumptions that have guided the formal fundamental physics enterprise over these past 45 years. SM+GR already encodes considerable experimental evidence. The question is how to be guided. It might be useful especially to question the truisms and mathematical idealizations that have governed the enterprise. It might be worthwhile to reconsider paths not taken.

1.1 Against Quantum Gravity



Especially questionable is the truism that General Relativity and Quantum Mechanics need to be reconciled in a theory of Quantum Gravity. On the contrary, there is no appreciable conflict between General Relativity and Quantum Mechanics at distances much larger than the Planck length ℓ_P. To suppose a conflict is to extrapolate the validity of Quantum Mechanics and

General Relativity over 16 orders of magnitude in distance from the smallest distance presently accessible to experiment $L_{\text{exp}} \sim (10^3\text{GeV})^{-1} = 10^{16}\ell_P$ down to ℓ_P. Such a presumptuous extrapolation beyond the physical evidence might be justifiable if it could produce a testable prediction. But it is implausible that any proposed theory of Quantum Gravity could be checked experimentally given that the smallest distance presently accessible to experiment is $10^{16}\ell_P$. There is no practical possibility of checking whether any proposed theory of Quantum Gravity actually describes the real world.

1.2 Against mathematical idealizations

Formal structures are used in physics for practical purposes, not as ideal mathematical forms. A quantum field theory is used in physics as an *effective* theory describing physics at distances greater than some UV cutoff at the short distance limit of evidence for the theory. Continuum QFT is a mathematical idealization which extrapolates far beyond the practical application. An effective QFT asserts nothing about distances smaller than the UV cutoff. It does not even suppose the existence of a space-time continuum.

An S-matrix is used in physics as an *effective* theory that describes physics at distances smaller than the size of the scattering region. The asymptotic S-matrix is a mathematical idealization which supposes ingoing scattering states are produced infinitely early in time and infinitely far from the scattering region, and outgoing scattering states are detected infinitely later in time infinitely far away. Actual scattering experiments take place within a finite region of space over a finite period of time. The asymptotic S-matrix extrapolates far beyond the practical application of the formalism.

The mathematical idealizations of continuum QFT and the asymptotic S-matrix can be useful for mathematical purposes. But for fundamental physics prudence suggests using the practical formal structures that encode the actual limits of physical knowledge — an *effective* QFT with a UV cutoff and an *effective* S-matrix with an IR cutoff.

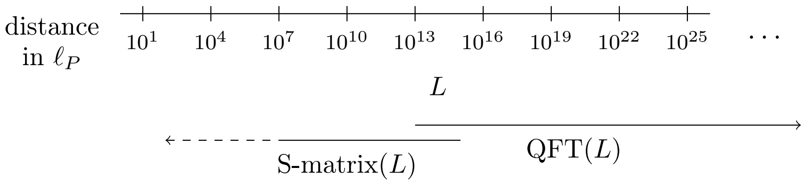
2 A minimal practical formal structure

A leading edge high energy experiment of size L probes physics at distances $< L$. The ingoing and outgong scattering states are states in the effective QFT that describes physics at distances $> L$. The scattering amplitudes describe physics at distances $< L$.

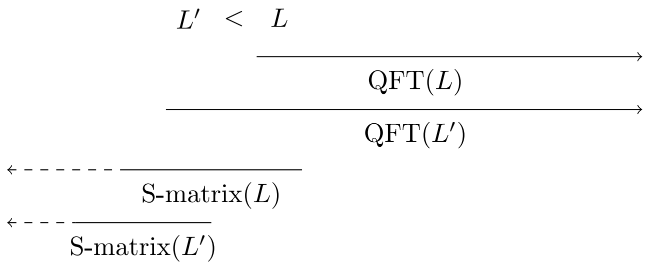
2.1 An effective QFT for distances $> L$, an effective S-matrix for distances $< L$, for observers at every scale $L \gg \ell_P$

A minimal practical formal structure expresses what is observable at each scale $L \gg \ell_P$. There is an effective QFT(L) with UV cutoff L and an effective

S-matrix(L) with IR cutoff L. QFT(L) describes the physics at distances $> L$. S-matrix(L) describes the physics at distances $< L$.



L is a sliding distance scale. The meaning of "short distance physics" depends on the scale of the observer. As progress pushes to a shorter distance $L' < L$ the descriptions of physics at the two distances must be consistent.



C1 QFT(L) must be derived from QFT(L') by the renormalization group acting on effective quantum field theories.

C2 At distances between L' and L, the scattering amplitudes derived from QFT(L') must agree with S-matrix(L).

C3 S-matrix(L') must be derived from S-matrix(L) by the "S-matrix renormalization group" acting on effective S-matrices.

2.2 The S-matrix RG

The S-matrix RG is the operation on effective S-matrices which takes an effective S-matrix with IR cutoff distance L to an effective S-matrix with smaller IR cutoff distance L' by using the scattering states at the larger distance L to make the scattering states at the smaller distance L'.

2.3 An S-matrix does not require Quantum Mechanics

The formal structure QFT(L) + S-matrix(L) expresses what is observable, depending on the scale L of the observer. There is no presumption of QFT or any other form of Quantum Mechanics all the way down to ℓ_P. An observer at scale L probes short distance physics by sending things in to smaller distances and measuring what comes out. There is only an effective S-matrix for short distance physics. Having an effective S-matrix does not require or imply that there must be an underlying quantum mechanical hamiltonian. The only implication is in the other direction. Given a microscopic hamiltonian, scattering amplitudes can be derived from it. But given an S-matrix, there is no necessity that it is or can be derived from a microscopic Quantum Mechanics. There is at least one construction of scattering amplitudes — string theory — which does not depend on a quantum mechanical hamiltonian.

The formal structure QFT(L) + S-matrix(L) is local in L. An observer at scale L makes only a modest extrapolation by supposing that the scattering amplitudes describing physics at scale L' somewhat smaller than L will be derived from a somewhat more fundamental effective QFT at scale $L' < L$.

3 A mechanism that produces such a formal structure

A search for a mechanism that would produce such a formal structure of fundamental physics began with [1, 2]. A mechanism was finally proposed in [3]. The line of thought is sketched in the Appendix.

3.1 Summary

The argument for the mechanism is summarized:

1. String theory provides a way to construct a self-consistent S-matrix for short distance physics without requiring a short distance QFT.

2. When the string worldsheet is described by an effective 2d-QFT with 2d cutoff distance Λ^{-1}, the string S-matrix constructed from this effective 2d-QFT is an effective S-matrix(L) with IR cutoff L given in dimensionless units by
$$L^2 = \ln(\Lambda/\mu) \tag{1}$$
where $\mu^2|dz|^2$ is the worldsheet metric. The condition $L \gg 1$ is the requirement $\Lambda^{-1} \ll \mu^{-1}$, i.e., the requirement that the 2d cutoff be insignificant at the 2d scale used in the S-matrix calculation.

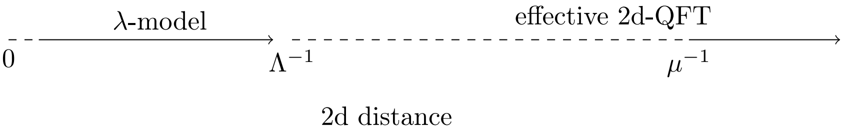
3. The effective string background is encoded in the local physics of the worldsheet at 2d distance Λ^{-1}.

4. The S-matrix RG acts on S-matrix(L) by integrating out the froth of small handles in the worldsheet. The IR cutoff L going down is the 2d UV cutoff Λ^{-1} going up.

5. The effects of small handles are replicated by a certain mathematically natural local 2d nonlinear model (2d-NLM) called the *λ-model*. Its target manifold is the space of effective 2d-QFTs of the worldsheet, parametrized by the effective 2d coupling constants $\lambda^i(\Lambda)$ at 2d scale

Λ^{-1}. These $\lambda^i(\Lambda)$ are the modes of the classical space-time fields with UV cutoff L. The target manifold is thus the space of classical space-time fields with UV cutoff L.

6. The froth of small handles is replaced by the λ-model. The λ-model acts at 2d distances $< \Lambda^{-1}$ to produce an effective 2d-QFT with cutoff Λ^{-1}.



7. Integrating out the λ-fluctuations has the same effect as integrating out the froth of small handles, so the 2d-RG of the λ-model implements the S-matrix RG.

8. The 2d-RG of the λ-model also produces a measure on the target manifold, the *a priori* measure of the 2d-NLM. A measure on the target manifold is a functional integral over the space-time fields with UV cutoff L, i.e., an effective quantum field theory QFT(L).

9. The quantum states of QFT(L) are the quantum string backgrounds.

10. The consistency conditions **C1-C3** on QFT(L) + S-matrix(L) are automatically satisfied. So the λ-model produces a consistent realization of the minimal practical formal structure described above.

11. The effective QFT(L) is produced by a 2d mechanism that does not necessarily correspond to canonical quantization. There are concrete possibilities of nonperturbative semi-classical 2d effects producing non-canonical degrees of freedom and non-canonical interactions in QFT(L). These 2d effects are 2d winding modes and 2d instantons coming from from nontrivial homotopy groups π_1 and π_2 of the target manifold of the λ-model, which is the space of space-time fields. These homotopy groups are nontrivial when the space-time fields include SU(2) and SU(3) gauge fields in four space-time dimensions.

    Thus there are concrete possibilities of testable conditional predictions of the form: *if QFT(L) contains SM+GR, then it predicts certain non-canonical degrees of freedom and certain non-canonical interactions beyond those of the canonically quantized quantum field theory.*

The last point is the main reason for considering the λ-model as a speculative proposal for formal fundamental physics.

3.2 The 2d-QFT of the string worldsheet

In the general renormalizable 2d nonlinear model

$$\int \mathcal{D}X\, e^{-\int d^2z\, g_{\mu\nu}(X)\partial X^\mu \bar\partial X^\nu} \qquad X(z) \in M \tag{2}$$

the field $X(z)$ takes values in a target manifold M. The 2d coupling constants are given by a Riemannian metric $g_{\mu\nu}(X)$ on M. The manifold M is taken to be compact and the metric is taken to have euclidean signature in order that the 2d-QFT will be well defined. The 2d-RG

$$\Lambda \frac{\partial}{\partial \Lambda} g_{\mu\nu}(X) = -R_{\mu\nu}(X) + O(R^2) \tag{3}$$

drives the 2d-NLM to a solution of $R_{\mu\nu} = 0$.

The 2d-QFT of the string worldsheet is an elaboration of the general 2d-NLM in which the target manifold M is space-time and the 2d coupling constants consist of the space-time metric $g_{\mu\nu}(X)$ as well as some non-abelian gauge fields, scalar fields, fermion fields, etc. on the space-time M. The equation $\beta = 0$ generalizing $R_{\mu\nu} = 0$ is a semi-realistic classical field equation which includes GR and potentially the SM.

Write, in abstract notation,

$$\begin{aligned} \lambda^i &= \text{the 2d coupling constants,} \\ \phi_i(z) &= \text{the corresponding spin-0 scaling fields of the 2d-QFT,} \\ |\phi_i\rangle &= \text{the corresponding states on the circle,} \\ G_{ij} &= \text{the natural metric } \langle\phi_i|\phi_j\rangle. \end{aligned}$$

The index i labels the modes of the space-time fields. For example, a mode of the space-time metric $h_{\mu\nu}(i)e^{ip_\mu(i)X^\mu}$ corresponds to the 2d field

$$\phi_i(z) = h_{\mu\nu}(i)\,\partial X^\mu \bar\partial X^\nu\, e^{ip_\mu(i)X^\mu(z)} \qquad i \leftrightarrow p_\mu(i),\, h_{\mu\nu}(i) \tag{4}$$

For each value of the coupling constants λ^i there is a perturbed 2d-QFT constructed by inserting in the worldsheet the perturbation of the action

$$e^{\int d^2z\, \lambda^i \phi_i(z)} \tag{5}$$

The coupling constants λ^i thus form a system of local coordinates on the space of 2d-QFTs.

The 2d scaling-dimensions are

$$\dim(\phi_i) = 2 + \delta(i) \qquad \dim(\lambda^i) = -\delta(i) \qquad \delta(i) = p(i)^2 \tag{6}$$

The 2d-RG

$$\Lambda \frac{\partial}{\partial \Lambda} \lambda^i = \beta^i(\lambda) \qquad \beta^i(\lambda) = -\delta(i)\lambda^i + O(\lambda^2) \tag{7}$$

drives the worldsheet 2d-QFT towards the $\beta = 0$ submanifold which is parametrized by the marginal couplings

$$\dim(\lambda^i) = -\delta(i) = 0 \tag{8}$$

3.3 The effective string S-matrix with IR cutoff L

Let $\mu^2|dz|^2$ be the 2d metric on the worldsheet. Impose a 2d UV cutoff $\Lambda^{-1} \ll \mu^{-1}$. The cutoff string propagator (the 2d-cylinder) is

$$\int_0^{\ln(\Lambda/\mu)} d\tau \sum_{i,j} \; |\phi_i\rangle \;\; e^{-\tau\delta(i)} G^{ij} \;\; \langle\phi_j| = \sum_{i,j} \; |\phi_i\rangle \;\; \frac{1 - e^{-L^2\delta(i)}}{\delta(i)} G^{ij} \;\; \langle\phi_j| \tag{9}$$

where

$$L^2 = \ln(\Lambda/\mu) \qquad e^{-L^2\delta(i)} = (\Lambda/\mu)^{-\delta(i)} \tag{10}$$

The only modes that propagate are those that satisfy

$$\delta(i) > L^{-2} \qquad \text{which is} \qquad p(i)^2 > L^{-2} \tag{11}$$

So the 2d UV cutoff Λ^{-1} puts an IR cutoff L on the string S-matrix. An effective 2d-QFT of the worldsheet gives an effective string S-matrix(L) with L given by $L^2 = \ln(\Lambda/\mu)$.

3.4 The effective 2d coupling constants $\lambda^i(\Lambda)$

The effects of the 2d coupling constants $\lambda^i(\Lambda)$ at 2d scale Λ^{-1} are suppressed by the 2d-RG running from Λ^{-1} up to μ^{-1}

$$\lambda^i(\mu) = (\Lambda/\mu)^{-\delta(i)} \, \lambda^i(\Lambda) = e^{-L^2\delta(i)} \lambda^i(\Lambda) \tag{12}$$

If $L^2\delta(i) > 1$ then $\lambda^i(\Lambda)$ is effectively irrelevant; its effects on the worldsheet are negligible. The only effective $\lambda^i(\Lambda)$ are the almost marginal couplings

$$\delta(i) < L^{-2} \qquad \text{which is} \qquad p(i)^2 < L^{-2} \tag{13}$$

So there is a UV cutoff distance L on the modes of the space-time fields that describe the effective 2d-QFT of the worldsheet.

The 2d UV cutoff Λ^{-1} separates the worldsheet fields $\phi_i(z)$ and the corresponding coupling constants λ^i into two subsets. The effectively marginal λ^i with $\delta(i) < L^{-2}$ are the perturbations of the classical string background. The effectively irrelevant $\phi_i(z)$ with $\delta(i) > L^{-2}$ are the vertex operators describing the propagating string modes in the effective S-matrix.

3.5 Implement the S-matrix RG

Consider the effect of a small handle in the worldsheet. A small handle is made by identifying the boundaries of two holes of radius r around two points z_1, z_2 which are close together in the worldsheet by

$$z \leftrightarrow z' \qquad (z - z_1)(z' - z_2) = r^2 e^{i\theta} \tag{14}$$

Integrate over the moduli z_1, z_2, r, θ and sum over states on the two boundary circles. The integral over θ projects on the spin-0 states. The small handle becomes the bi-local insertion

$$\frac{1}{2}\sum_{i_1,i_2}\int d^2z_1\,\phi_{i_1}(z_1)\int d^2z_2\,\phi_{i_2}(z_2)\int_{\Lambda^{-1}}^{\frac{1}{2}|z_1-z_2|} dr\; r^{-1-\delta(i_1)-\delta(i_2)}\;g_{\mathrm{str}}^2 G^{i_1 i_2} \tag{15}$$

where $G_{i_1 i_2}$ is the natural metric on the space of 2d-QFTs and g_{str} is the string coupling constant. The cutoff dependent contribution comes from the effectively marginal fields with $\delta(i)\sim 0$

$$\frac{1}{2}\sum_{\delta(i_{1,2})\sim 0}\int d^2z_1\,\phi_{i_1}(z_1)\int d^2z_2\,\phi_{i_2}(z_2)\;g_{\mathrm{str}}^2 G^{i_1 i_2}\ln\left(\Lambda|z_1-z_2|\right) \tag{16}$$

The cutoff dependence of the small handle can be canceled by letting the effectively marginal 2d coupling constants λ^i fluctuate locally on the worldsheet. Make the λ^i sources $\lambda^i(z)$ so the worldsheet perturbation becomes

$$e^{\int d^2z\,\phi_i(z)\lambda^i(z)} \tag{17}$$

Then set the $\lambda^i(z)$ fluctuating with 2-point correlation function

$$\langle\,\lambda^{i_1}(z_1)\;\lambda^{i_2}(z_2)\,\rangle = -g_{\mathrm{str}}^2 G^{i_1 i_2}\ln\left(\Lambda|z_1-z_2|\right) \tag{18}$$

The cancellation of the single small handle by the gaussian λ-fluctuations holds around every background 2d-QFT. Therefore the cutoff dependence of the entire froth of small handles is canceled by λ-fluctuations governed by the 2d-NLM

$$\int \mathcal{D}\lambda\;e^{-\int d^2z\,g_{\mathrm{str}}^{-2}G_{ij}(\lambda)\partial\lambda^i\bar\partial\lambda^j}\;e^{\int d^2z\,\phi_i(z)\lambda^i(z)}\qquad \lambda(z)\in\mathcal{M} \tag{19}$$

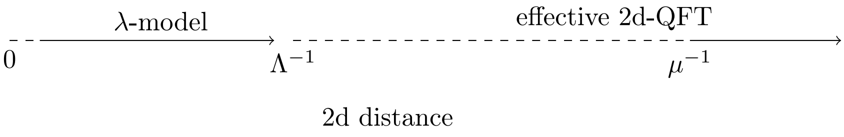
This is the λ-model. Its target manifold $\mathcal{M}$ is

$$\begin{aligned}\mathcal{M} &= \text{the manifold of effective worldsheet 2d-QFTs with 2d cutoff } \Lambda^{-1}\\ &= \text{the manifold of classical space-time fields with UV cutoff } L\end{aligned}$$

where L is related to Λ^{-1} by $L^2=\ln(\Lambda/\mu)$.

The entire sum over handles does not depend on the arbitrary choice of Λ^{-1} so the sum over λ-fluctuations at 2d distances $<\Lambda^{-1}$ has the same effect as the sum over small handles, since either sum cancels the Λ^{-1} dependence of the sum over handles at 2d distances $>\Lambda^{-1}$. So the sum over λ-fluctuations can replace the sum over small handles at 2d distances $<\Lambda^{-1}$.

The λ-fluctuations at 2d distances $<\Lambda^{-1}$ produce an effective 2d-QFT with UV cutoff Λ^{-1} giving an effective string S-matrix(L) with IR cutoff L. By replicating the froth of small handles, the λ-model implements the S-matrix RG. The 2d-RG of the λ-model operates from a smaller 2d distance



Λ'^{-1} up to a larger 2d distance Λ^{-1} by integrating out the λ-fluctuations at 2d distances between Λ'^{-1} and Λ^{-1}. Integrating out the λ-fluctuations is equivalent to integrating out the froth of small handles at 2d distances between Λ'^{-1} and Λ^{-1}, taking the effective S-matrix(L') with the larger IR cutoff L' to the effective S-matrix(L) with the smaller IR cutoff L.

3.6 Production of an effective QFT with UV cutoff L

Like any 2d-NLM, the λ-model is specified by two pieces of data

- the metric $g_{\rm str}^{-2} G_{ij}(\lambda)$ on the target manifold $\mathcal{M}$
- the *a priori* measure $d\lambda\, \rho(\lambda)$ on the target manifold $\mathcal{M}$ which gives the functional volume element

$$\int \mathcal{D}\lambda = \prod_z \int_{\mathcal{M}} d\lambda(z)\, \rho(\lambda(z)) \tag{20}$$

$d\lambda\, \rho(\lambda)$ is called the *a priori* measure because it arises from the fluctuations that have gone before. A point z in the effective worldsheet represents a 2d block of dimensions $\Lambda^{-1} \times \Lambda^{-1}$. The measure $d\lambda(z)\, \rho(\lambda(z))$ summarizes the short distance λ-fluctuations inside the block that have already been integrated out.

The measure $d\lambda\, \rho(\lambda)$ evolves under the 2d-RG, diffusing in the target manifold $\mathcal{M}$ because of the λ-fluctuations. Simultaneously, the λ^i flow along $\beta^i(\lambda)$ towards the $\beta = 0$ submanifold. So the measure $d\lambda\, \rho(\lambda)$ evolves under a driven diffusion process. Taking $d\lambda$ to be the metric volume element, $\rho(\lambda)$ becomes a function on $\mathcal{M}$. The driven diffusion equation is

$$\Lambda \frac{\partial}{\partial \Lambda}\, \rho(\lambda) = \nabla_i \left(g_{\rm str}^2 G^{ij} \partial_j + \beta^i \right) \rho(\lambda) \tag{21}$$

Integrating out the λ-fluctuations at all 2d distances up to Λ^{-1} drives $d\lambda\, \rho(\lambda)$ to the equilibrium measure

$$d\lambda\, \rho(\lambda) \to d\lambda\ e^{-g_{\rm str}^{-2} S(\lambda)} \qquad \text{where} \quad \beta^i = G^{ij} \partial_j S$$

The λ^i are the space-time field modes with UV cutoff L so $d\lambda\, \rho(\lambda)$ is the functional integral of an effective QFT(L) with classical action $g_{\rm str}^{-2} S(\lambda)$.

The λ-model thus produces an effective S-matrix(L) and an effective QFT(L) at every $L \gg 1$. The consistency conditions **C1-C3** are automatically satisfied. The S-matrix RG acts on the S-matrix(L) because the λ-model is designed precisely to accomplish that. The QFT RG acts on the QFT(L) because the decoupling of the effectively irrelevant $\lambda^i(\Lambda)$ is a fundamental property of the 2d RG. Agreement between S-matrix(L) and QFT(L) on scattering amplitudes at scales $\sim L$ is guaranteed because the scattering amplitudes of S-matrix(L) near the IR cutoff L are given by the 2d correlation functions of vertex operators near the 2d UV cutoff Λ^{-1} which are determined by the *a priori* measure $d\lambda\, \rho(\lambda)$ which is QFT(L).

3.7 Possible non-canonical degrees of freedom and couplings in SU(2) and SU(3) gauge theory

The λ-model is a nonperturbative 2d-NLM with possibilities of nonperturbative semi-classical effects: winding modes associated to $\pi_1(\mathcal{M})$ and 2d instantons associated to $\pi_2(\mathcal{M})$, where the target manifold $\mathcal{M}$ is the manifold of space-time fields [4]. Supposing that the λ-model produces effective quantum string backgrounds with four macroscopic space-time dimensions and with $SU(N)$ gauge fields on space-time, the mathematical result

$$\pi_k \text{ of the manifold of } SU(N) \text{ gauge fields on } \mathbb{R}^4 \cup \{\infty\} = \pi_{k+3}(SU(N))$$

implies that there are winding modes ($k = 1$) when $\pi_4(SU(N)) \neq 0$ and there are 2d instantons ($k = 2$) when $\pi_5(SU(N)) \neq 0$. In particular, the mathematical results

$$\pi_4(SU(2)) = \mathbb{Z}_2 \quad \pi_4(SU(3)) = 0 \quad \pi_5(SU(2)) = \mathbb{Z}_2 \quad \pi_5(SU(3)) = \mathbb{Z} \tag{22}$$

imply that there are winding modes when the gauge group is $SU(2)$ and 2d-instantons when the gauge group is $SU(2)$ or $SU(3)$. These nonperturbative semi-classical effects in the λ-model offer possibilities of conditional predictions of the form

If the λ-model produces SM+GR then it also produces

- *non-canonical degrees of freedom from the $\mathbb{Z}_2$ winding mode in the manifold of $SU(2)$ gauge fields on $\mathbb{R}^4$ and*
- *non-canonical interactions from the 2d instanton in the manifold of $SU(2)$ gauge fields and the 2d instantons in the manifold of $SU(3)$ gauge fields on $\mathbb{R}^4$.*

The $\mathbb{Z}_2$ winding mode in the manifold of $SU(2)$ gauge fields in $\mathbb{R}^4$ can be described explicitly. Let $A(x_+, x_-, u)$ be the $SU(2)$ gauge field consisting of a zero-size instanton at x_+ and a zero-size anti-instanton at x_- with relative orientation matrix u. The relative orientation u is in the adjoint representation $SU(2)/\mathbb{Z}_2$. The winding mode is the topologically nontrivial closed geodesic loop in $SU(2)/\mathbb{Z}_2$

$$\theta \mapsto A(x_+, x_-, u(\theta)) \qquad u(0) = 1 \quad u(2\pi) = -1 \tag{23}$$

This is the shortest topologically nontrivial loop in the manifold of $SU(2)$ gauge fields; it has zero length. The winding mode is bi-local in space-time, depending on the two space-time points x_+, x_-. It also depends on the fermionic zero-modes localized in the intanton and in the anti-intanton.

The 2d instantons are minimal topologically nontrivial 2-spheres that are found in slightly more complicated configurations of zero-size instantons and anti-instantons. For $SU(3)$, the relative orientations between a zero-size instanton and a zero-size anti-instanton are parametrized by $SU(3)/U(1)$ in which there is a topologically non-trivial 2-sphere. For $SU(2)$, the minimal nontrivial 2-sphere is in the space of gauge configurations consisting of two zero-size instantons and two zero-size anti-instantons.

3.8 To do

Most urgent is to determine if the λ-model can in fact make conditional predictions of observable non-canonical effects in $SU(2)$ and $SU(3)$ gauge theory in 4 dimensions. This requires figuring out how to calculate semi-classical corrections to the *a priori* measure of the λ-model coming from the zero length winding modes and from the zero area 2d instantons. If such conditional predictions can be made and if they can be checked, there will be motivation for further investigation of the λ-model.

The $\mathbb{Z}_2$ winding mode in the λ-model for $SU(2)$ gauge theory offers a possibility. There is a unique trajectory of the $SU(2)$ Yang-Mills flow running from the aligned zero-size instanton/anti-instanton configuration $A(x_+, x_-, 1)$ down to the gauge field with zero curvature which is the classical ground state. Along this trajectory, the instanton and the anti-instanton grow from zero size, merge, and annihilate. A 2d operator product of an even number of $\mathbb{Z}_2$ winding modes will be an untwisted 2d field localized at the zero-size instanton/anti-instanton configurations. It will evolve under the 2d-RG along the downward trajectory towards the classical ground state. Supposing that some of the *a priori* measure is deposited in the twisted sector, the downward trajectory will produce nontrivial 1-point functions in the effective QFT. The 1-point functions will be determined by the direction along which the downward trajectory approaches the classical ground state.

There are also many basic questions about the λ-model, but the most interesting of these seem difficult and open-ended. Expending effort does not seem worthwhile unless and until a successful conditional prediction provides motivatation.

The top-down construction of effective QFT is tantalizing. The λ-model operates at 2d distances from near 0 up to Λ^{-1}. The corresponding space-time distance is given by $L^2 = \ln(\Lambda/\mu)$. So the λ-model builds effective QFT from the largest distances in space-time *down* to L (nonetheless guaranteeing that the QFT RG is satisfied). Can this top-down mechanism of construction of QFT give natural explanations of unnaturalness?

The *a priori* measure $d\lambda\rho(\lambda)$ of the λ-model is a measure on the target manifold which is the space of effective 2d-QFTs of the string worldsheet, the space of effective classical string backgrounds. In the extreme large distance limit $L \to \infty$, only the marginal 2d couplings λ^i fluctuate. The target manifold becomes the space of worldsheet 2d conformal field theories, the idealized classical string backgrounds. The λ-model dynamically produces a measure on this huge space of exact backgrounds. The *a priori* measure $d\lambda\rho(\lambda)$ is the functional integral of an effective QFT only locally where it concentrates near a background which is a very large macroscopic space-time. Exploring all the possible variety of places this measure might concentrate is an intimidating task. A more practical approach is to assume that some part of the measure concentrates near a large 4-manifold space-time with the space-time fields of the SM. If conditional on this assumption predictions can be made of testable non-canonical effects, and if they can be checked against experiment, then it might be worth asking such questions as, is there anything in the λ-model mechanism that would cause the dynamically generated *a priori* measure $d\lambda\,\rho(\lambda)$ to concentrate near backgrounds with macroscopic space-time dimensions, four in particular?

Wick rotation is an after-thought. Space-time has been taken to be compact with euclidean signature in order that the 2d-NLM of the worldsheet be a well defined effective 2d-QFT. Only finitely many modes $\lambda^i(\Lambda)$ of the space-time fields fluctuate and their fluctuations are governed by a positive definite metric $G_{ij}(\lambda)$. The λ-model is an effective mechanism. Wick rotation is to be carried out in the effective QFT(L) and in the effective S-matrix(L) if and when the *a priori* measure concentrates at a macroscopic space-time. Wick rotation is then only approximate, up to corrections that are small at distance scales well away from L. Is there any explanation of Wick rotation to Minkowski signature?

How is cosmology to be done? Perhaps the space-time scale L can be related inversely to cosmological time, so cosmology at later times and larger length scales is governed by QFT(L), while cosmology at earlier times and smaller length scales is described by an outgoing scattering state in S-matrix(L).

There is much foundational technical work to be done. In particular, systematic technical treatments are needed for the effective S-matrix, for the S-matrix RG, and for the basic claim that integrating out the froth of small handles gives the string S-matrix RG.

Appendix. Notes on the line of thought

A.1 Search for a mechanism that produces QFT

The ideas expressed in this note were developed during the period 1977–2002 in the process of searching for and eventually formulating a mechanism that would produce quantum field theory.

The 2d-RG as a mechanism for space-time physics (1977–79)

The line of thought began with the renormalization of the general renormalizable 2d nonlinear model (2d-NLM)

$$\int \mathcal{D}X \; e^{-\int d^2z \; g_{\mu\nu}(X)\partial X^\mu \bar{\partial} X^\nu} \qquad X(z) \in M \tag{24}$$

whose coupling constants are given by a Riemannian metric $g_{\mu\nu}(X)$ on a manifold M. The 2d-RG

$$\Lambda \frac{\partial}{\partial \Lambda} \, g_{\mu\nu}(X) = -R_{\mu\nu}(X) + O(R^2) \tag{25}$$

drives the 2d-NLM to a solution of $R_{\mu\nu} = 0$. This was very exciting (at least for me). The 2d-RG appeared as a *mechanism* that *produces* solutions of a GR-like space-time field equation $R_{\mu\nu} = 0$. It suggested the possibility that a mechanism — the 2d-RG — might actually answer questions like

> *Where does space-time field theory come from?*

or even

> *Where do the laws of physics come from?*

The goal became a mechanism that produces the actual laws of physics, rather than a "fundamental" principle that would determine the laws of physics.

It had become clear by the late 1970s that there are too many effective QFTs. A mechanism was needed that would produce effective QFT more selectively than the QFT renormalization group. The 2d-RG seemed promising as a novel mechanism for the purpose that at least produced *classical* field theory. The goal then became a mechanism that produces *quantum* field theory and that has the 2d-RG as its classical limit. The 2d-NLM had two other shortcomings as a mechanism for space-time physics: the $\beta = 0$ equation $R_{\mu\nu} = 0$ is not quite Einstein's equation; and the RG fixed points, the solutions of $\beta = 0$, have unstable directions along which the RG flow diverges from the fixed point rather than converges to it.

The 2d-RG incorporated into string theory (1981–85)

In the early 1980s it was realized that the 2d-RG fixed point equation $\beta = 0$ (i.e., 2d scale invariance) is a consistency condition for calculating the string S-matrix from a worldsheet 2d-QFT. The string worldsheet is a supersymmetric 2d-QFT containing additional 2d degrees of freedom besides $X(z)$. The 2d coupling constants become, in addition to the space-time metric $g_{\mu\nu}(X)$, a collection of non-abelian gauge fields, scalar fields, fermion fields, etc. on space-time. The supersymmetric string worldsheet resolved two of the problems with the basic 2d-NLM. The 2d supersymmetry of the string worldsheet eliminates the unstable directions at the fixed points (tachyons in the S-matrix). The worldsheet $\beta = 0$ equation generalizing $R_{\mu\nu} = 0$ is a semi-realistic classical field equation that includes GR and potentially the SM.

In the mid-1980s, several assumptions and mathematical idealizations about string theory became more or less truisms:

1. The string S-matrix was seen as an asymptotic S-matrix without IR cutoff — a "theory of everything".

2. The string backgrounds were taken to be the conformally invariant worldsheet 2d-QFTs from which such asymptotic string S-matrices were derived: the exact worldsheet solutions of $\beta = 0$ given by Calabi-Yau manifolds ($R_{\mu\nu} = 0$) and generalizations.

3. It was *assumed* that the low momentum physics of string theory is the (supersymmetric) QFT that happens to have the same low momentum scattering amplitudes as the asymptotic string S-matrix. The string backgrounds were conflated with those supersymmetric QFTs.

4. It was assumed that there must exist a QFT or some other kind of fundamental microscopic quantum mechanical system from which the string S-matrix could be derived.

Questions (1987)

Circa 1987, the key questions seemed to be:

1. How does the 2d-RG act in string theory as a *mechanism*? The fixed point equation $\beta = 0$ is a only consistency condition for the string recipe.

2. Where does *quantum* field theory come from in string theory? What produces a functional integral over space-time fields?

3. What is the *quantum* string background? It should be given by a quantum state of a QFT in space-time.

The λ-model (1988-2002)

The attempt to answer these questions was a long-drawn-out process. One seed was the idea that the string background is encoded in the local 2d physics of the worldsheet. Another seed was the vague notion that nonperturbative effects in string theory might come from infinite genus worldsheets. Eventually, these were combined in the idea that a froth of small handles would contribute to the local 2d physics of the worldsheet and thus to the string background. This motivated the calculation of the log divergence in the contribution of a single small handle (as in section 3.5 above), which took the form of a bi-local insertion in the worldsheet. This was a strong signal. The infrared log divergence of the scalar field 2-point function plays a fundamental role in 2d-QFT. Thus the idea of setting the 2d coupling constants λ^i fluctuating as 2d scalar fields $\lambda^i(z)$, with the 2d-IR log divergences of the λ-fluctuations cancelling the 2d-UV log divergences of the small handles.

The essential role of a 2d distance scale Λ^{-1} as a 2d UV cutoff in the string S-matrix calculation and as the 2d IR cutoff on the λ-fluctuations required abandoning the idealized asymptotic string S-matrix "of everything" for an effective string S-matrix with IR cutoff L (as in section 3.3 above). Integrating out the froth of small handles became the S-matrix RG.

Recognizing that the *a priori* measure $d\lambda\rho(\lambda)$ on the λ-fluctuations would govern the local worldsheet physics led to identifying the *a priori* measure as the effective space-time QFT that is the quantum string background (as in sections 3.4 and 3.6 above).

At this point the task became to identify semi-classical 2d effects in the λ-model that might lead to checkable predictions.

A.2 Pragmatism and the S-matrix philosophy

At several points in the history of Quantum Mechanics when QFT seemed to have hit a wall, the S-matrix was proposed as an alternative formal structure for fundamental physics. A version of the history is recounted in [5]. Heisenberg proposed in the 1940s using the S-matrix as fundamental formalism in response to the divergences of perturbative QED and the difficulty of accounting for cosmic ray showers. The S-matrix bootstrap program was proposed in the 1960s in response to the plethora of mesons and baryons and their strong couplings. In the end, on both occasions, QFT overcame its difficulties and the S-matrix proposals lapsed. The third occasion was the string S-matrix proposal of the early 1970s which attracted interest at least in part because of the incompatibility between GR and QFT when extrapolated down to the Planck length.

Heisenberg's explicit rationale for using the S-matrix was the principle that fundamental physics should be expressed in terms of what is actually observed. This principle has had notable successes in fundamental theoretical physics. For example, the route of Bohr and Heisenberg to Matrix Mechanics was guided by a focus on observable transitions. But the principle was not followed literally. Matrix Mechanics in the end described the world by quantum states and transition amplitudes, which are not themselves observable. Only their absolute squares are observable. The strategy of focussing on what is observable led to a formalism, Matrix Mechanics, that reliably produces observable quantities. The pragmatic strategic principle might be: use the minimal formal machinery that is useful to produce the observable quantities of physics. Quantum Mechanics is so successful at producing observable quantities that it can be considered the formalism for "what is observable" at distances greater than about $(10^3\mathrm{GeV})^{-1}$.

The S-matrix philosophy proposed replacing Quantum Mechanics and QFT with an asymptotic S-matrix. But the asymptotic S-matrix is a fantastic idealization of what is observable. All the useful work of physics at distances larger than the elementary particle scale uses Quantum Mechanics or its effective approximation, Classical Mechanics. Is it feasible, for example, to describe the behavior of a galaxy in terms of an S-matrix? If fundamental physics were formulated as an S-matrix, what would effective QFT or effective Quantum Mechanics or effective Classical Mechanics be derived from?

On the other hand, a pragmatic version of the S-matrix philosophy seems reasonable. Scattering amplitudes describe what is observable at short distances where 'short' is relative to the size of the observer. An effective S-matrix with an IR cutoff L is a practical formulation of what we can actually observe at distances smaller than the limit of our best quantum mechanical model.

From this point of view, string theory is interesting not because it offers an S-matrix "theory of everything", but rather because it is a way to construct a self-consistent S-matrix for short distance physics without requiring a short distance QFT. A short distance S-matrix that does not require a short distance QFT is entirely suitable in a pragmatic version of the S-matrix philosophy. Before string theory, the only successful way to construct an S-matrix was by deriving it from a QFT, but that does not mean that every S-matrix must derive from a microscopic QFT.

The pragmatist philospher C. S. Peirce (a contemporary of Ernst Mach) proposed that the symbolic tools of science take their significance from the work that they do. (This might be a selective, idiosyncratic reading of Peirce.) A pragmatic strategy is to shape the formalisms of fundamental physics for the work they need to perform. The pragmatic view argues against pursuit of mathematical beauty, against pursuit of beautiful fundamental principles, against attempting to extrapolate to an absolutely fundamental theory based on absolutely fundamental principles. Eventually, a successful fundamental theory may be based on beautiful principles and may be formulated in beautiful mathematics. But there is no telling how far away that is or in what direction. There is no telling in advance which mathematically beautiful forms will prove useful for fundamental physics. Meanwhile, a practical strategy is to seek incremental changes in the formalisms of fundamental physics, incremental improvements that can actually do useful work in describing the fundamental physics of the real world.

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References

[1] D. Friedan, "Nonlinear Models in $2 + \epsilon$ Dimensions," *Phys. Rev. Lett.* **45** (1980) 1057.

[2] D. H. Friedan, "Nonlinear Models in Two + Epsilon Dimensions," *Annals Phys.* **163** (1985) 318. PhD thesis, U.C. Berkeley, 1980, Lawrence Berkeley Laboratory Report LBL-11517 (1980).

[3] D. Friedan, "A tentative theory of large distance physics," *JHEP* **10** (2003) 063, `arXiv:hep-th/0204131`.

[4] D. Friedan, "A loop of $SU(2)$ gauge fields stable under the Yang-Mills flow," in *Perspectives in mathematics and physics: Essays dedicated to Isadore Singer's 85th birthday*, T. Mrowka and S.-T. Yau, eds., vol. 15 of *Surveys in Differential Geometry*, pp. 131–204. International Press of Boston, 2010. `arXiv:hep-th/1008.1189 [hep-th]`.

[5] J. T. Cushing, *Theory construction and selection in modern physics THE S MATRIX*. Cambridge University Press, 1990.