Graphs

This SageMath notebook plots numerical calculations for the paper *A theory of the dark matter*.

The section numbering follows the paper. Equation numbers refer to equations in the paper.

Preamble

In [1]:
```
%display latex
LE = lambda latex_string: LatexExpr(latex_string);
```

In [2]:
```
from mpmath import mp
from mpmath import mpf,mpc
import sage.libs.mpmath.all as a
mp.pretty = True
mp.dps=400
#
binary_precision=mp.prec
Reals = RealField(binary_precision+10)
RealNumber = Reals
myR = Reals
```

In [3]:
```
lambdaH = var('lambdaH',latex_name=r"\lambda_H")
gsq = var('gsq',latex_name=r"g^2")
K_EW = var('K_EW',latex_name=r"K_{\mathrm{EW}}")
ahat_EW = var('ahat_EW',latex_name=r"\hat a_{\mathrm{EW}}")
m = var('m')
#m = var('m',latex_name=r"k^2")
K = var('K')
EoverK = var('EoverK',latex_name=r"\frac{E}{K}")
```

In [4]:
```
lambdaH_val = 0.5080829235055462
gsq_val= 0.4262847210445738
K_EW_val =  myR(mp.ellipk(mpf(1/2)))
ahat_EW_val = myR(sqrt(6*pi)/(4*K_EW_val))
```

In [5]:
```
Kdot = K*(EoverK/(1-m) -1)/2
EoverKdot = EoverK -1/2 -EoverK^2/(2*(1-m))
def dot(fn):
    return m*fn.derivative(m) + Kdot*fn.derivative(K) + EoverKdot*fn.derivative(EoverK)
```

In [6]:
```
#
latex_name={}
latex_name['alpha']    = r"\alpha"
latex_name['bsq']      = r"\langle b^2\rangle"
latex_name['musq']    = r"\mu^2"
latex_name['ECGF']    = r"E_{\mathrm{EW}}"
latex_name['ahatsq']    = r"\hat a^2"
latex_name['rhohat']    = r"\hat \rho"
latex_name['ahat']    = r"\hat a"
latex_name['phat'] = r"\hat p"
latex_name['ahat_norm']    = r"\frac{\hat a}{\hat a_{\mathrm{EW}}}"
latex_name['phisq_norm']    = r"\frac{(\phi^\dagger\phi)_0}{v^2/2}"
latex_name['w'] = r"w"
for vstr,form in latex_name.items():
    var(vstr,latex_name=latex_name[vstr])
#
```

In [7]:
```
formula={}
formula['alpha']       = ((K/K_EW)*2*(1-m+(2*m-1)*EoverK))^(1/3)
formula['bsq']         = (m-1+EoverK)/alpha^2
formula['musq']        = (1-2*m)/alpha^2
formula['ECGF']        = (m*(1-m)/2)/alpha^4
formula['ahatsq']      = 3*bsq/2 + (4*lambdaH^2/gsq)*musq
formula['rhohat']      = ((3/gsq)*ECGF+(9/(32*lambdaH^2))*bsq^2)/ahatsq^2
formula['phat']      = ((1/gsq)*(ECGF-musq*bsq)-(9/(32*lambdaH^2))*bsq^2)/ahatsq^2
formula['w']          = phat/rhohat
```

```
formula['ahat']        = (ahatsq)^(1/2)
formula['ahat_norm']  = ahat/ahat_EW
formula['phisq_norm'] = 1 - (3/2)*bsq/ahatsq
for vstr,form in formula.items():
    pretty_print(LE(latex_name[vstr]),' = ', formula[vstr], hold= True)
    print('\n')
```

$$\alpha \;=2^{\frac{1}{3}}\left(\frac{\left(\frac{E}{K}(2\,m-1)-m+1\right)K}{K_{\mathrm{EW}}}\right)^{\frac{1}{3}}$$

$$\langle b^2\rangle \;=\frac{\frac{E}{K}+m-1}{\alpha^2}$$

$$\mu^2 \;=-\frac{2\,m-1}{\alpha^2}$$

$$E_{\mathrm{EW}} \;=-\frac{(m-1)m}{2\,\alpha^4}$$

$$\hat{a}^2 \;=\frac{4\,{\lambda_H}^2\mu^2}{g^2}+\frac{3}{2}\,\langle b^2\rangle$$

$$\hat{\rho} \;=\frac{3\left(\frac{32\,E_{\mathrm{EW}}}{g^2}+\frac{3\,\langle b^2\rangle^2}{{\lambda_H}^2}\right)}{32\,{\hat{a}^2}^2}$$

$$\hat{p} \;=-\frac{\frac{32\,\left(\langle b^2\rangle\mu^2-E_{\mathrm{EW}}\right)}{g^2}+\frac{9\,\langle b^2\rangle^2}{{\lambda_H}^2}}{32\,{\hat{a}^2}^2}$$

$$w \;=\frac{\hat{p}}{\hat{\rho}}$$

$$\hat{a} \;=\sqrt{\hat{a}^2}$$

$$\frac{\hat{a}}{\hat{a}_{\mathrm{EW}}} \;=\frac{\hat{a}}{\hat{a}_{\mathrm{EW}}}$$

$$\frac{(\phi^\dagger\phi)_0}{v^2/2} \;=-\frac{3\,\langle b^2\rangle}{2\,\hat{a}^2}+1$$

In [8]:

```
alpha       = ((K/K_EW)*2*(1-m+(2*m-1)*EoverK))^(1/3)
bsq         = (m-1+EoverK)/alpha^2
musq        = (1-2*m)/alpha^2
ECGF        = (m*(1-m)/2)/alpha^4
ahatsq      = 3*bsq/2 + (4*lambdaH^2/gsq)*musq
rhohat      = ((3/gsq)*ECGF+(9/(32*lambdaH^2))*bsq^2)/ahatsq^2
phat      = ((1/gsq)*(ECGF-bsq*musq)-(9/(32*lambdaH^2))*bsq^2)/ahatsq^2
w            = phat/rhohat
```

```
ahat       = (ahatsq)^(1/2)
ahat_norm  = ahat/ahat_EW
phisq_norm = 1 - (3/2)*bsq/ahatsq
```

In [9]:

```
def paramsub(vstr):
    temp = eval(vstr).subs(lambdaH=lambdaH_val,gsq=gsq_val,ahat_EW=ahat_EW_val,K_EW=K_EW_val)
    return temp
```

In [10]:

```
#
alphan       = paramsub('alpha')
bsqn         = paramsub('bsq')
musqn        = paramsub('musq')
ECGFn        = paramsub('ECGF')
ahatsqn      = paramsub('ahatsq')
rhohatn      = paramsub('rhohat')
phatn      = paramsub('phat')
wn      = paramsub('w')
ahatn        = paramsub('ahat')
ahat_normn   = paramsub('ahat_norm')
phisq_normn  = paramsub('phisq_norm')
```

In [11]:

```
def msub(v,m_val):
    # v is one of the physical variables, m is a Sage real
#    m_mpf = mpf(m_val)
    m_R = myR(m_val)
    K_val = myR(elliptic_kc(m_R))
    E_val = myR(elliptic_ec(m_R))
    EoverK_val = E_val/K_val
#    print(RDF(m_R),RDF(EoverK_val-1), RDF(E_val-K_val),RDF(K_val-myR(pi)/2))
    temp = v.subs({EoverK:EoverK_val,m:m_R,K:K_val})
    return myR(temp)
def fn(v):
    return lambda marg: msub(v,marg)
```

In [12]:

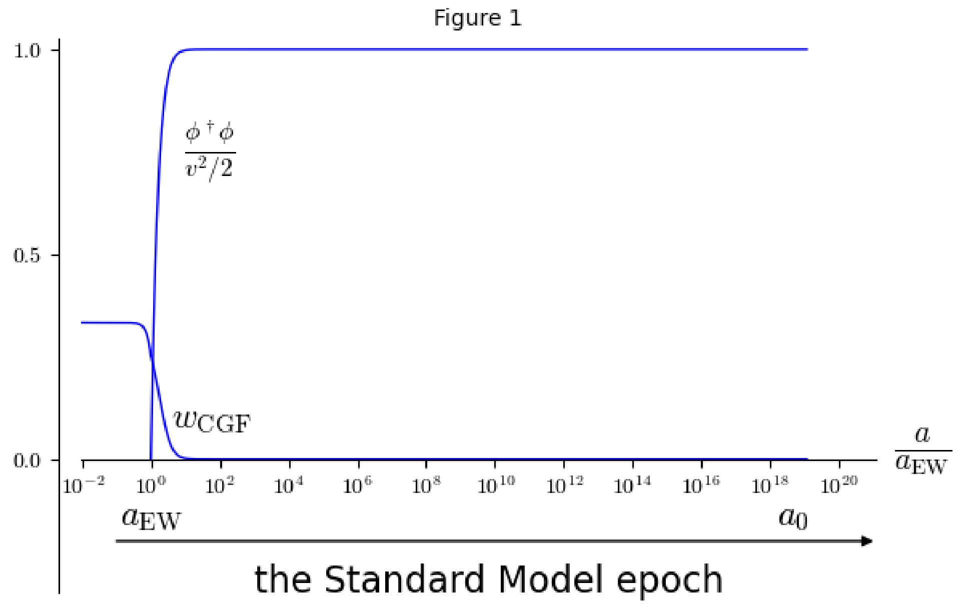
```
t = var('t')
ahat_t = lambda t: fn(ahat_normn)(10^(-t)/2)
w_t    = lambda t: fn(wn)(10^(-t)/2)
phisq_t    = lambda t: fn(phisq_normn)(10^(-t)/2)
#
plt1 = parametric_plot([ahat_t ,w_t],(t,0,56),scale="semilogx")
plt2 = parametric_plot([ahat_t,phisq_t],(t,0,56),scale="semilogx")
```

In [13]:

```
anorm=var('anorm')
coeff_low = ahat_EW_val^4 * gsq_val/(lambdaH_val^2)
w_low = lambda anorm: -1+4/(3+anorm^4*coeff_low)
plt3=plot(w_low,(anorm,1e-2,1),scale='semilogx')
```

In [14]:

```
plt=plt1+plt2+plt3
#  +line([(1e-2,1),(100,1)],linestyle=":")\
plt = plt\
+text(r"$w_{\mathrm{CGF}}$",(6e1,.1),fontsize=16,color='black')\
+text(r"$\frac{\phi^\dagger\phi}{v^2/2}$",(5e1,.75),fontsize=16,color='black')\
+text(r"the Standard Model epoch",(10^3,-.3),fontsize=16,color='black',\
      horizontal_alignment='left',fontweight=300)\
+text(r"$a_0$",(4.54e18,-.14),fontsize=16,color='black')\
+text(r"$a_{\mathrm{EW}}$",(1,-.14),fontsize=16,color='black')
#plt.set_aspect_ratio(1)
plt.set_axes_range(xmin=10^(-2),xmax=10^(22),ymin=0)
plt = plt+ arrow((.1,-.2),(10^21,-.2),width=1,arrowsize=2,color='black')
plt.axes_labels([r"$\frac{a}{a_{\mathrm{EW}}}$",""])
plt.axes_labels_size(2)
#plt.axes_labels(["",""])
show(plt,title="Figure 1",\
     ticks=[[10^(-2),1,10^2,10^4,10^6,10^8,10^(10),10^(12),10^(14),10^(16),10^(18),RDF(1.0e20)],[0,1.
     tick_formatter='latex')
plt.save('plots/SMepoch.pdf',dpi=600,\
    ticks=[[10^(-2),1,10^2,10^4,10^6,10^8,10^(10),10^(12),10^(14),10^(16),10^(18),RDF(1.0e20)],[0,1/
     tick_formatter='latex')
```



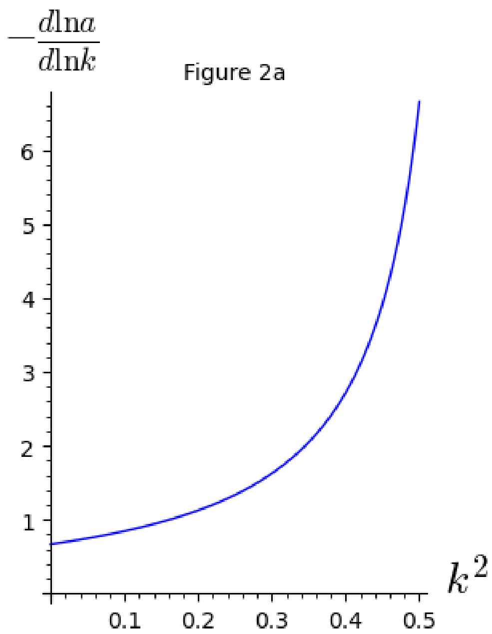
4.3 a as a function of k^2

Figure 1 showing that a increases monotonically from a_{EW} to ∞ as k^2 decreases from $\frac{1}{2}$ to 0.

In [15]:
```
y=-(dot(ahatsq)/ahatsq)
yn = paramsub('y')
```

In [16]:
```
plt=plot(fn(yn),m,1e-8,0.5)
```

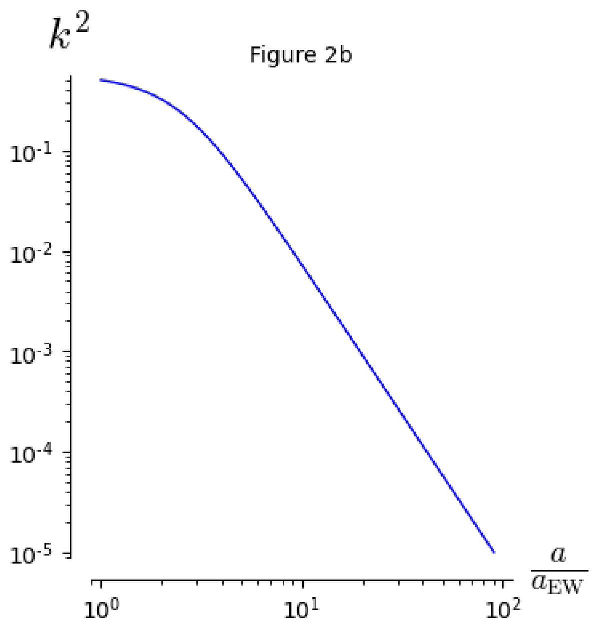
In [17]:
```
plt.set_axes_range(ymin=0)
plt.set_aspect_ratio(0.1)
plt.axes_labels([r"$k^2$",r"$-\frac{d \ln a}{d\ln k}$"])
plt.axes_labels_size(2)
show(plt,title="Figure 2a")
plt.save('plots/aofkmonotonic.pdf',dpi=600)
```



In [18]:
```
plt = parametric_plot([fn(ahat_normn),m],(m,1e-5,0.5),scale="loglog")
```

In [19]:
```
plt.set_aspect_ratio(0.5)
plt.set_axes_range(xmin=1,xmax=100,ymax=0.5)
plt.axes_labels([r"$\frac{a}{a_{\mathrm{EW}}}  $",r"$k^2$"])
plt.axes_labels_size(2)
```

```
plt.show(title="Figure 2b")
plt.save('plots/ksqofa.pdf',dpi=600)
```

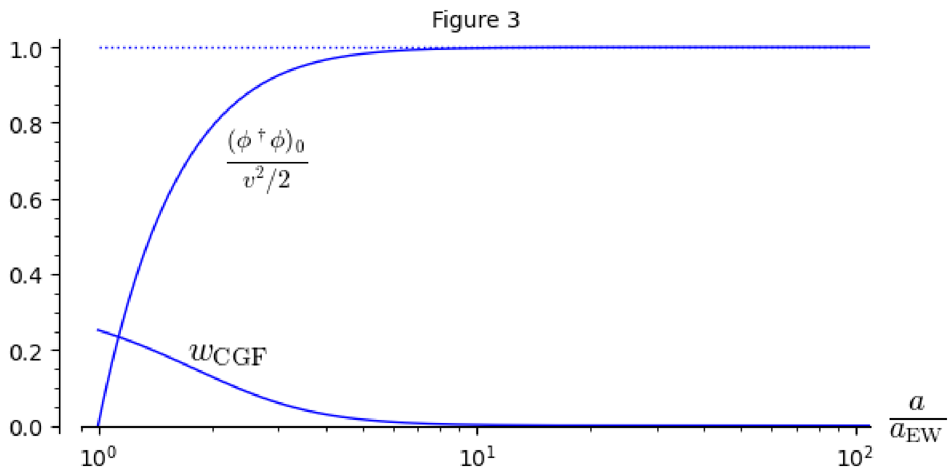


5.2 w_CGF=0

Figure 2 shows w_{CGF} and the progress of the electroweak transition in the first 2 ten-folds of expansion after a_{EW}.

In [20]:
```
plt1 = parametric_plot([fn(ahat_normn),fn(wn)],(m,1e-6,0.5),scale="semilogx")
plt2 = parametric_plot([fn(ahat_normn),fn(phisq_normn)],(m,1e-6,0.5),scale="semilogx")
```

In [21]:
```
plt=plt1+plt2\
+line([(1,1),(100,1)],linestyle=":")\
+text(r"$w_{\mathrm{CGF}}$",(2.2,.2),fontsize=16,color='black')\
+text(r"$\frac{(\phi^\dagger\phi)_0}{v^2/2}$",(2.8,.7),fontsize=16,color='black')
plt.set_aspect_ratio(1)
plt.set_axes_range(xmin=1,xmax=100)
plt.axes_labels([r"$\frac{a}{a_{\mathrm{EW}}}$",""])
plt.axes_labels_size(2)
show(plt,title="Figure 3")
plt.save('plots/wandphisq.pdf',dpi=600)
```

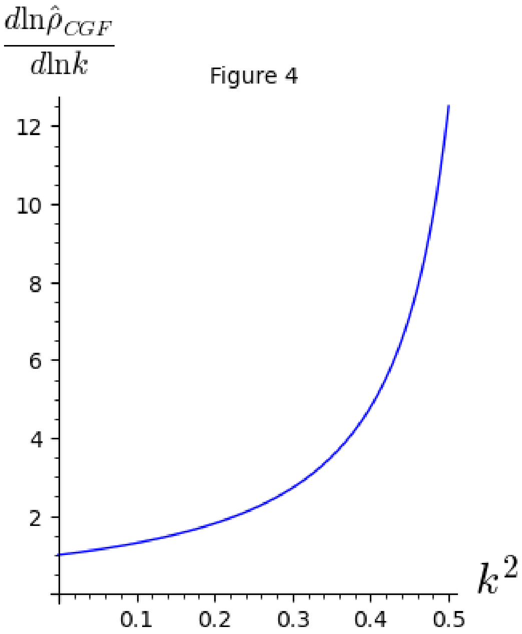


5.4 CGF equation of state

In [22]:
```
y=(dot(rhohat)/rhohat)
yn = paramsub('y')
```

In [23]:
```
plt=plot(fn(yn),m,1e-8,0.5)
```

In [24]:
```
plt.set_axes_range(ymin=0)
plt.set_aspect_ratio(0.05)
plt.axes_labels([r"$k^2$",r"$\frac{d \ln \hat\rho_{CGF}}{d\ln k}$"])
plt.axes_labels_size(2)
show(plt,title="Figure 4")
plt.save('plots/rhoofkmonotonic.pdf',dpi=600)
```



5.5 Adiabatic condition for $a \geq a_{\mathrm{EW}}$

Figure 4 showing a bound on the ratio of time scales, verifying the adiabatic condition.

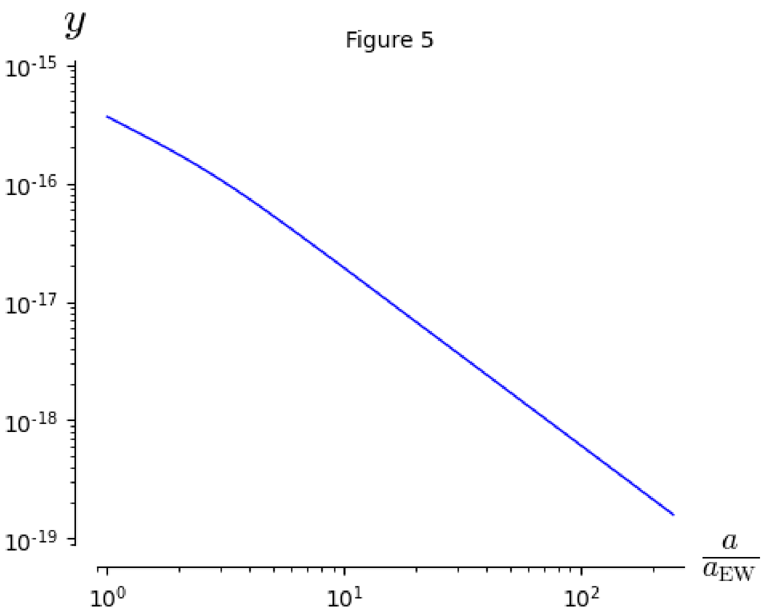
In [25]:
```
r_gH= 2.6387686437244608e-33
r_HH= 1.3207956122976951e-88
Omega_Lambda = 0.685
pretty_print(LE(r"r_{\mathrm{gH}}=\frac{t_{\mathrm{grav}}^2}{t_{\mathrm{Higgs}}^2}="),\
            RDF(r_gH),LE(r"\qquad r_{\mathrm{HH}}=\frac{t_{\mathrm{Higgs}}^2}{t_{\mathrm{Hubble}}^2
           LE(r"\qquad \Omega_\Lambda="),RDF(Omega_Lambda))
```

$$r_{\mathrm{gH}} = \frac{t_{\mathrm{grav}}^2}{t_{\mathrm{Higgs}}^2} = 2.6387686437244608 \times 10^{-33} \qquad r_{\mathrm{HH}} = \frac{t_{\mathrm{Higgs}}^2}{t_{\mathrm{Hubble}}^2} = 1.3207956122976951 \times 10^{-88} \qquad \Omega_\Lambda = 0.685$$

In [26]:
```
bound = 4*K*alpha*sqrt(r_gH*ahatsq*rhohat/3 + r_HH*Omega_Lambda*ahatsq)
boundn = paramsub('bound')
```

In [27]:
```
plt= parametric_plot((fn(ahat_normn),fn(boundn)),(m,5e-7,0.5),scale="loglog")
```

In [28]:
```
plt.set_aspect_ratio(.5)
plt.set_axes_range(ymax=1e-15,ymin=1e-19)
plt.axes_labels([r"$\frac{a}{a_{\mathrm{EW}}}  $",r"$y$"])
plt.axes_labels_size(2)
show(plt,title="Figure 5")
plt.save('plots/adiabaticbound.pdf',dpi=600)
```



6.4 Temperature after $a_{\rm EW}$

CGF temperature $k_{\rm B}T_{\rm CGF} = \frac{\hbar}{4K'\alpha a} = \frac{m_{\rm Higgs}}{4K'\alpha \hat{a}}$

redshifted to the present $k_{\rm B}T_{\rm rs} = \frac{a}{a_0}k_B T_{\rm CGF} = \frac{m_{\rm Higgs}}{4K'\alpha \hat{a}_0}$

use numbers from the Arithmetic notebook and from above

In [29]:
```
T_CMB = mpf(2.7255) # K
K_EW_mpf = mp.ellipk(1/2)
m_Higgs_mpf= mpf(125.1)     # GeV
kB_mpf = mpf(8.61733326214518e-14)  # GeV/K
ahat0_mpf = mpf(2.6592893046343716e18)   # s
ahat_EW_mpf = mpf(0.5854143283037644)    # s
lambda_mpf = mpf(lambdaH_val)
gsq_mpf = mpf(gsq_val)       # g^2
```

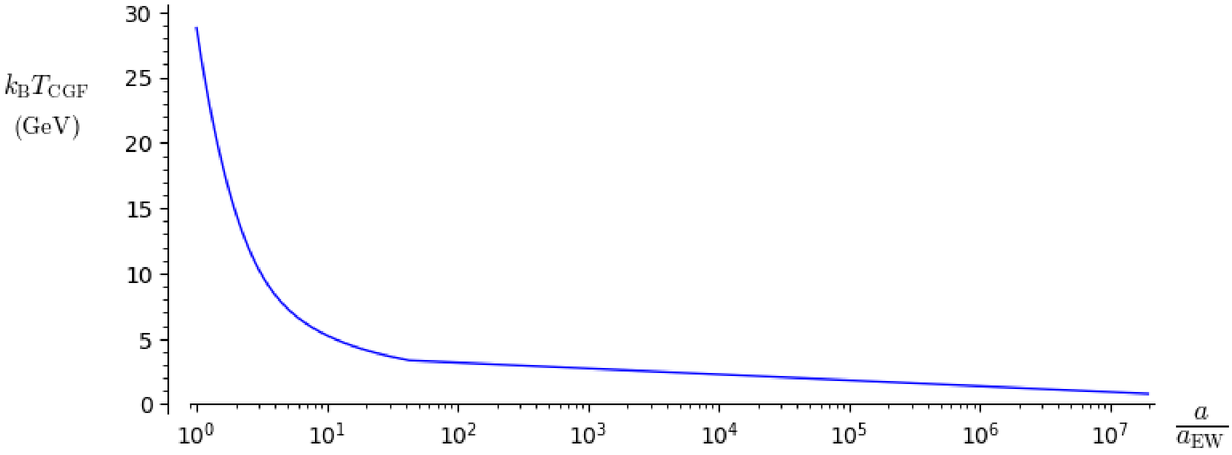
define functions in mpmath

In [30]:
```
def kT_CGF(ma):
    m=mpf(ma)
    K = mp.ellipk(m)
    Kp = mp.ellipk(1-m)
    E = mp.ellipe(m)
    EoverK = E/K
    alpha = mp.power( (K/K_EW_mpf)*2*(1-m+(2*m-1)*EoverK) ,1/3)
    ahatsq = ((3/2)*(m-1+EoverK)+(4*lambda_mpf^2/gsq_mpf)*(1-2*m))/alpha^2
    ahat = mp.sqrt(ahatsq)
    return m_Higgs_mpf/(4*Kp*alpha*ahat)
def T_rs(ma):
    m=mpf(ma)
    K = mp.ellipk(m)
    Kp = mp.ellipk(1-m)
    E = mp.ellipe(m)
    EoverK = E/K
    alpha = mp.power( (K/K_EW_mpf)*2*(1-m+(2*m-1)*EoverK) ,1/3)
    return m_Higgs_mpf/(4*Kp*alpha*ahat0_mpf*kB_mpf)
def ahat_over_ahat0_mp(ma):
    m=mpf(ma)
    K = mp.ellipk(m)
    E = mp.ellipe(m)
    EoverK = E/K
    alpha = mp.power( (K/K_EW_mpf)*2*(1-m+(2*m-1)*EoverK) ,1/3)
    ahatsq = ((3/2)*(m-1+EoverK)+(4*lambda_mpf^2/gsq_mpf)*(1-2*m))/alpha^2
    ahat = mp.sqrt(ahatsq)
    return ahat/ahat_EW_mpf
```

In [31]:
```
plt=parametric_plot((ahat_over_ahat0_mp,kT_CGF),(m,1e-21,0.5),scale="semilogx")
plt.set_aspect_ratio(.1)
plt.set_axes_range(ymin=0,ymax=30,xmin=1,xmax=1e7)
```

```
yaxis_label = Graphics()+text(r"$ k_\mathrm{B} T_{\mathrm{CGF}}$",(-0.15,0.80),axis_coords=True,font
yaxis_label += text(r"$ (\mathrm{GeV})$",(-0.15,0.70),axis_coords=True,fontsize=11,color='black')
xaxis_label = Graphics()+text(r"$\frac{a}{a_{\mathrm{EW}}}  $",(1.05,-0.04),axis_coords=True,fontsiz
plt2 =  plt+yaxis_label+xaxis_label
plt.axes_labels_size(2)
show(plt2,title="Figure 6",figsize=[8,20],ymax=30,xmin=1)
plt2.save('plots/kT_after_a_EW.pdf',figsize=[8,20],ymax=30,xmin=1,dpi=600)
```

Figure 6



Solve $T_\mathrm{rs}(k^2) = T_\mathrm{CMB}$

In [32]:

```
m_dc=mp.findroot(lambda ma: T_rs(ma)-T_CMB,1e-17,solver='anewton')
T_rs_dc=T_rs(m_dc)
ahatnorm_dc = ahat_over_ahat0_mp(m_dc)
kT_dc = kT_CGF(m_dc)
```

In [33]:

```
pretty_print(\
    LE(r"k^2 =  "), RDF(m_dc).n(prec=20),\
    LE(r"\qquad \frac{a}{a_0} k_B T_{\mathrm{CGF}} = "),RDF(T_rs_dc).n(prec=20),LE(r"\,\mathrm{K}"),
    LE(r"\qquad \frac{a}{a_0} =  "),RDF(ahatnorm_dc).n(prec=20),\
    LE(r"\qquad k_B T_{\mathrm{CGF}} =  "),RDF(kT_dc).n(prec=20),LE(r"\,\mathrm{GeV}"))
```

$$k^2 = 5.1168 \times 10^{-18} \qquad \frac{a}{a_0} k_B T_\mathrm{CGF} = 2.7255\,\mathrm{K} \qquad \frac{a}{a_0} = 1.1305 \times 10^6 \qquad k_B T_\mathrm{CGF} = 0.94371\,\mathrm{GeV}$$