

THE SPEED OF LIGHT

OBJECT: To determine the speed of light and to learn the operation and use of a laser beam expander (Gaussian optics), an acousto-optic modulator, a half wave plate, and a beam splitter cube.

INTRODUCTION: *Historical Note:* Galileo tried to measure the speed of light by timing the round trip time of light flashes from lanterns, one held by him and the other by an assistant located a distance away -- all he measured was the human reaction time.

In 1676 Ole Römer noticed that the timing of the occultations and transits of the Jovian moons varied with the distance between Jupiter and the Earth. He deduced that the observed variations in the observed eclipse times was due to the different time intervals required for light to travel from Jupiter to Earth as their separation changed. He calculated that light required 22 minutes to traverse the diameter of the Earth's orbit; modern measurements put this at 17 minutes.

Fizeau in 1849 used a rotating toothed wheel with 720 teeth and a path length of about 17 km; he measured the speed of light as 3.15×10^8 m/s.

In 1860 Foucault used a rotating wheel with faceted mirrors. Albert Michelson redid this experiment in 1883 at Mount Wilson Observatory using a path length of 44 miles. He found $c = 2.9980 \pm 0.0002 \times 10^8$ m/s. By continually improving his experiment he finally arrived at $c = 299,774 \pm 11$ km/s. Michelson was America's first Nobel laureate in physics.

Today the accepted value of the speed of light is $c = 2.99792458 \times 10^8$ m/s.

Theory: The experimental procedure is to amplitude modulate a beam of light, split it into two beams and send them over two different paths that vary greatly in length. The first beam, the reference beam, travels over a very short path to a detector. The second beam, the signal beam, is sent over a long path before it is detected and compared to the reference beam. When the two beams were first split the phase of the amplitude modulation of the two beams was naturally the same. However, when you compare the signals at the detectors at a specific moment, the signal beam you are comparing with the reference started out earlier in order to have traveled over the long path to reach the detector at that instant. Thus, for sinusoidal modulation, its phase leads that of the reference beam. By measuring the phase difference between the reference and signal beams

you can calculate Δt , the time it takes the signal beam to travel the extra distance ΔL . The speed of light is readily calculated from $c = \Delta L / \Delta t$.

APPARATUS: The apparatus is shown in Fig. 1. The **Helium-Neon laser** (4 mW) puts out a very narrow, intense polarized beam. However, because of its very narrow waist diameter, this beam would expand excessively by the time it had traveled back and forth between the mirrors resulting in a very weak signal from the detector. To reduce the expansion, the beam is passed

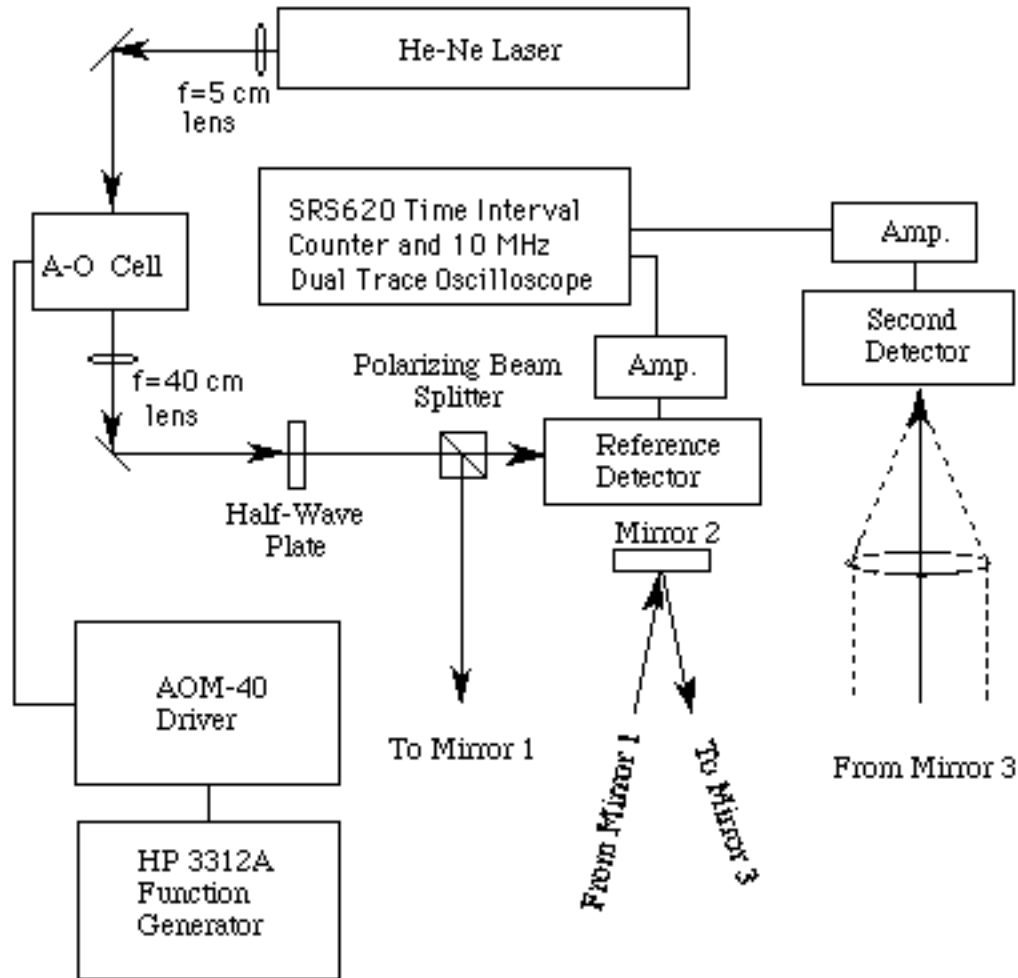


Figure 1: The Speed of Light Apparatus

through two lenses with focal lengths of 5 and 40 cm which together act as a **Beam Expander**. As you found (will find) in the experiment on **Gaussian Optics** when the waist diameter of a beam is enlarged, the rate of expansion is reduced. The focal length and locations of the two lenses are chosen to minimize the beam waist when the signal beam arrives at the signal detector. After passing through the first lens, the beam is directed into an **Acousto-Optic Modulator**, which sinusoidally amplitude modulates the laser beam at a

frequency (0 to 10 MHz) determined by the highly stable function generator. The AOM is discussed in Appendix 1.

The beam is split into the signal and reference beams by passing it through the **Cube Beamsplitter**. The cube beamsplitter transmits any light that is polarized vertically; this is the reference beam. It reflects at 90 degrees any light that is polarized horizontally; this is the signal beam. Before entering the cube beamsplitter, the modulated beam passes through a **Half-Wave Plate** that rotates the plane of polarization of the laser beam (see Appendix 2). The amount of rotation can be changed by rotating the half-wave plate. The purpose of the half-wave plate is to serve as an adjustable attenuator. The half-wave plate is rotated so that the signal and reference beams have the same amplitude as seen on the dual trace oscilloscope. (Because much of the signal beam is lost due to beam expansion and reflection losses, the half-wave plate is adjusted so that the signal beam when it first starts out is much more intense than the reference.) The theory behind the beam splitter cube and the half wave plate are discussed in the readings.

The signal beam travels down the hallway to **Mirror 1** which, in turn, reflects the beam back to **Mirror 2** on the optical table, which reflects the beam to **Mirror 3**. This mirror returns the beam to the optical table where a large collecting lens focuses the diverged beam onto a detector. The electrical signals from the **Reference and Signal Detectors** are amplified and passed to both a SRS 620 Universal Time Interval Counter and an oscilloscope. There are actually two amplifiers for each signal -- an amplifier attached directly to the detector and a SRS model SR445 amplifier, which amplifies by x10 or x100 depending on the number of stages used. The two detectors, amplifiers and coaxial cable lengths should be as close to identical as possible so that any delays they introduce are the same and cancel out.

The oscilloscope and the SRS 620 will be used to independently measure the time interval Δt between maxima in the modulation of the two signals. The oscilloscope will give the least precise value for Δt . You will first adjust the half-wave plate so that the two signals have the same amplitude. Then you will use the "add" mode on the oscilloscope to add the signals, and adjust the modulation frequency f so that the sum is zero and the two signals are then exactly out of phase, $\Delta\phi = \pi$. Then using the proportion

$$\frac{\Delta\phi}{2\pi} = \frac{\Delta t}{T} = \Delta t f, \quad (1)$$

we see that $\Delta t = 1/(2f)$. To get another value for Δt , you will "invert" one of the signals, again add them and adjust f so that the sum is zero. The two signals are

then exactly in phase, $\Delta\phi = 2\pi$, and $\Delta t = 1/f$. If the path difference were longer or the AOM could operate at higher frequencies, you could get additional points for $\Delta\phi = 3\pi, 4\pi, \dots$. But for the current setup you will have to be content with only two oscilloscope measurements of Δt . However, this is not a real problem since we can instead use the SRS 620 to measure Δt with high precision. Moreover the SRS can repeat the measurement for up to one million times and calculate the average time interval and standard deviation (jitter). The SRS 620 is a highly versatile (and complicated) instrument whose operation we will discuss in the procedure section of this write-up.

PROCEDURE: Important warnings!!

The 4 mW laser can cause serious damage to your eyes. Even the diffusely reflected light should be avoided.

Be sure to replace the dust covers on the three mirrors whenever you are finished using them.

Be sure to turn off the batteries on the photoelectric detectors. The on/off switch is on the casing side. The battery will die if left on for a few days.

1. Measure ΔL , the difference in path length for the two beams from the cube beam splitter to the reference and signal detectors. This difference is about 40 m. Make your measurement as carefully as possible since the accuracy of your value for c will depend on the quality of this measurement. Estimate the error in ΔL .
2. Turn on the photodiode bias batteries for the two photodetectors and make sure each amplifier is supplied with 5 to 10 V. **Do not apply more than 10 V.** Turn on the AOM-40 driver and HP 33120A function generator.
3. Make sure the relay rack containing the AOM-40 driver and HP 33120A Function generator is not in contact with the optical table since the instruments generate considerable vibration which can degrade the quality of your signal. Set the function generator to sine wave with the peak-to-peak voltage equal to 1 V and the frequency to about 2 Hz. Place a white card in the laser beam on the output side of the AOM, next to the 40 cm lens. The 2 Hz signal amplitude modulates on and off the 40 MHz sound wave in the AOM which is acting as a diffraction grating. This has the effect of turning on and off the diffracted beam at the 2 Hz rate. Thus on the white card you should see a spot flickering at 2 Hz. This is the first order diffracted beam. If you see a steady spot, it is the primary (zeroth order) beam. Adjust the alignment of the beam as it passes through the AOM to maximize the flickering spot and minimize the steady spot. This is the most difficult and critical adjustment you will need to make for this experiment!

The manufacturer claims that with careful adjustment up to 90% of the laser energy can be diffracted into the first order beam. When you have maximized the first order beam, adjust the small beam stop to eliminate the zeroth order beam while passing the first order.

4. Remove the white card and adjust the beam so that it strikes the reference detector. Make additional adjustments of the AOM alignment and beam alignment to maximize the reference signal on the oscilloscope. The scope signal should be a sine wave with no ripples or distortion. Applying a modulating voltage from the HP much greater than 0.25 V will give you distortion and erroneous results. Rotate the half-wave plate and note how the amplitude of the reference signal changes.

5. Adjust the three mirrors shown in the figure to place the signal beam on the signal detector and then to maximize the signal you see on the oscilloscope. It pays to spend time maximizing this signal since the quality of your results will depend on having a large signal with little jitter. You will need the two signals to be at least 1 V peak-to-peak, and preferably larger.

6. Rotate the half-wave plate so that the signal beam and the reference beam traces on the oscilloscope have the exactly same amplitude. Now increase the modulation frequency to the range you will use for the experiment -- 1 to 8 MHz. The low modulation frequency you used for steps 3 to 5 was simply to make it possible to visually adjust the AOM.

7. Set the scope to "add" and adjust the modulation frequency so the sum is exactly zero. [You may see a small residual signal at twice or three times the modulation frequency. These harmonics are due to distortion produced by over modulation of the AOM. You can reduce it by reducing the amplitude of the driving voltage of the AOM.] Record this frequency. Estimate how much error there is in determining this frequency. Next set the scope to subtract the two signals and readjust the frequency to give a zero sum. Record this frequency and estimate the error (by repeating the settings a number of times and calculating the mean and standard deviation). Convert these frequencies to Δt as discussed in the previous section. Calculate your experimental value for the speed of light and your estimated experimental error.

8. Now repeat your measurement of Δt using the SRS 620. You will be making your measurement using both the phase mode and the time interval mode. After the SRS boots make the following settings for a phase reading:

Mode: Phase
Source: A

Gate: + Time	[This means a pulse arriving at Channel A will start the counting which will continue until there is a pulse at B.]
Sample Size: AUTO 1x10 ² .	[Increase the sample size for your final data to decrease the error.]
Display: Mean	[This is the mean of the (100) samples the SRS has taken. To get the standard deviation choose jitter. To get the trigger levels choose trigger.]]
A Input: AC 50 Ohm	
A TRG level: adjust to give 0.20 V	[The SRS will begin counting time when the A signal reaches + 0.2 V]
B Input: AC 50 Ohm	
B TRG level: adjust to 0.20 V. It MUST be identical to the A TRG level setting.	[The SRS will stop counting time when give the A signal reaches + 0.2 V]

To avoid serious systematic errors, it is essential that the SRS be properly triggered: The SRS works by beginning to count time when the signal applied to channel A reaches a certain trigger level and stopping when the signal at channel B reaches a separately adjustable trigger level. It is crucial that these trigger levels be adjusted properly so that A and B trigger at exactly the same point in the sine wave. Otherwise there will be a possibly large systematic error. The SRS has two trigger modes. The automatic mode is set by turning the trigger level knobs all the way counter-clockwise. This mode sets the trigger at exactly half the peak-to-peak voltage, which for a sine wave is at zero volts. Unfortunately, the detectors or amplifiers in the speed of light apparatus are not exactly linear. So the signals are not perfect sine waves and the automatic trigger levels do not fall exactly at zero making a systematic error possible. To avoid this, you should use the manual trigger mode, which allows you to set the A and B trigger levels individually. **To get good results use the half wave plate to make the amplitudes of the signal and reference voltages on the oscilloscope identical (about 1 V peak-to-peak, with no visible distortion)** and setting the A and B trigger levels to exactly the same value, about 0.2 V, which can be seen when the display is set to show trigger levels.

9. When the signal has stabilized, record the mean value for $\Delta\phi$, the standard deviation, and the modulation frequency. Increase the sample size to decrease the error but note that the value of the jitter does not change. This is because the jitter or standard deviation gives the error you could expect if you made one more measurement. The error of the mean -- the standard deviation of the mean -- is related to the jitter σ by

$$\sigma_{mean} = \sigma / \sqrt{N}$$

where N is the number of measurements you have averaged, i.e. the sample size -- 100 to 10,000. The reason you might not want to make N very large is that the measuring time also becomes very large as N increases. Also note that at some point the error in ΔL will dominate the error and there is no point in improving the time measurement. Repeat your measurement for about 10 different modulation frequencies. Note the SRS readings become unreliable when the phase difference is 180° (and 360°) due to ambiguity in whether the phase is + or - 180° . Avoid these frequencies.

Question (answer in your notebook and mention in report)

- For what frequencies will the phase difference be 180° and 360° ?
- Can one tell from the SRS the difference between say 15° and $375^\circ = 360^\circ + 15^\circ$?

Make sure to have the correct phase reading!

Calculate Δt using the reasoning given for Eq. (1). Average the values of Δt you obtained for the various modulation frequencies. Simple averaging assumes the error (jitter) is the same for all readings, which is not true. A weighted average will give a more reliable value for the mean. Estimate the error in your mean value for Δt

Calculate the speed of light and estimate your measurement error.

REPORT: One of the most important goals of this experiment is to give you experience in treating various kinds of measurement errors in a scientifically valid manner.

1. In your report you will derive a value for c and its error as determined using the oscilloscope to measure elapsed time and a separate value using the SRS 620. For both measurements make a plot of the phase shift versus frequency and use a linear fit to obtain the speed of light. Show the fit and error bars on the plots.
2. Carefully estimate the random error in the measurement and discuss the sources and probable magnitude of systematic errors in your measurement. In your discussion differentiate between the precision and accuracy of your measurements.
3. Use your error estimates to compare your results with the defined speed of light.
4. What would be the percent error in c if the two coaxial cables (on the reference and signal sides) differed in length by 2 cm? [A signal on a 50 ohm coaxial cable travels with a speed of about $2c/3$.]

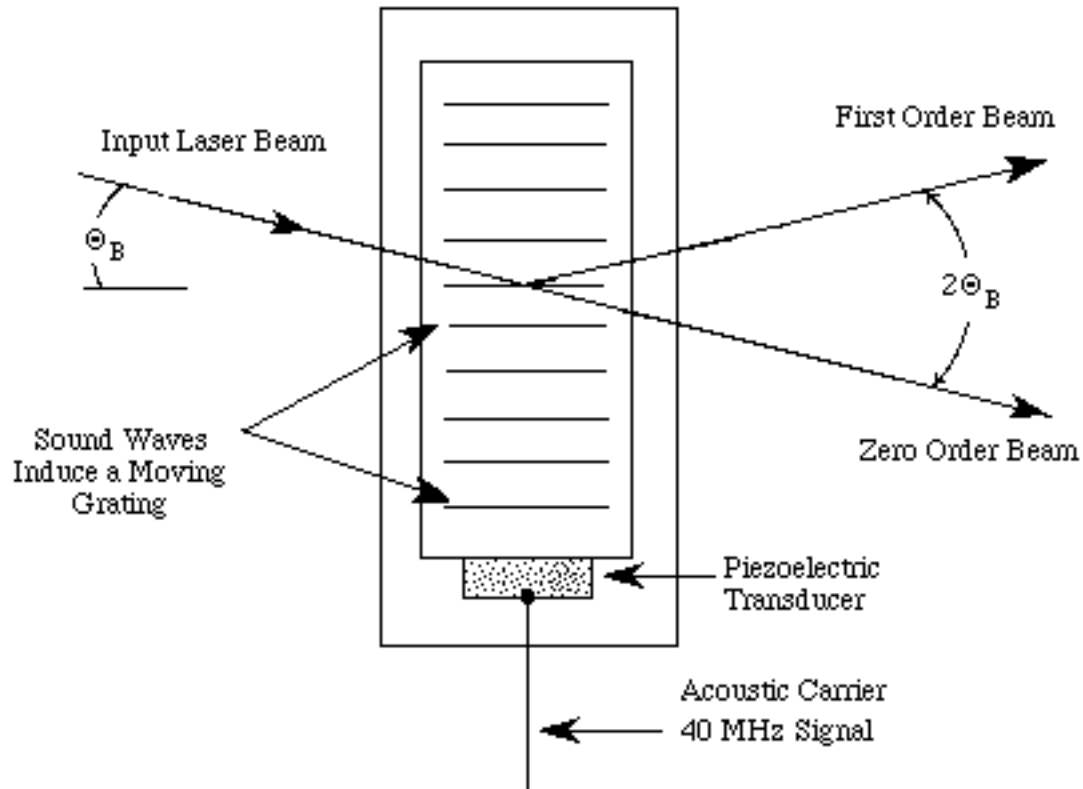
5. Does the wave-nature of light play a role in this experiment. If yes explain how. If not discuss under what circumstance will the finite wavelength of light become important.

REFERENCES: There is a binder in the lab with the following readings:

- “Precision Measurement of the Speed of Light” Hamsberger et al. This gives a description of the experiment as done at Bethel College and reproduced here at Rutgers.
- Model AOM-40 Acousto-Optic Modulator- Instruction Manual - InterAction Corp.
- “The Acousto-Optic Modulator” excerpt from a lab manual written by Kevin Jones for the Modern Physics lab at Middlebury College.
- High Speed Silicon Detector, ThorLabs. Data sheet for the photo detectors used in this experiment.
- “Gaussian Beam Optics” excerpt from Optics Guide 5 published by Melles Griot giving a tutorial on Gaussian beam optics and beam expanders.
- “Cube Beamsplitters” excerpt from Optics Guide 5 published by Melles Griot.
- “Retardation Plates” excerpt from Optics Guide 5 published by Melles Griot giving a tutorial on the half-wave plate.
- “Time-of-flight measurement of the speed of light using an acousto-optic modulator’ Deblaquiere et al. American Journal of Physics 59, 443 (1991). This journal article describes a similar but somewhat different use of the AOM to measure c .

APPENDIX 1: The Acousto-Optic Modulator consists of lead zirconate titanate piezoelectric transducer that is epoxy bonded to a (flint) glass plate. The transducer responds to a 40 MHz electric signal by expanding and contracting which generates an acoustic signal in the glass. The ultrasonic waves vary the pressure and density in the glass which minutely changes the

Acousto-Optic Modulator



refractive index of the glass and produces a moving diffraction grating. The diagram shows a light beam incident at nearly grazing angle to the wavefronts of the sound waves. (This angle is exaggerated in the diagram.) It is deflected by the sound field through twice the Bragg angle,

$$\sin \Theta_B \approx \Theta_B = \frac{\lambda}{2\Lambda},$$

where λ is the wavelength of the light and Λ is the wavelength of the sound wave. Energy is transferred with maximum efficiency into the first order beam when the angle of incidence of the light beam onto the grating equals the Bragg angle. The speed of sound in the glass is about 3800 m/s. So for a 40 MHz sound wave $\Lambda = 0.095$ mm. For a HeNe laser $\lambda = 633$ nm. Thus

$$\Theta_B = .0067 \text{ rad} = 0.38^\circ,$$

and we see that very careful, delicate alignment is needed to optimize the intensity of the first order beam.

In order to modulate the intensity of the first order deflected beam, the 40 MHz driver voltage is modulated by a signal generator, which effectively turns the 40 MHz grating in the medium on and off at the modulation frequency. Note that you have three frequencies here -- the very high frequency of the laser, the 40 MHz frequency of the sound wave that produces the diffraction grating and the variable frequency 0 to 10 MHz voltage that amplitude modulates the sound wave, turning the diffraction grating on and off, which in turn amplitude modulates the laser beam. The Acousto-Optic Modulator is made by IntraAction Corporation, model; AOM-40. The 40 MHz driving signal is produced by their model ME-40 driver. The amplitude of this 40 MHz signal is modulated by a lower frequency signal that you input to the unit. The maximum modulation frequency at which the AOM will operate is about 10 MHz. A very stable 1 MHz to 10 MHz function generator is needed to provide the 0.1 V sinusoidal drive signal.

APPENDIX 2 --Half Wave Plate: The half-wave plate is a very simple optical device that rotates the direction of polarization of linearly polarized light. It is made of a single crystal of an anisotropic or birefringent material like quartz or mica and its operation depends on that fact that the index of refraction in these materials depends on the direction of the polarization of the light relative to the crystalline axes. It is very easy to see how the HWP works as follows. Suppose the light is traveling in the z direction and strikes a HWP of thickness d. Let the crystal axes be along x and y and the index of refraction be n_x and n_y respectively with $n_y > n_x$. Assume the polarization makes an angle ϕ with the x axis. Then the electric field is:

$$E = A \cos \omega t$$

and

$$E_x = A_x \cos \omega t \text{ and } E_y = A_y \cos \omega t$$

where

$$A_x = A \cos \phi \text{ and } A_y = A \sin \phi.$$

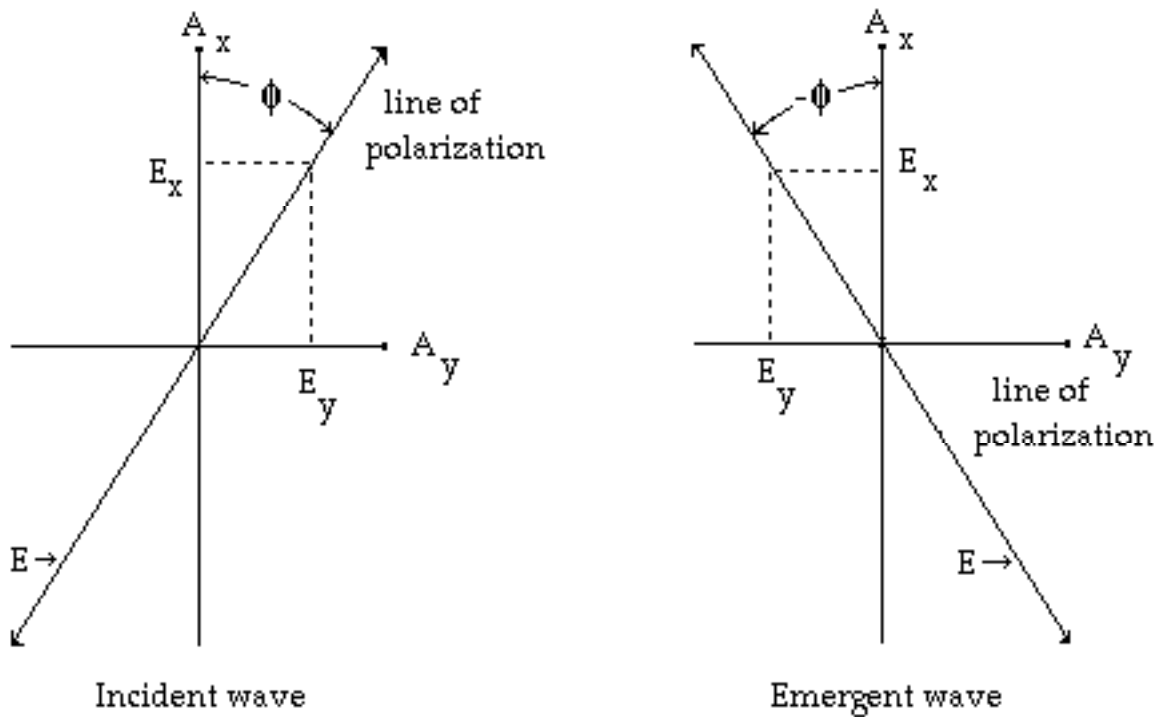
After passing through the plate the two components are no longer in phase:

$$E_x = A_x \cos(\omega t - 2\pi \frac{n_x d}{\lambda}) \text{ and } E_y = A_y \cos(\omega t - 2\pi \frac{n_y d}{\lambda})$$

where $\omega \lambda = 2\pi c$. The emerging wave is in general elliptically polarized rather than linearly with E_y lagging behind E_x by an angle

$$\theta = 2\pi \frac{d}{\lambda} (n_y - n_x).$$

But if the thickness of the HWP is chosen so that $d(n_y - n_x) = \lambda/2$, then $\theta = \pi$ and the emerging wave is again linearly polarized but with the angle of polarization now rotated by 2ϕ from the original direction as shown in the figure. Thus the polarization of a linearly polarized wave can be rotated to any desired direction simply by rotating the HWP to change the angle of the incident wave with its crystalline axes.



Principle of Operation of a Half-Wave Plate
(wave traveling into the paper)

JP 7/99