The Compton Effect

Introduction

In this experiment we will study two aspects of the interaction of photons with electrons. The first of these is the Compton effect named after Arthur Holly Compton who received the Nobel Prize for physics in 1927 for its discovery. The other deals with the radiation emitted when a tightly bound electron from a heavy element is kicked out by a photon. This gives rise to “characteristic” X-rays that can be used to identify the element.

Kinematics of the Compton Effect

If a photon with energy E_0 strikes a stationary electron, as in Figure 1, then the energy of the scattered photon, E, depends on the scattering angle, Θ, that it makes with the direction of the incident photon according to the following equation:

$$\cos\Theta = 1 - m_e c^2 \left[\frac{1}{E} - \frac{1}{E_0} \right] \tag{1}$$

where m_e is the mass of the electron.

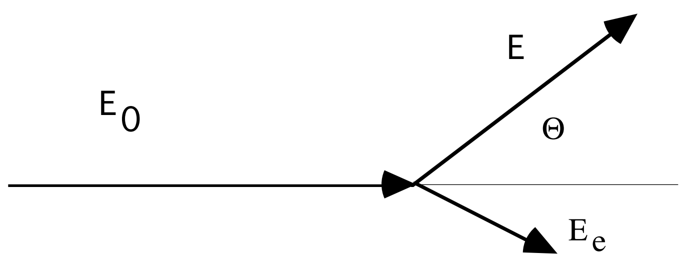

Fig. 1: Schematic diagram of Compton Effect kinematics.

For instance, the lowest energy for the scattered photon results when it emerges at 180 degrees with respect to its original direction, in which case Eq. 1 shows that the incident and scattered photon energies are related as:

$$\frac{1}{E} - \frac{1}{E_0} = \frac{2}{m_e c^2} \tag{2}$$

The total energy of the electron E_e is the sum of its kinetic energy T_e and its rest energy $m_e c^2$, i.e. $E_e = T_e + m_e c^2$. The total energy of the recoiling electron can be computed from energy conservation in the reaction and is given by:

$$E_e = E_0 + m_e c^2 - E \tag{3}$$

or equivalently:

$$T_e = E_0 - E$$

Clearly the electron energy achieves its maximum value in this scattering where the photon is back scattered.

The Klein-Nishina Formula

While Equations 2 and 3 tell us how to compute the energies of the scattered photon and electron in terms of the photon's angle, they do not tell us anything about the likelihood of finding a scattered photon at one angle relative to another. For this we must analyze the scattering process in terms of the interactions of electrons and photons.

The electron-photon interaction in the Compton effect can be fully explained within the context of our theory of Quantum Electrodynamics or QED for short. This subject is beyond the scope of this course and we shall simply quote some results. We are interested particularly in the angular dependence of the scattering or the differential cross-section and the total cross-section both as a function of the energy of the incident photon.

First the differential cross-section, also known as the Klein-Nishina formula:

$$\frac{d\sigma}{d\Omega} = \frac{1/2 r_0^2 \left[1 + \cos^2 \Theta\right]}{\left[1 + 2\varepsilon \sin^2 \Theta / 2\right]^2} \left\{1 + \frac{4\varepsilon^2 \sin^4 \Theta / 2}{\left[1 + \cos^2 \Theta\right]\left[1 + 2\varepsilon \sin^2 \Theta / 2\right]}\right\} \tag{4}$$

where $\varepsilon = E_0/m_ec^2$ and r_0 is the "classical radius of the electron" defined as e^2/m_ec^2 and equal to about 2.8×10^{-13} cm. The formula gives the probability of scattering a photon into the solid angle element $d\Omega = 2\pi \sin \Theta \, d\Theta$ when the incident energy is E_0.

We illustrate this angular dependence in Figure 2 for three energies of photons, where the vertical scale is given in units of cm^2.

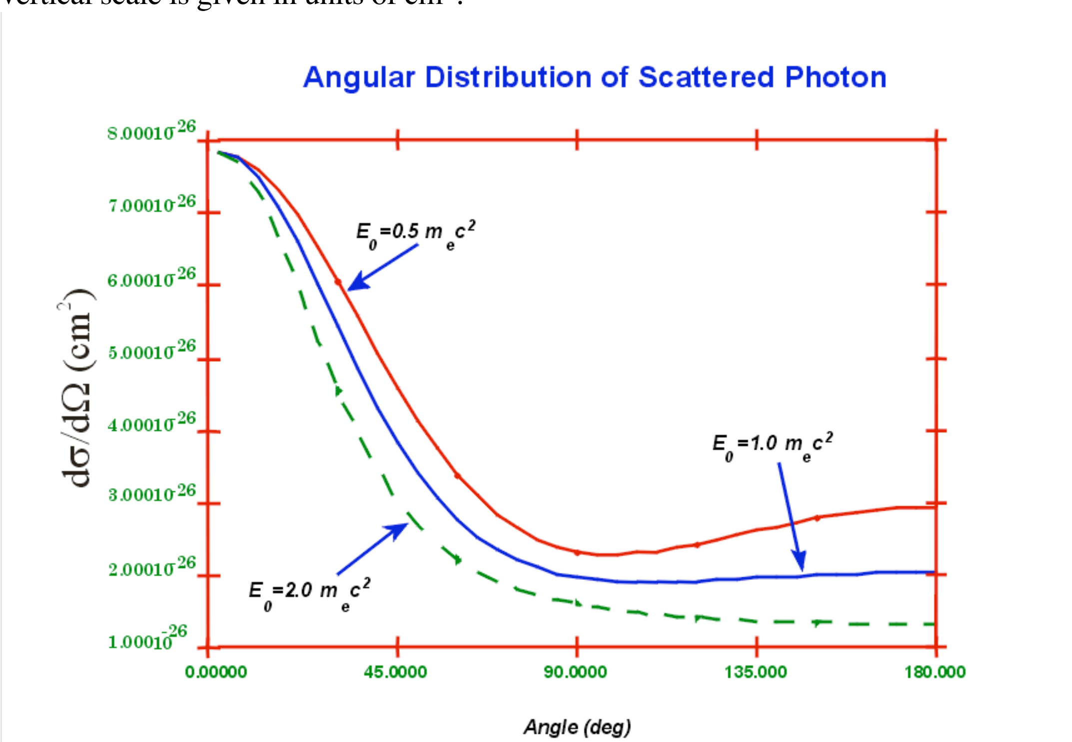

Fig. 2: Differential Cross-section of Compton scattering vs. angle

Note that the most likely scattering is in the forward direction and that the probability of scattering backward is relatively constant with angle.

It will be of interest to us in this experiment to know the probability of measuring electrons with a given kinetic energy $T = E_e - m_e c^2$. We can readily get this expression by substituting for the angle Θ in Eq. 4 via Equations (2) and (3) and noting that:

$$\frac{d\sigma}{dT} = \frac{d\sigma}{d\Omega} \cdot \frac{d\Omega}{dT} = \frac{d\sigma}{d\Omega} \cdot \frac{2\pi}{(\varepsilon - T)^2} \tag{5}$$

In Figure 3 we plot this energy dependence for an incident photon with energy equal to the rest mass of an electron.

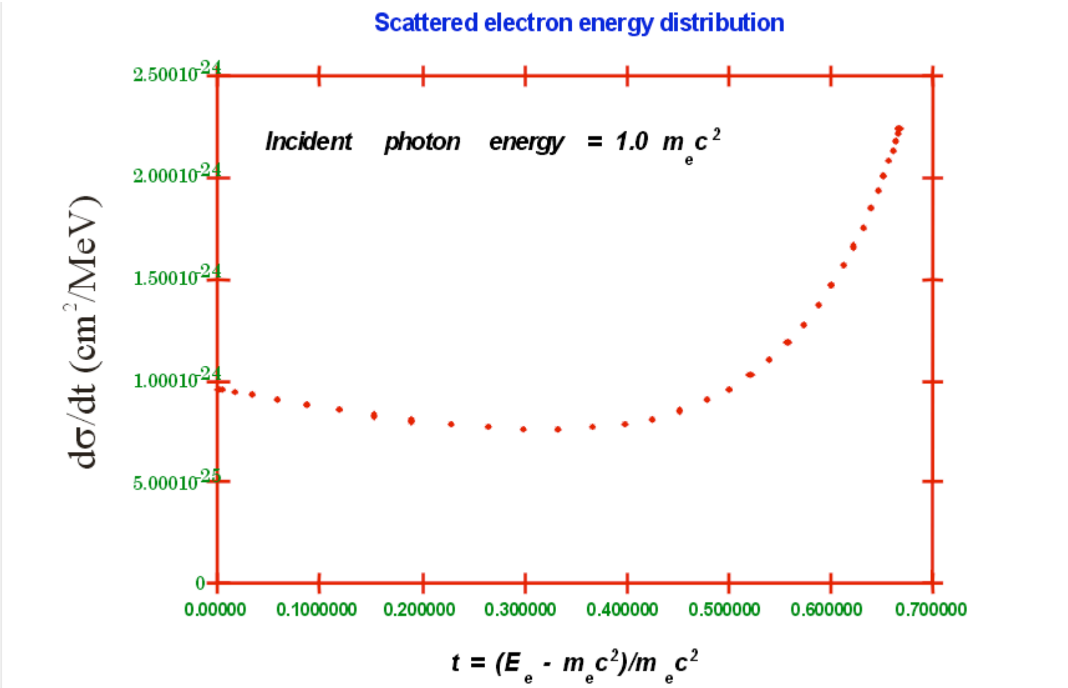

Fig. 3: The probability of finding an electron with reduced kinetic energy t for a photon with incident energy $E_0 = m_e c^2$.

Note the rise in the cross-section with increasing kinetic energy up to the kinematic limit where it abruptly falls to zero. In our experiment we will be looking for this edge.

Energy dependence

The Klein-Nishina formula can be integrated to yield the total cross-section which displays the energy dependence for the process:

$$\sigma = 2\pi \cdot r_0^2 \left\{ \frac{1+\varepsilon}{\varepsilon} \left[\frac{2+2\varepsilon}{1+2\varepsilon} - \frac{\ln(1+2\varepsilon)}{\varepsilon} \right] + \frac{\ln(1+2\varepsilon)}{2\varepsilon} - \frac{1+3\varepsilon}{(1+2\varepsilon)^2} \right\}$$