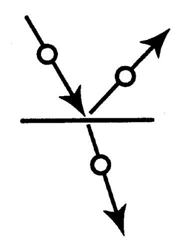
Fresnel Equations and Electromagnetic Boundary Conditions

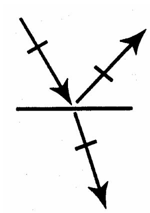
Note: Prior to reading this write-up you should read the **Background information** on Fresnel's equations from the course webpage.

Goal:

- to test experimentally Fresnel equations for the case of a non-conducting, nonmagnetic transparent medium. To this end you will measure the angle and polarization dependence of the reflection and transmission of light at the interface between air and an unknown transparent material.
  a) You will measure the angle dependence of the reflected and refracted beams to prove Snell's law and to obtain the index of refraction of a transparent medium.
  b) You will measure the power of the reflected and refracted beams as a function of the incident angle for S and P polarizations. By fitting your data with Fresnel's equations you will obtain the following: i) the index refraction ii) the critical angle iii) Brewster's angle. Note: the relation between power, P, (which you will measure) and intensity, I, is: I = P/A, where A is the beam area.



Perpendicular (S) polarization sticks up out of the plane of incidence.



Parallel (P) polarization is parallel to the plane of incidence.

Theory: Read the chapter on boundary conditions and Fresnel equation in any E&M textbook as well as the supplementary material on the course webpage (For a detailed description of theory and experimental pitfalls in this experiment see Phil's E&M file.)

Apparatus: We will use the new PASCO setup. It is equipped with an optics rail, red 650nm diode laser, a polarizer, a primary lens, an angle table with interchangeable semicircular samples, a secondary lens, and a silicon photodetector attached to a wattmeter to measure absolute power. A labeled image is shown in Figure 1.

The lenses are in place to make sure that as much of the laser as possible goes to the detector. They can be adjusted at any time to make sure the beam focuses into the detector. This should not affect the results as the detector measures absolute power, and not the size of the beam with respect to its power. While you are adjusting the lenses, you might notice the contrary. The reason that the power measurement changes is because the alignment of the lenses is not perfect, and as you move them they might be rotated slightly. This will either spill some of the beam outside of the detector, or make it hit the side of the detector.

The polarizer. The polarization states of S and P are relative to the sample. When the polarizer is at zero degrees relative to the top of the optics brace, it is allowing vertically polarized light through. That means, by the time it hits your sample it will be S polarized. If you rotate it 90 degrees it will come out horizontally polarized and then it will be P polarized at the interface. Make sure to keep track of the polarization state in your lab manual.

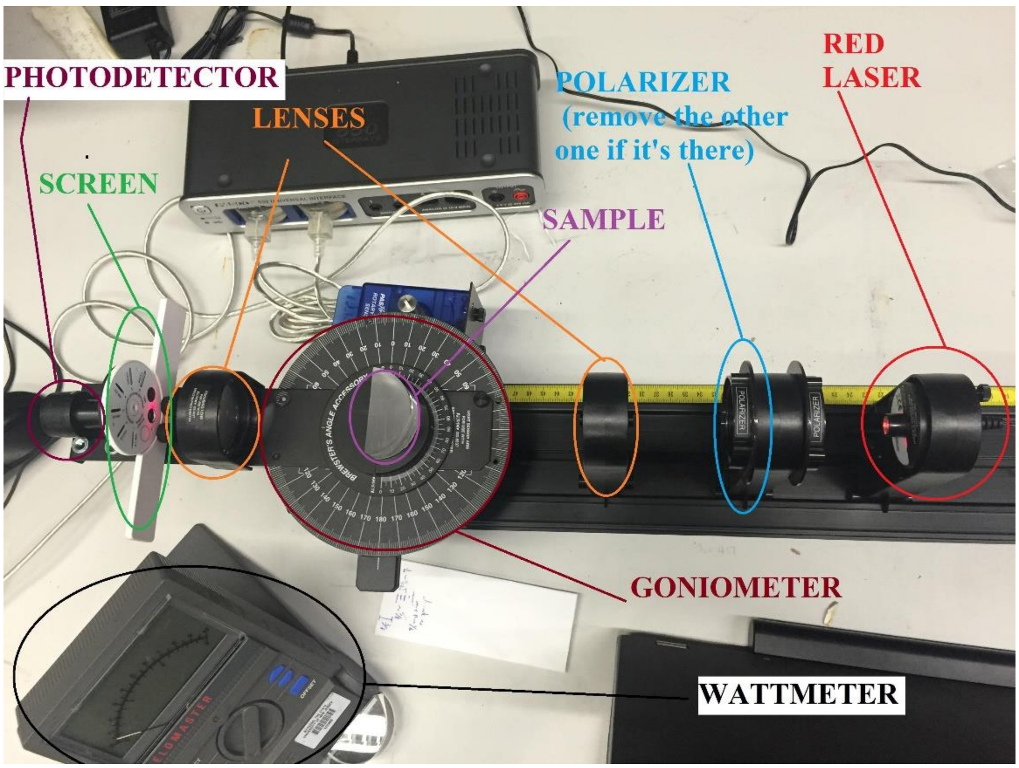


Figure 1. Photograph of measurement setup.

Procedure:

- **Be careful not to look into the laser beam.**

- Make sure the optical elements are in order from right to left: laser, polarizer, track lens, sample, magnetic lens. Then align the 180 degree mark on the outer goniometer (angle measurer) to the line on the extended platform. Then align the magnetic stage so that the white line lines up with 90 degrees on the inner goniometer. Note that there are two white lines, and you can start lining up either, but make sure to put the sample on the higher part of the magnetic stage later.

- Now with all of the optical elements in place besides the sample, line up the laser. There are two thumbscrews on the laser which control the vertical and horizontal direction of the beam. You can find out very quickly which one does which. Make sure to rotate the screen so as to see the laser inside of the small white circle on the screen. Try to align it as close to the center of the circle as you can, and adjust the lenses to focus the beam at that point as best you can.
- Now add the semicircular sample. The important thing is to not only line up the sample on the stage so its semi-center (this is the center of an imaginary circle created by completing the semicircle) is at the center of the circular stage, but also to have the laser still hitting the same spot on the screen while also passing through the semi-center of the sample. Doing this by eye is the easiest. Try to make sure the laser hits the top of the arc of the semicircle and passes through to the semi-center. Make sure to record any realignments or lens movements in your lab manual, so you know EXACTLY how it affects your data!

  Note that adding the sample introduces dispersion losses by. How would you correct for this problem or take this into account?
- This alignment process is the first source of systematic error, but it is not the largest, so as long as you did your best to align it do not worry too much about it. Just make sure not to disturb the sample's relative position on the magnetic stage, or any optical elements during your mmeasurements. This includes adjusting the thumbscrews on the laser. It is difficult to estimate the error induced by these mistakes, so be careful!

  Note: If you are having difficulties seeing the beam try using the black felt blanket, or negotiate turning off the lights for a few minutes.

1. **Initial observation**. Recall that the sample must be centered on its turntable to avoid angle changes on exit. See schematics in figure 2. To familiarize yourself with the setup start by detecting the beam on a piece of paper wrapped around the turntable. This will allow you to see the positions of both reflected and transmitted beams. Observe what happens as you change the incident angle. You should be able to measure Brewster’s angle and the critical angle. How will you know that you reached the critical angle? How will you identify Brewster’s angle? Record the values - how do these compare to the expected values? These are rough estimates that will be

useful for comparison to values that you will obtain later from more precise measurements. Repeat for both sample configurations and for both polarizations. Record your result and include in your report.

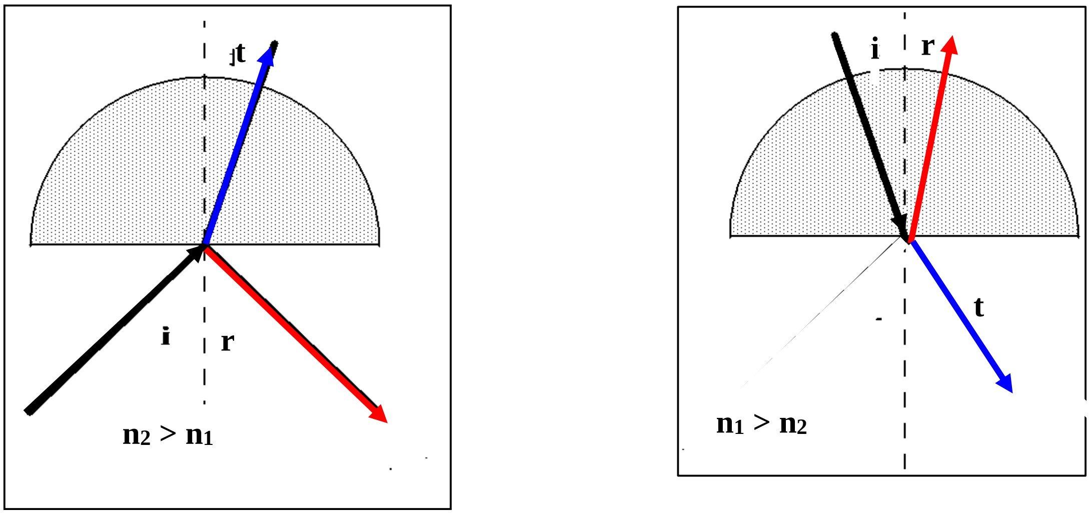


Figure 2.Incoming reflected and transmitted beams for the two experimental configurations.

2. Snell's Law. Measure the angle of the incident beam and the refracted beam for both the air to glass and glass to air interface, as depicted in Figure 2. Record the angle of refraction **t** as a function of the incident angle **i**. Plot sin(**t**) versus sin(**i**) and obtain (n_{glass}/n_{air}) from the slope of a linear fit. Find the critical angle. Do these measurements for both the semicircular glass and semicircular plastic sample. Take at least 5 measurements, but the more measurements you take the less error you should have. If done perfectly, the highest source of error should be from the goniometer itself.

Hints: Keep in mind that the numbers on the goniometers are not necessarily in line with the convention derived in the equations: zero angle of incidence means the ray is perpendicular to the surface. You may have to move the lens in order to find the true focus point of the beam at larger angles. If you need to do so, move the lens slowly, so you can monitor the horizontal deviation of the beam and hopefully reduce it to none.

3. Brewster's angle and Fresnel equations for air/dielectric interface (n_2>n_1).

In this part you will measure the reflected angle (same as incident angle), the refracted angle, and the corresponding power of the reflected and transmitted beams for both parallel and perpendicular polarization, as a function of the incident angle. These measurements will be done for the air to glass interface (depicted on the left in Figure 2) for both the semicircular glass and plastic sample. Repeat this measurement for 10-15 values of **i**. For better precision increase the density of point in the regions of interest near Brewster's angle.

For transmission measurements make sure the beam exits normal to the second interface (semicircular geometry with initial incidence at the center of the flat side of the semicircular sample). Otherwise the power on emergence into air is reduced by any reflection at the second interface.

a. Observe and record I_0 (the laser beam power at zero degrees <u>before</u> sample placement). Make sure the beam is aligned and focused as well as possible on the blank screen. Then rotate the screen so that it is open to the beam. Turn on the wattmeter attached to the photodetector. You will need to aim the laser with the thumbscrews to get a maximum power reading. This will be the reference power reading.
b. Measure the power of the beam with your sample on the stage. Place the semicircular sample on the holder so that the flat side of the sample faces the incident beam and the beam hits the center of the flat side.
c. Measure the angle and power of the reflected beam I_R and the transmitted beam I_T. Do this for both the glass and plastic sample and both parallel and perpendicular polarization. For parallel incident polarization, measure the Brewster's angle. Make sure to have enough data around the angle because reflectance will be low. At these low reflection intensities direct observation of the beam may be more difficult with the linear meter than with the non-linear eye. A fit of the data both above and below the Brewster's angle may allow you to determine it more easily than direct observation. For the analysis, compare and record Brewster's angle from your best fit and compare to the result obtained in part 1.

4. Brewster' s angle , Total internal reflection and Fresnel equations for dielectric/air interface: (n_1>n_2)

Repeat the measurement for section 4 for the incident wave propagating through the denser medium (glass), again for both directions of polarization and for both reflected and transmitted light. This is the glass to air interface, depicted on the right in Figure 2. In this geometry, the systematic error due to multiple reflections will be larger. Make corresponding corrections on the basis of your estimates of losses for the normal incidence. Pay special attention to the angle of the total internal reflection (critical angle). Does it depend on polarization?

Be sure that the incident beam enters the sample radially. This way the beam will not refract due to the first (air to glass) interface and you can measure the angle corresponding to the second (glass to air) interface, which we are interested in. For parallel incident polarization observe quantitatively the transmitted and the reflected beam which can be seen inside the sample, as the incidence angle is varied. Note the occurrence of Brewster's angle and of a (larger) angle of critical reflection at the second surface. Record these angles.

Take measurements of the incident angle, refracted angle, and their corresponding power for both directions of polarization, just like you did in part **4.** In this geometry, the systematic error due to the multiple reflections will be larger. Make corresponding corrections on the basis of your estimate of losses for the normal incidence. Pay special attention to the angle of total internal reflection. Does it depend on polarization? Determine the internal Brewster's angle as well as possible, and the corresponding relative index of refraction.

For both the air to glass (part **4.**) and glass to air (part **5.**) interface, when measuring the power of the transmitted beam, you introduce some systematic error (why?). From comparison of the measured and calculated intensities of the transmitted beam (T=1-R, where T and R and the normalized intensities of the transmitted and reflected beams), estimate the losses when the beam passes through the air to glass interface at normal incidence. Do the same for the glass to air interface. Are the transmitted beam power losses the same for the air to glass and glass to air interfaces?

Hints:

Make sure in parts 3 and 4 to measure BOTH the reflected and transmitted beams.

- If the alignment is done correctly and the lenses aren't disturbed significantly in the process, the largest source of error should come from the wattmeter and the goniometer.
- Sketch the results while taking the data and compare them to the theoretical curve to make sure that you are on the right track.
- You can eliminate the area dependence by taking the projections of the laser on the surface (a

cosine of the beam angle times area term). This gives you a formula for power ratios in terms of the angle.

Report.

- Discuss the physics of Fresnel's equations and how they are verified by your experiment. Define Brewster's angle and the angle of total internal reflection and how they are determined in this experiment.
- Explain the mechanism of suppression of the (P polarized) reflected wave close to Brewster's angle on the basis of molecular optics (reradiation of the light waves by induced atomic or molecular dipoles).
- Briefly describe the measurement procedure.
- Include the plot obtained in part 2 of sin(t) versus sin(i) together with the mean square linear fit from which you determined the index of refraction and error.
- Plot relative powers of the reflected and refracted beams as a function of the incident angle for both parallel and normal polarization for both experimental geometries. Fit the data with Fresnel's equations. From your fit, estimate the systematic error caused by inaccuracy of the initial alignment of the sample. If necessary, make corresponding corrections to the incident angle, and replot the dependences. On you plot indicate Brewster's angle and the angle of total internal reflection .
- Show a zoom into the data around the Brewster angle and mark its location.
- Find the index of refraction *n* for the sample from your fits and, separately, from the Brewster's angle and the angle of total internal reflection. Compare the results for *n* obtained by the different methods, (including part 2). Discuss your conclusions.
- Explain why the incident and refracted beams obey Snell's law regardless of their polarization. (What properties of the electromagnetic waves are used in the derivation of Snell's law? Is it important that the light waves are transverse?).

References:

1. P. Lorrain, D. P. Corson, and F. Lorrain, Electromagnetic Fields and Waves. 3rd ed. W.H.Freeman and Co., New York 1988.
2. D. J. Griffiths, Introduction to Electrodynamics, 3rd ed., Prentice Hall, Upper Saddle River 1999.
3. M. Dekker *et al*., Quantitative investigation of Fresnel reflection in the electromagnetism laboratory. *Am. J. Phys*. **67** (1999) 606.
4. For semiconductor lasers and light-emitting diodes, see Handbook of Optics, vol.1, McGraw-Hill, New York 1996 (available in the Physics Labriry)