Electromagnetic Boundary Conditions

Object

To test experimentally Fresnel equations for the case of a non-conducting, nonmagnetic transparent medium and to measure the index of refraction, n, for an unknown transparent material.

References

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Apparatus:

Laser diode (negative 3.0 volts; -3.2 absolute maximum), silicon detector (+6 to 18 volts), digital voltmeter, angle table, semicircular glass and plastic samples

I Introduction

In a uniform medium an electromagnetic wave plane wave travels without bending; at an interface reflection (at the mirror angle) and refraction of the incident wave (according to Snell's law) occur. In the steady state the frequencies of all waves (incident, reflected and refracted) must be the same; the wavelengths v must differ in media of different propagation velocities (index of refraction n = c/v, where c is the free space wave velocity).

Reflection and refraction at the interface between dissimilar media are governed by boundary equations implied by Maxwell's equations, and are expressed by Fresnel's equations which specify reflected and refracted field strengths relative to those of an incoming electromagnetic wave. Two fundamental field quantities are involved, the electric field E and magnetic induction B, and two auxiliary fields involved in relating E and B to their sources, the displacement field D and the magnetic field intensity H. Properties of the propagation media involved are parameterized by (frequency dependent) material constants of linear relations among D and E ($D = \varepsilon E$), and B and H ($B = \mu H$). Since the boundary conditions are statements of continuity between certain tangential and normal components of these four vector quantities, geometric quantities such as the angle of incidence and the polarization state (E vector direction, by convention) of the incident wave enter into Fresnel's equations.

The observed quantities, intensities of reflected and refracted waves, follow (quadratically) from the amplitudes of the E and H vectors of the traveling waves, as specified by the Poynting vector.

A. Static theories

Classical (non-string) sources are considered to be point; the net effect of a distributed collection of point sources can be expanded (for distant field point) into a convergent series of multipoles with increasing powers of inverse distance. In mks units, forces between point charges located within a homogeneous, isotropic medium are given by

$$F = q_1 q_2 (4\pi\varepsilon r^2)$$

or, defining the electric field in terms of force,

$$E_{12} = F_{12}/q_2 = q_1 (4\pi\varepsilon r^2)$$

where the dimensionless <u>permittivity</u> constant ε characterizes the polarizibility of the medium, as discussed below and, typically, $\varepsilon > \varepsilon_0$, the free space value. The incorporation of the 4π into the definition of the proportionality constant ε is a matter of choice to make more convenient the form of Gauss' law. The ratio $\varepsilon/\varepsilon_0 = K_e$ is called the dielectric constant of the medium; the quantity $\chi = K_e-1 = \varepsilon/\varepsilon_0 - 1 = (\varepsilon-\varepsilon_0)/\varepsilon_0$ (the fractional difference in dielectric constants between the medium and free space) is called the electric susceptibility.

Macroscopic distributions of "free" charges and currents are possible, which can be treated by summation or integration, or with a truncated multipole expansion but, in addition, the existence of microscopic, distributed charge and current in neutral matter frequently plays a profound role , as with dielectric capacitors or ferromagnetic yokes. This charge is bound, but polarizable (non-polar molecules, diamagnetism) or orientable (in the case of permanent, intrinsic moments); its contribution to the distant field quantities is that of volume and surface dipole distributions (with the above caveat concerning the near range effect of magnetic sources being not that of a dipole constructed of two monopoles (since there is no evidence for magnetic monopoles in ordinary life), but that of an elementary current loop). (See Corson & Lorrain.)

The distributed polarization, induced or intrinsic, is expressed in terms of vectors P or M, representing electric or magnetic dipole moment per unit volume. Different field vectors are defined to correspond to the contributions of these **two source types of quite different character: "free" charges and currents** , and **"bound" (distributed dipole, bulk-neutral matter distributions)** . These latter involve the macroscopic effects of the polarization properties of matter, expressed by numerical parameters which are frequency dependent in general, and temperature dependent in some cases. These parameters are simple scalars for isotropic materials, but may have several matrix components in case of anisotropy. For our materials, they will pertain to homogeneous materials and will be the same at all points of the material.

For electricity the field quantities are defined according to the two major source types with the net field E given by

$$E = D/\varepsilon_0 - P/\varepsilon_0$$

where D/ε_0 represents the field due to "free" charges only (in that D field lines begin and end only on free charge) and $-P/\varepsilon_0$ represents the field due to bound charges, with E field lines beginning and ending on either free or bound charge. (See Tralli, p 78; Corson & Lorrain, p 98.)

For common, isotropic and homogeneous, dielectrics the relation between P (dipole moment per unit volume) and the polarizing field E can be parameterized linearly by

$$P = \varepsilon_0\chi E = \varepsilon_0(K_e-1)E \qquad \text{whence}$$

$$D = \varepsilon E = \varepsilon_0 K_e E.$$

where χ , as discussed earlier, is the fractional difference between the permittivities of the medium and of free space, and K_e is the electric permittivity ratio $\varepsilon/\varepsilon_0$.

As discussed above, in classical (non-string) field theory, the elementary electric field point source has monopole (point) character, and $1/r^2$ separation dependence. In contrast, for the magnetic field there are no apparent elementary monopole sources; the elementary magnetic point source is a current loop with distant dipole field character (1/r^3 strength) but non-dipole near character (field loops surrounding the loop current). (See the contrasting E and B field patterns in Fig. 7-6, p267, Corson & Lorrain.) Again, the static theory relating electric fields and sources is expressed in terms of vector quantities E, D and P, where

D represents the field due to "free" charges only (typically charge on a conductor, in a beam, etc.)

P represents the field due to dipole polarization of a dielectric, and

$\varepsilon_0 E$ represents the field due to all charges.

Here, ε_0 is a constant of proportionality in the basic free-space force law

F_E = (1/4pe0) q1q2/r^2

and the relations between the vectors are given by

$$D = \varepsilon E \; P = \varepsilon_0(K_e\text{-}1)E$$

where ε is the permittivity defined above from the point force equation, $K_e = \varepsilon/\varepsilon_0$ is the dielectric constant and $(K_e\text{-}1)$ is the electric susceptibility.

The magnetic properties of materials and the magnetic fields defined are parameterized in a manner similar to the electric case:

B is the basic magnetic induction field (units webers/meter^2 = tesla, weber = volt second) defined by the force between two

$$dF = \mu_0 \, I_1 I_2 \, dl_1 dl_2/(4\pi)$$

with the free space permeability proportionality constant μ_0 defined arbitrarily as $4\pi x10^7$ newton/ampere.

Similar to this treatment of the electric field vectors, the (magnetic) dipole moment M per unit volume of matter, induced or permanent, is related to the polarizing field B and an auxiliary magnetic field intensity vector H by

$$B = \mu_0 H + M = \mu H$$

with dimensionless magnetic susceptibility χ_m = and relative permeability $K_m = \mu/\mu_0 = 1+\chi_m$.

B. Dynamic theories

Maxwell's equations unify the theories of electricity and magnetism, and successfully predict the existence and propagation velocity of coupled free space waves of electricity and magnetism, mutually generating one another through their time variation (Faraday's law, displacement "currents").

With parameterization of material properties as in the static case (but with different and frequency dependent) values of the dielectric and magnetic susceptability, these equations also specify changes in the E and B field magnitudes for a traveling wave incident on an interface between media with different properties. Application of these "boundary conditions" specifies the field magnitudes for the transmitted and reflected waves; these, in turn, allow the transmitted and reflected intensities (quadratic in the fields) to be calculated.

Traveling electromagnetic waves have their E and B fields perpendicular to the direction of propagation (for isotropic media), so there are two possible directions for the "polarization" direction, conventionally the direction of the electric field. For plane wave incident on an interface, the propagation direction unit vector and the normal to the interface determine a plane; it is useful to define this plane as that of "parallel" (P) polarization and the perpendicular plane as that of "perpendicular" or norma (N) polarization. These two electric field polarizations have quite different transmission and reflection properties at boundaries.

Application of the theoretical boundary conditions gives E and B field ratios , separately for the parallel and perpendicular polarization components to the incident radiation. These and the corresponding expressions for the reflected and transmitted energies are referred to as the Fresnel equations, derived by him in a less general form (based on an elastic theory of light) in 1823, before Maxwell's work.

Useful observable quantities are incident, reflection and transmission (refraction) angles (i, r, and t), and the incident and reflected intensities, which are proportional to the squares of the theoretically treated quantities (electric and magnetic fields). (Since these intensities are measured in the same medium (air) there are no material complications (for non-absorbing materials) in calculation of the Poynting vector, which gives the flux of electromagnetic energy, and which is proportional to the product of electric and magnetic field oscillation amplitudes. In vacuum (and in air to excellent approximation) E and B are equal so $ExB = E^2$ (or B^2)

(Determination of the refracted intensity would require measurement inside the refracting material, whereas inference from the intensity of the refracted wave exiting into air would be complicated by multiple reflections between the entrance and exit interfaces which might, however, be calculated from theory.)

Assumption of steady state implies that the refracted and reflected waves have the same frequency as the forcing incident wave. Since the propagation velocity is different in the refracting medium the wavelengths differ, and node matching at the interface requires refraction according to Snell's law

$$n_1 \sin i = n_2 \sin t \, .$$

Most texts on electromagnetic theory derive and list the Fresnel equations. In words, the applicable boundary conditions are:

a, b) The normal components of D and B, and

c,d) The tangential components of E and H

are continuous across an interface between two dissimilar materials.

Employing the material parameterization constants μ and ε (or the corresponding K_e and K_m), with $\eta_{21} = n_2/n_1$, Fresnel's equations are derived:

Set 1

For perpendicular (normal) polarization (N):

$$(E_{0r}/E_{0i})_N = (\mu_2 \cos i - \eta_{21}\, \mu_1 \cos t)/(\mu_2 \cos i + \eta_{21}\, \mu_1 \cos t)$$

$$(E_{0t}/E_{0i})_N = (2\, \mu_2 \cos i)/(\mu_2 \cos i + \eta_{21}\, \mu_1 \cos t)$$

and for parallel polarization (P):

$$(E_{0r}/E_{0i})_P = (\eta_{21}\, \mu_1 \cos i - \mu_2 \cos t)/(\eta_{21}\, \mu_1 \cos i + \mu_2 \cos t)$$

$$(E_{0t}/E_{0i})_P = (2\, \mu_2 \cos i)/(\eta_{21}\, \mu_1 \cos i + \mu_2 \cos t)$$

For non-magnetic materials ($\mu_1 = \mu_2 = \mu_0$) these reduce to:

Set 2

$$(E_{0r}/E_{0i})_N = (\cos i - \eta_{21} \cos t)/(\cos i + \eta_{21} \cos t)$$

$$(E_{0t}/E_{0i})_N = (2 \cos i)/(\cos i + \eta_{21} \cos t)$$

$$(E_{0r}/E_{0i})_P = (\eta_{21} \cos i - \cos t)/(\eta_{21} \cos i + \cos t)$$

$(E_{0t}/E_{0i})_P = (2 \cos i)/(\eta_{21} \cos i + \cos t)$

or, utilizing $\eta_{21} = n_2/n_1 = \sin i/\sin t$ (Snell's law) and trigonometric identities:

Set 2'

$(E_{0r}/E_{0i})_N = -\sin(i-t)/\sin(i+t)$

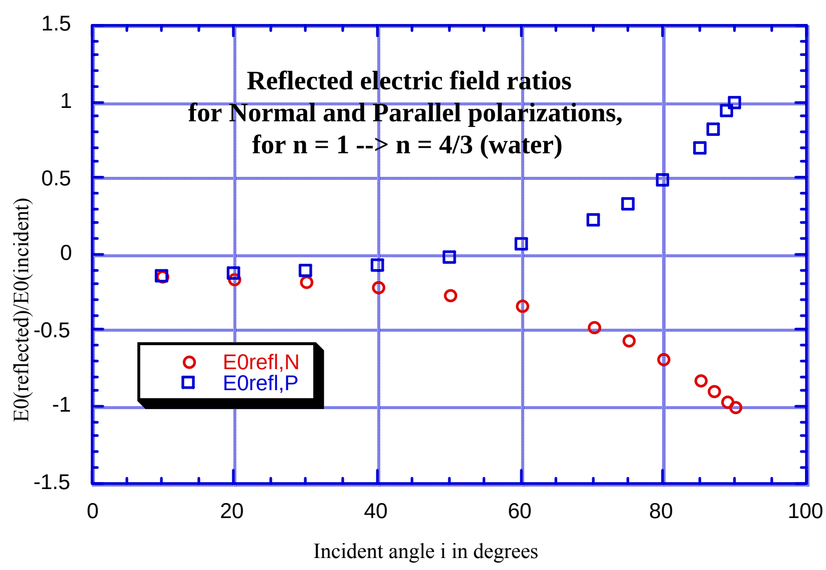
$(E_{0t}/E_{0i})_N = 2 \sin t \cos i/\sin(i+t)$

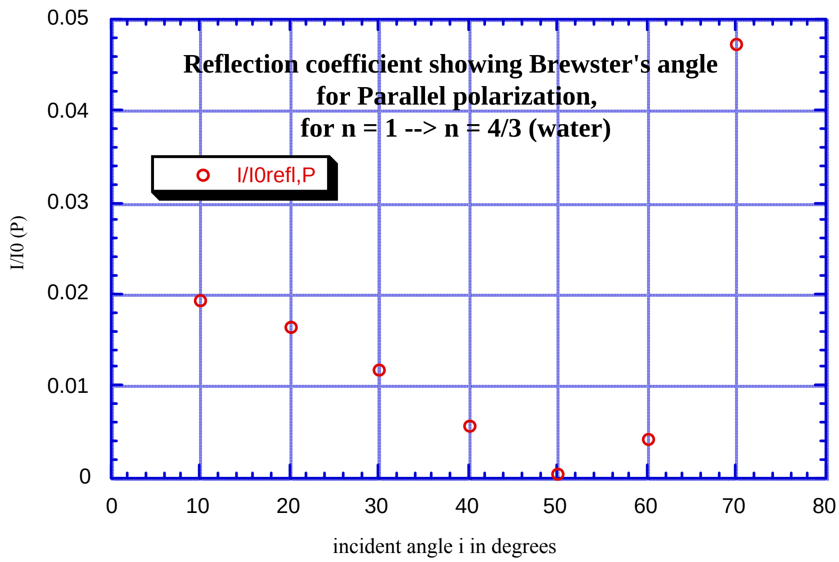
$(E_{0r}/E_{0i})_P = -\tan(i-t)/\tan(i+t)$

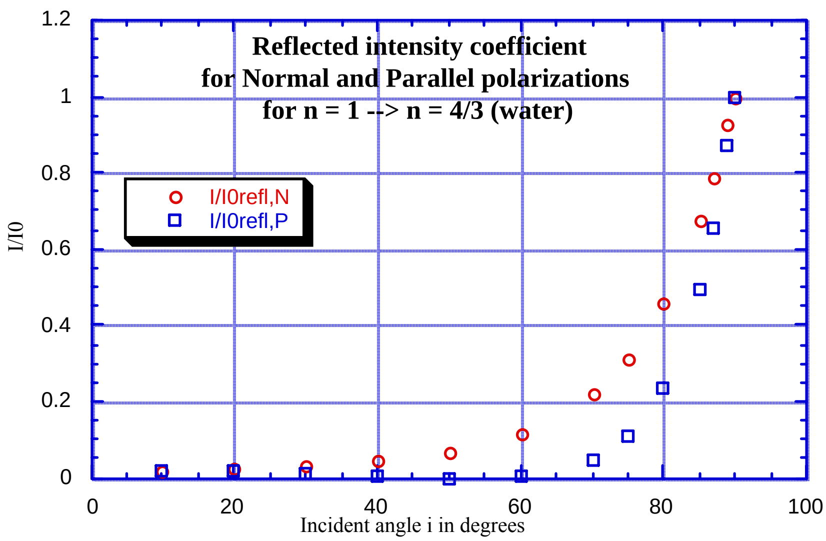
$(E_{0t}/E_{0i})_P = 2 \sin t \cos i/(\sin(i+t) \cos(i-t))$.

Both E_{0r} and E_{0t} are real quantities, so their electric vector phases relative to that of the incident wave E_{0i} is either 0 or 180 degrees (field ratios above $\pm$ 1).

From these forms we see that, for perpendicular polarization, $(E_{0r})_N$ has a 180 degree phase relation to the incoming wave and that, for parallel polarization, $(E_{0r})_P$ can have either zero or 180 degree relative phase (E_0's are wave amplitudes). This is illustrated below for an air-->water interface ($\eta_{21} = n_2/n_1 = 4/3$):







The general case for incident polarization can be treated by a linear superposition of these results. For unpolarized incident beam (equal mixture of perpendicular and parallel polarizations) the reflected and transmitted waves are polarized in general, since the reflection coefficients above differ for the two incoming polarizations. This is particularly evident for near-grazing incidence, leading to the choice of orientation for Polaroid sunglasses.

The Poynting (energy density flow) vector gives relative intensities (average energy per unit time per unit area = energy density x phase velocity) as $S_{av} = [(E_{r0} x H_{r0})/2] / [(E_0 x H_0)/2]$ and $[(E_{t0} x H_{t0})/2] / [(E_0 x H_0/2]$, respectively for reflection and transmission, where the factors of 1/2 represent the time average of $\sin^2$ and the x represents the cross product of two vectors. Equivalent expressions are:

$$S_{av} = 1/2\ [(\varepsilon/\mu)^{1/2}]\ E_0^2 = 1/2\ [1/((\varepsilon\mu)^{1/2})]\ \varepsilon E_0^2 = 2.65x10^{-3}\ [(K_e/K_m)^{1/2}]\ E^2{}_{rms}$$

where E_0 is the electric field amplitude, $E^2{}_{rms} = 1/2\ E_0^2$, $K_e = \varepsilon/\varepsilon_0$, $K_m = \mu/\mu_0$,

$[1/((\varepsilon\mu)^{1/2})]$ is the phase velocity and $[(K_e/K_m)^{1/2}]$ is the index of refraction n.

These expressions give energy flow rate per unit area. In calculating reflection (R) and transmission (T) coefficients we are interested in the total energy flow rate: S_{av} x (laser beam area). For the simple case of reflection at an interface the beam areas are the same for the incident and reflected beams, as are the material parameters, and it suffices to express the simple ratio $E_{0r}{}^2/E_0{}^2$.

This is not so for the transmitted beam; the beam area changes. The consideration of beam areas in calculating energy flow instead of energy flow per unit area introduces an additional geometrical factor multiplying the Poynting vector expression, giving the following expressions for reflected and transmitted energy ratios (R and T), for the two incident polarizations:

Energy reflection and transmission coefficients:

$$R_N = [(\eta_{12}\text{cosi-cost})/(\eta 12\text{cosi+cost})]^2$$

$$T_N = (4\eta_{12}\text{cosi cost})/((((\eta_{12}\text{cosi+cost}))))^2$$

$$R_P = [(\text{-cosi}+\eta_{12}\text{cost})/(\text{cosi}+\eta_{12}\text{cost})]^2$$

$$T_P = (4\eta_{12}\text{cosi cost})/(\text{cosi}+\eta_{12}\text{cost})^2$$

where $\eta_{12} = n_1/n_2$ and $R_N + T_N = 1$, $R_P + T_p = 1$.

The non-magnetic set 2 form of Fresnel's equations show that the single parameter $n_2/n_1 = \eta_{21}$ governs not only the refraction angle in terms of incidence angle, via Snell's law, but also all electric field reflection and transmission ratios, and therefore the reflection and transmission intensity coefficients R and T. A single measurement of Snell's law would suffice to specify the entire behavior. We will seek to check the theory by non-linear least-square fit to multiple measurements (KaleidaGraph or equivalent). Since we expect the single governing parameter $\eta_{21} = n_2/n_1$ to be over determined, it may be best in fitting to leave the incoming intensity I_0 as a multiplicative fit parameter to be compared with the measured quantity. (This is equivalent to an unknown, constant multiplier of the theoretical energy ratio. We would expect the ratio of the best fit value to the initially measured measured zero-degree intensity (with sample removed) to be close to one. The absolute value of I_0 will depend on laser intensity, etc.)

Further, refraction angle and reflected intensity will form independent data sets for fit to η_{21}. We might also introduce, on a trial basis, a systematic angle offset in the refracted or reflected angle.

In general $i \neq t$, except for $i = t = 0$ degrees. For $i = t = 0$, the a plane of refraction is undefined and any incident polarization lies in the interface plane, so $(E_{0r})_P$ and $(E_{0t})_P$ are zero. Then set 1 gives

Special case 1: normal incidence

$$(E_{0r}/E_{0i})_N = (\mu_2 \cos i - \eta_{21}\, \mu_1 \cos t)/(\mu_2 \cos i + \eta_{21} \cos t)$$

$$\rightarrow (\mu_2 - \eta_{21}\, \mu_1)/(\mu_2 + \eta_{21}\, \mu_1)$$

$$\rightarrow (1 - \eta_{21})/(1 + \eta_{21}) \qquad (\text{for } \mu_1 = \mu_2 = 1)$$

$$(E_{0t}/E_{0i})_P = (2\, \mu_2 \cos i)/(\mu_2 \cos i + \eta_{21}\, \mu_1 \cos t)$$

$$\rightarrow (2\, \mu_2)/(\mu_2 + \eta_{21}\, \mu_1)$$

$$\rightarrow 2/(1 + \eta_{21}) \qquad (\text{for } \mu_1 = \mu_2 = 1)$$

For the singular angle given by $(i+t) = \pi/2$ radians, for non-magnetic media, set 2' becomes

Special case 2: Brewster's angle for non-magnetic media

$$(E_{0r}/E_{0i})_N = - \sin(i-t)/\sin(i+t) \qquad \rightarrow -\sin(i-t)$$

$$(E_{0t}/E_{0i})_N = 2 \sin t \cos i/\sin(i+t) \qquad \rightarrow 2 \sin t \cos i$$

$$(E_{0r}/E_{0i})_P = \tan(i-t)/\tan(i+t)\ ^{*} \qquad \rightarrow 0$$

$$(E_{0t}/E_{0i})_P = 2 \sin t \cos i/(\sin(i+t) \cos(i-t)) \rightarrow 2 \sin t \cos i/\cos(i-t).$$

(* The numerator and denominator both become zero for the uninteresting case of normal incidence: $i = t = 0$. Brewster's angle is found for infinite denominator.)

Application of Snell's law, with $(i+t) = 90$ degrees, gives Brewster's angle in terms of the relative indices of refraction:

$$n_2/n_1 = \eta_{21} = \sin i_B/\sin t_B = \sin i_B/\cos i_B = \tan i_B.$$

At Brewster's angle, any reflected wave is polarized only in the perpendicular direction, whatever the incident wave polarization.

For a particular pair of media there are two Brewster's angles, for the two different arrangements of incidence and transmission material. For a plane wave incident on a planar dielectric slab at Brewster's angle the entire wave is transmitted through the slab if the incident wave is polarized in the plane of polarization, since the Brewster conditions are satisfied both at wave entry and exit of the slab (entrance condition is $\tan i_B = \eta_{21}$, exit condition is $\tan i'_B = 1/\eta_{21}$), as can be seen by application of Snell's law. Thus Brewster angle windows allow loss-less laser beam exit from a lasing medium into air (for proper polarization), where adjustment of the reflecting cavity mirrors is easy.

Where $\mu 1 \neq \mu 2$, there can be a Brewster's angle situation for both incident polarizations. In this case the numerator can be zero while denominator remains finite. Setting the Set 1 reflection numerators equal to zero, we obtain $(E_{0r}/E_{0i}) = 0$ for:

Special case 3: Brewster's angles for magnetic media

$$(E_{0r}/E_{0i})_N = (\mu_2 \cos i - \eta_{21}\, \mu_1 \cos t)/(\mu_2 \cos i + \eta_{21}\, \mu_1 \cos t)$$

$$\text{-->}\ 0 \text{ for } \cos i = (\eta_{21}/K_m) \cos t$$

$$(E_{0r}/E_{0i})_P = (\eta_{21}\, \mu_1 \cos i - \mu_2 \cos t)/(\eta_{21}\, \mu_1 \cos i + \mu_2 \cos t)$$

See the footnote on page 369, Corson and Lorrain, for an interesting refutation of a common explanation of Brewster's angle polarization for non-magnetic materials.

Special case 4: Total internal reflection and evanescent wave

Where $n_2 > n_1$ ($\eta_{21} = n_2/n_1 < 1$) there is an critical incidence angle i_c ($i_c = \text{invsin}(\eta_{21})$) beyond which Snell's law gives $\sin t > 1$. For greater angles the beam energy is totally reflected back into medium 1 at the usual mirror angle i. Application of Maxwell's equations and corresponding boundary conditions shows that there must be in addition a shallow wave penetration into the "forbidden" dielectric 2, the wave amplitude decaying exponentially with distance from the interface boundary and consisting of an wave traveling unattenuated, and without energy loss, in the direction parallel to the interface with a wavelength $\lambda_2 = \lambda_0/\sin i$ ($> \lambda_0$), oscillating without energy absorption (once established) somewhat like a resonant LC circuit.

The amplitude of penetration into medium 2 in this case damps as $e^{-\delta}$ where

$$\delta_2 = [(\lambda_2/(2\pi)]/(\eta_{12}{}^2 \sin^2 i - 1)^{1/2}\ .$$

This shallow penetration implies, for a sufficiently thin film of medium 2, the possibility of coupling energy through "forbidden" medium 2 to form again a traveling wave in a third medium, just as in quantum mechanical barrier penetration situations (electron tunneling, alpha decay). Such "leakage" (e.g., through a thin air gap between two glass dielectrics) has been observed.

Absorptive materials

For perfectly transparent (i.e ., non-absorptive) materials, the single parameter $\eta_{21} = n_2/n_1$, which gives the relative wave numbers (via the relative phase velocities) in the two media completely specifies the reflected and refracted relative wave amplitudes and energies. When the second medium is absorptive description of the transmitted wave requires specification, not only of the wave number (phase velocity) in the absorptive medium, but also of an (exponential) spatial rate of damping perpendicular to the interface, corresponding to energy loss from the wave into other forms. Thus two parameters, rather than one, are required. Correspondingly, the relative index of refraction n' (and the dielectric constant of absorbing medium 2) become complex numbers and Snell's law implies imaginary angles, though there is a real angle of refraction. Nertheless, the Maxwell boundary conditions can be satisfied and reflection and transmission coefficients can be expressed in term of two constants (see Ditchburn's chapter on the theory of reflection and dispersion):

$$n' = n(1+i\kappa) \qquad \text{where } n \text{ and } \kappa \text{ are real numbers.}$$

(For a real dielectric constant $\varepsilon > 0$, there is wave propagation without damping. For real $\varepsilon < 0$, or for complex ε, there is damped propagation in the medium. See Kittel for further discussion.)

Snell's law and Fresnel's equations in the form of set 2 are formally employed, with

$$\cos t = (1/n')\,(\mathrm{sqrt}(n'2-\sin^2 i)) \quad \text{and} \quad \tan t = \sin i/(\mathrm{sqrt}(n'2-\sin^2 i))\ .$$

For $n^2+n^2\kappa^2 = n'n'^*$ (square of modulus of the complex index n') >> 1 (strong absorption) the reflection coefficients for perpendicular (normal) and parallel polarization are, respectively (Ditchburn; see also Palik, p26 for an equivalent expression) on dividing through:

$$R_N = [n^2(1+\kappa^2) - 2n\cos i + \cos^2 i]/[n^2(1+\kappa^2)\cos^2 i + 2n\cos i + \cos^2 i] \text{ and}$$

$$R_P = [n^2(1+\kappa^2)\cos^2 i - 2n\cos i + 1]/[n^2(1+\kappa^2)\cos^2 i + 2n\cos i + 1]\ .$$

where n and κ are the real numbers representing the real and imaginary parts of the complex index of refraction, $n' = n+i\kappa n$.

R_P can show a minimum, as for Brewster's angle for non-absorptive dielectrics, but not a zero minimum.

Special case 5: Strong absorption in metals

Here the absorption, (for a good conductor) can be so strong that the penetrating, damped wave in the metal travels perpendicular to the interface, for all angles of incidence. Nevertheless, despite the damping, the penetration depth δ is small ($\delta \approx \lambda$, the wavelength in the metal, which is much smaller than λ_0, the wavelength outside)), and there is little energy loss, most of the energy being reflected. The expression for δ is (neglecting a small correction)

$$\delta = \mathrm{sqrt}(2/(\omega\sigma\mu))$$

($\omega = 2\pi f$) where δ indicates the exponential spatial damping constant for the transmitted electric field amplitude (energy damping as $\exp(-\delta^2)$).

Corson and Lorrain give the reflection amplitude coefficients (for E_i normal (N) to the plane of incidence):

$$\{[(E_{0r}/E_{0i})N = [n_1K_{m2} \cos i-(\lambda/(2\pi\delta))(1-j)]/[n_1K_{m2} \cos i+(\lambda/(2\pi\delta))(1-j)]\} \approx -1,$$

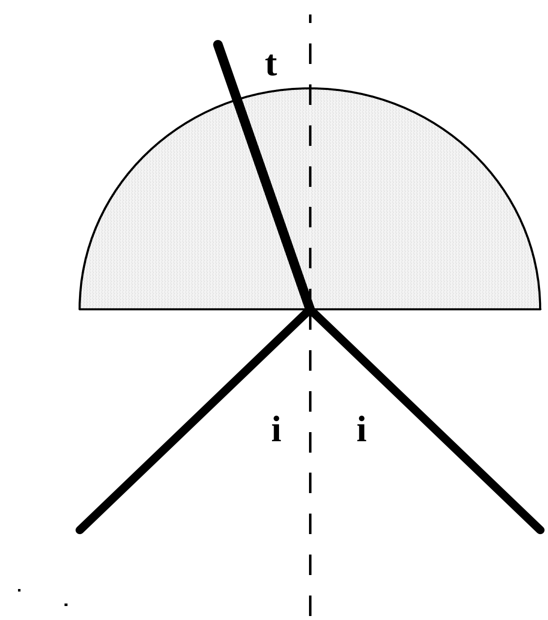
and similarly for E_i parallel (P) to the plane of incidence.

Procedures and analysis

<u>Be careful not to look into the laser beam or into any reflected or refracted beam</u>. <u>Choose a stool which puts your eyes well above the beam reflection plane!!</u> <u>Avoid shining refracted or reflected beams in the direction of neighboring experimenters.</u>

<u>Never exceed 3 volts in the laser supply</u>. The voltage to the power meter is not critical, 6 - 18 volts.

1. Snell's law



Observe and record corresponding angles i and t and plot sini/sint for a pair of refracting materials (e.g., air - glass). Obtain (n_{medium}/n_{air}) from the slope of a linear fit. Submit these plots , showing equations and errors in slope and intercept. (A least square KaleidaGraph fit equation displays immediately the uncertainties in slope and intercept, somewhat more conveniently than the coefficient R from which these errors may be derived.) Note that the semicular sample must be centered on its turntable to avoid angle changes on exit.

2. Reflection coefficients: air --> dielectric ($n_2 > n_1$)

The data polarization order (N or P) is immaterial.

A simple way to take data is to loosen the detector arm locking screw and rotate it slightly from its previous position, without attempting to set at a precise angle. (Grip the arm, not the detector. Be careful not to move the laser.) Then rotate the sample table until the reflected laser beam is visible on the photodetector, lock the detector arm and adjust its angle for maximum meter voltage with the fine control screw. Read the coarse angle and vernier. For detector angle readings R to the right of zero, angle i is (180-R)/2; to the left, i = (R-180)/2. (Check that these make sense.) Record read angles R and let the fitting program (e.g., KG) calculate later the corresponding values of i.

It is not practical to measure the intensity of the transmitted wave within the second medium, and the intensity on emergence into air is reduced by any reflection at the second interface, although this may be small for exit normal to the second interface (semicircular geometry with initial incidence at the center of the diameter), especially if the second interface is metallized.

The reflected wave, for air incidence, is not subject to multiple reflections. Observe and record I_0 (laser beam intensity at zero degrees before sample placement), then I_r (reflected beam intensity) vs. incidence angle i, separately for perpendicular and parallel polarization. (For parallel incident polarization, note the occurrence of a Brewster's angle. Reflectance will be low near this angle. therefore, though the relative index η_{21} is sensitive to Brewster's angle, direct observation may be more difficult with the linear meter than with the non-linear eye. Therefore a fit to data observations both below and above Brewster's angle may well determine it more easily than a direct observation. Calculate and record Brewster's angle from your best fit.

Calculate and plot separately the corresponding reflection coefficients R_P and $R_N = I_r/I_0$, as a function of incidence angle i. By conservation of energy (for non-absorptive material), we expect that (R + T) = 1, for corresponding reflection and transmission coefficients.

For the reflected wave, both I and I_0 are measured in air, and the reflection coefficient R(i) is given simply by the square of the electric field ratios, in either of two equivalent forms (set 2 or 2'). From the set 2' form of Fresnel's equations:

$$[(E_{0r}/E_{0i})_N{}^2 = [-\sin(i-t)/\sin(i+t)]^2$$

$$[(E_{0r}/E_{0i})_P]^2 = [-\tan(i-t)/\tan(i+t)]^2 \cdot$$

These expressions involve both incidence and transmission angles i and t; a theoretical fitting function of incidence angle i alone is needed. Angle t is expressed as a function of angle i via Snell's law

t = invsin(sin(i)/m3) = invsin(sin(m0)/m3)

if m3 is chosen for the KG parameter referring to the relative index $\eta_{21} = n_2/n_1$ of refraction of the two dielectrics. (Note that η_{21} will be > 1 for air --> plastic, and will have the inverse value (< 1) for plastic --> air; this is relevent for starting parameter specification in least square curve fitting.)

(Alternatively, if the set 2 form of Fresnel's equations are used cost appears, which can be expressed as sqrt[1-$\sin^2$t] = sqrt[1-$\eta_{12}{}^2\sin^2$i]) which becomes, in KG notation:

cost = sqrt(1-$m_3{}^2\sin^2$i).

KaleidaGraph allows for nine independent search parameters (format: m1 to m9), with m0 reserved for the independent variable (angle i, here).

Use KaleidaGraph to plot as a function of incident angle i the reflected intensity ratio R = I/I_0, separately for parallel (P) and perpendicular (N) polarization. (<u>KG requires plotting before it will fit</u>.) From the R_P fit (parallel incident polarization), determine Brewster's angle as well as you can, and thence the relative indices of refraction.

Perform, for each polarization, non-linear least square fits (for KG, choose <u>General</u> under <u>Curve Fit</u>, enter function to be fit after clicking <u>Define</u>) to the appropriate form (N or P) representing $(E/E0)^2$. Submit the plots, showing equations and best fit parameters and errors. KG <u>Define</u> fit-functions for the two (perpendicular and parallel incident polarization) forms, with some possible starting search parameters, are (set 2' forms):

$[(E_{0r}/E_{0i})_N]^2 = [\sin(i-t)/\sin(i+t)]^2$ --> (KaleidaGraph notation)

```
m1+m2*((sin(m0-invsin(sin(m0)/m3)))
/(sin(m0+invsin(sin(m0)/m3))))^2; m1=.002; m2=1;m3=1.5
```

and

$[(E_{0r}/E_{0i})_P]^2 = [\tan(i-t)/\tan(i+t)]^2$ --> (KaleidaGraph notation)

```
m1+m2*((tan(m0-invsin(sin(m0)/m3)))
/(tan(m0+invsin(sin(m0)/m3))))^2; m1=.002; m2=1;m3=1.5 .
```

These can be copied directly from this Word file and pasted into the definition box of the KaleidaGraph general fit. Remember that a better starting m3 parameter for plastic --> air would be 1/1.5. Starting parameters too far from actual values may result in a "false" (non-global chi-square minimum) fit, with corresponding erroneous best-fit parameters. The Snell's law η_{21} value should be a good stating value for m_3.

(The corresponding set 2 KG forms for N and P polarizations respectively are somewhat longer but give identical fits, as they should:

m1+m2*(((1/m3)*cos(m0)-cos(invsin(sin(m0)/m3)))
/((1/m3)*cos(m0)+cos(invsin(sin(m0)/m3))))^ 2; m1=.002;m2=1;m3=1.5
and

m1+m2*((-cos(m0)+(1/m3)*cos(invsin(sin(m0)/m3)))
/(cos(m0)+(1/m3)*cos(invsin(sin(m0)/m3))))^ 2; m1=.002;m2=1;m3=1.5
).

In these expressions m1 represents a possible constant offset in I/I_0, m_2 is a scale factor (expected to be 1, if the data follows theory) and m_3 is the relative index of refraction η_{12}, written in the denominator so as to represent ($n_{glass}/n_{air} > 1$).[1,2] <u>Be sure to select</u>

[1] m1, and the deviation of m2 from the expected value of 1, might represent the effects of constant, additive readings to the true values E^2 and E_0^2 : $E^2 \to E^2+e$, $E_0^2 \to E_0^2+e_0$. The effect on the ratio is then, using the binomial expansion of the denominator:

$R' = (E^2+e)/(E_0^2+e_0) = [(E^2/E_0^2+e/E_0^2)/(1+e_0/E_0^2)] \to [R+e/E_{02}]\,[1-e_0/E_0^2+2(e_0/E_0^2)^2+ ---]$

$\to [R+e']\,[1-e''+2e''^2+ ---] \to [e'-e'e''+2e'e''^2+ ---] + R[1-e''+2e''^2+ ---]$. Keeping only first order small quantities, $R' = e' + R[1-e'']$, where m1 above = e' and m2 = [1-e''] (R' = observed data ratio, R = theoretical ratio, $e'=e/E_0^2$, $e''=e_0/E_0^2$).

To investigate the fit with the assumption that $e = e_0$ (same additive constant for E^2 and E'^2) only two fit parameters should be used, not three, since then e' = e'' and m2 = (1-m1). If m1 and the deviation of m2 from 1 have the same sign and approximate magnitude (for a three parameter fit), the corresponding two parameter fit mightn be tried. The relative sizes of errors and fit parameters must be considered in deciding whether a one, two or three parameter fit is appropriate. Formation of an estimated random error data column (perhaps constant error or constant percentage error, or other) will permit use of a weghted least square fit and will give a meaningful best-fit chi square value. Dividing by the number of degrees of freedom to obtain a reduced chi square for the best fit will then provide further guidance as to whether the fit is reasonably related to the random errors, or whether systematic effects or another fit function should be considered.

degrees instead of radians before OK'ing the plot and fit, if this is your angular data unit. Never specify 0.0 as a starting search parameter; use a small number instead. KaleidaGraph is unable to proceed with its initial operations for a zero initial parameter value.

If the error in the best-fit m_1 value is comparable to m_1, include also in your report general least squares fits with m_1 set equal to zero. (It is not necesary to remove the corresponding starting values from the fit definition; KG will ignore them.)

List together the various values of η_{21} obtained from: Snell's law, Brewster's angle, R_N and R_P best fits. For easy comparison list $1/\eta_{21\ fit}$ for the plastic --> air fits.

3. Reflection coefficients for dielectric --> air ($n_2 < n_1$); total internal reflection

The data polarization order (N or P) is immaterial.

Reverse the semicircular glass or plastic sample. Use a sample for which you took previous external reflection data. (Mount the semi-circular glass or plastic sample so that the incident beam enters and exits radially. Then the beam will not refract at these points (ignoring finite beam width effects) and the internal angle of incidence can be simply determined.) For parallel incident polarization observe qualitatively the doubly transmitted beam exiting into air, and the reflected beam exiting normally into air, as the incidence angle is varied. Note the occurrence of a Brewster's angle and of a (larger) angle of critical reflection at the second surface. Record these angles.

Take quantitative I_r (internal) vs. i data for the P (parallel incident polarization) reflected at the glass (or plastic)-air interface. Plot observed intensities for $i < i_{critical}$ and fit as before, with the intensity represented by a multiplicative search parameter. Examine the non-linear best fit parameters and their associated errors.

---

[1] Possible sources of such additive data constants are a zero offset of the meter reading (may be + or -), or room light in addition to the laser light (+ only). If these are the only sources, a negative fit value (outside of errors) fit for m1 would indicate dominance by zero offset; room light effect could still be present.

[2] If a slight mis-orientation is suspected, the fit function could be augmented by addition of the form for the second (small) polarization component, multiplied by a search parameter m4, the sum of both terms being multiplied by m2 (or (1-m1) , depending on whether it is assumed that e=e0 or e ≠e0. The expression would then correspond to

m1+m2*[Rperpendicular(m3)+m4*Rparallel(m3)];m1 = ,m2 = ;m3 = ;m4= ,

where the R's are the theoretical expressions in KG syntax and numerical starting search values (≠0) are specified for m1, m2, m3 and m4.

The theory suggests a sharp onset for total reflection, but the data may show a steep, finite rise to a constant value. This may be due to laser beam divergence and/or non-normal incidence and exit at the circular face. The width of such a rise is of practical interest (please report it), but corresponding points must be excluded from the fit. Further, while the data for $i < i_{critical}$ should contain information on the laser beam intensity, a multiplicative fit parameter (intensity parameter) may be poorly determined. Fit also with a numerical value for this obtained by averaging the "flat" total reflection intensities.

(Note that transmission losses in entrance and exit from the circular face will be a constant fractional correction, to the extent that all rays are normal. Then the shape of the data curve will be unaffected. However, while the observed total reflection intensities also contain this common loss factor, a zero degree intensity measurement does not and would therefore not be an appropriate intensity normalization, in contrast to the case for external reflection.)

Determine the internal Brewster's angle as well as possible, and the corresponding relative index of refraction.

Repeat for the N beam (perpendicular incident polarization).

Indices for some common transparent glasses are given in the table below, taken from Monk. The numerical part of this table can be copied and pasted directly into a KaleidaGraph data file. As discussed in the Faraday effect experiment instructions, Cauchy found that the dispersion curve (n vs. λ) for most materials is well fit by a power series in inverse even powers of the wavelength. For the Faraday effect, the derivative of this curve is of interest.

λ– A.U	**Light Crown**	**Dense crown**	**Light Flint**	**Dense Flint**	**Heavy Flint**	**Fused Quartz**	**Fluorite**
4000	1.5238	1.5854	1.5932	1.6912	1.8059	1.4699	1.4421
4600	1.5180	1.5801	1.5853	1.6771	1.7843	1.4655	1.4390
5000	1.5139	1.5751	1.5796	1.6770	1.7706	1.4624	1.4366
5600	1.5108	1.5732	1.5757	1.6951	1.7611	1.4599	1.4350
6000	1.5085	1.5679	1.5728	I.6542	1.7539	1.4581	1.4336
6500	1.5067	1.5651	1.5703	I.6503	1.7485	1.4566	1.4324
7000	1.5051	1.5640	1.5684	1.6473	1.7435	1.4553	1.4318

7500	1.5040	1.5625	1.5668	1.6450	1.7389	1.4542	1.4311

4. Reflection coefficients for absorbing media

A KG fit form, adapted from an approximation shown in Ditchburn (1953 edition, p 444, for parallel polarization (adjust scale factor m1 to data), is

R_P = m1*(m2^ 2*(1+m3^ 2)*(cos(m0))^ 2-2*m2*cos(m0)+1)

/(m2^ 2*(1+m3^ 2)*(cos(m0))^ 2+2*m2*cos(m0)+1)

;m1=1;m2=1.5;m3=1.00

and for perpendicular polarization

R_N = m1*(m2^ 2*(1+m3^ 2)-2*m2*cos(m0)+(cos(m0))^2)

/(m2^ 2*(1+m3^ 2)+2*m2*cos(m0)+(cos(m0))^2)

;m1=1;m2=1.5;m3=1.00

(m1 could be adjustedto the plot data).

In Ditchburn's notation the damped, travelling wave propagating in the z direction in the absorbing medium can be described by the expression

$$E_x = A_0 \{\exp\{ -\alpha z +i(\omega t-\kappa z)\}, \text{ or}$$

$$E_x = A_0 \{ \exp[i\omega(t-(n_{complex}/c)z]$$

where the "complex index of refraction" $n_{complex} = n(1-i\chi)$

with $n = \kappa c/\omega$ and $\chi = \alpha/\kappa$.

Ditchburn approximation condition is

$n^2 + n^2\chi^2 >> 1$ or, in the KG notation above

$m2^2 + m2^2\ m3^2 >> 1$.

which can be checked after a fit has been performed.

Using the parameters given by Ditchburn for his Figure 15.1 (p 444) (n = 1.5, c = 1.00, reflection of 1 at 90 degrees for both polarizations, data for the figure can be simulated by KG (Formula Entry, with incidence angles in column c0 and putting the parallel and normal theoretical intensities in columns c1 and c2 respectively), using the expressions below derived from the KG forms above:

for parallel (P) incident polarization

c1 = 1*(1.5^ 2*(1+1.00^ 2)*(cos(c0))^ 2-2*1.5*cos(c0)+1)

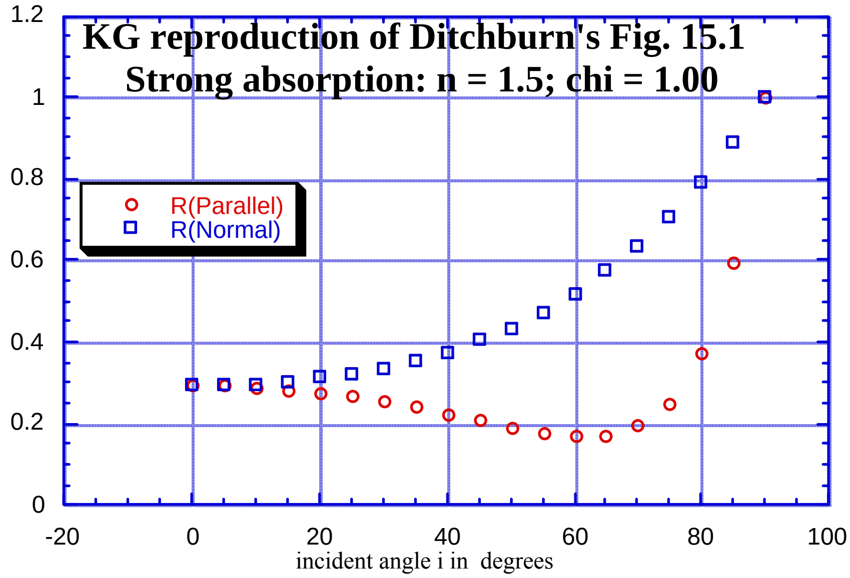
/(1.5^ 2*(1+1.00^ 2)*(cos(c0))^ 2+2*1.5*cos(c0)+1)

and for perpendicular (Normal) polarization

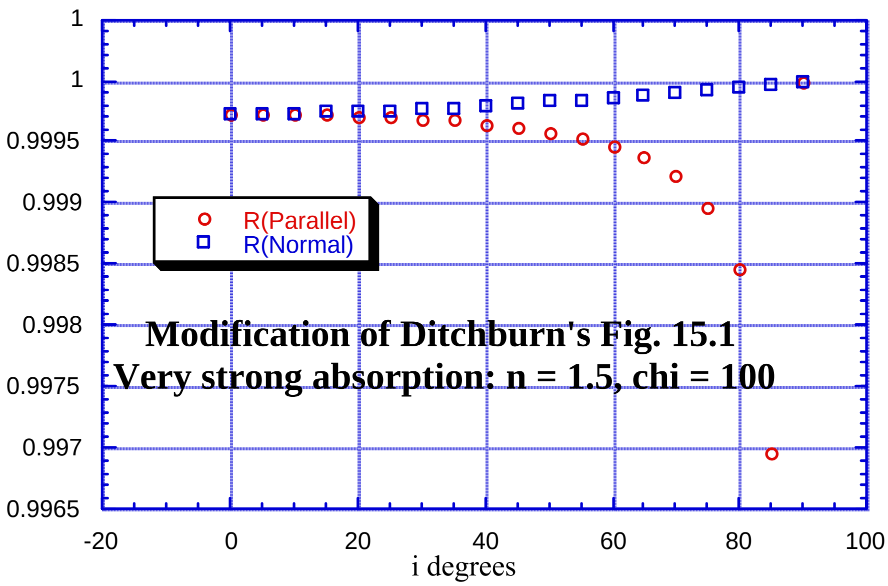
c2 = 1*(1.5^ 2*(1+1.00^ 2)-2*1.5*cos(c0)+(cos(c0))^2)

/(1.5^ 2*(1+1.00^ 2)+2*1.5*cos(c0)+(cos(c0))^2) .

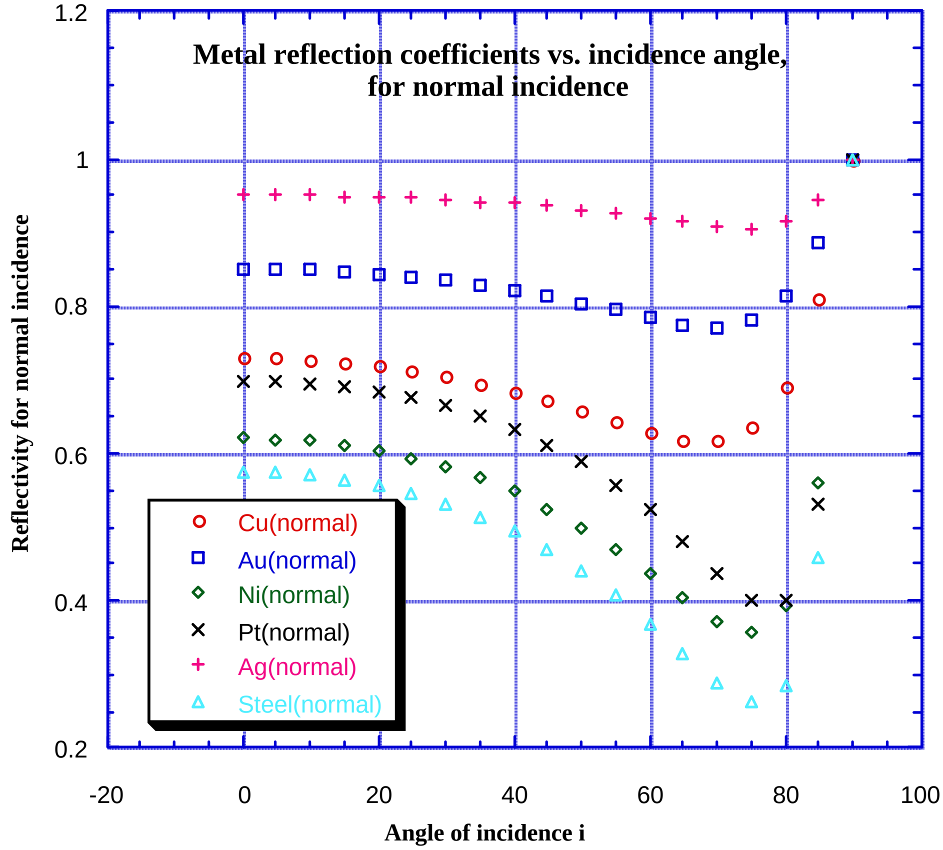
The corresponding plots are shown below

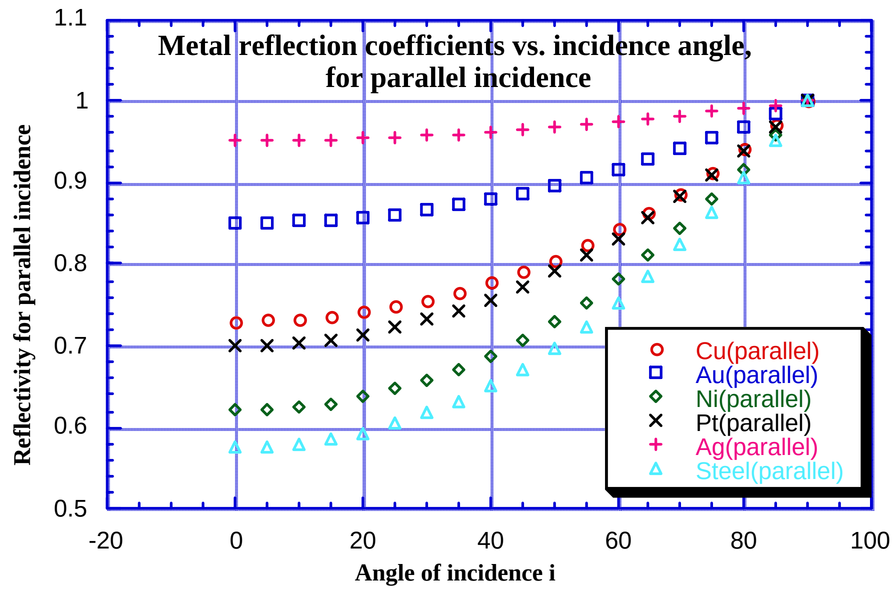


For strong absorption (n still 1.5, but c = 100) the simulation approcahes the near 100% reflection of metals, for both polarizations:



KG plots and data formula forms for several metal reflections are given below, using Dirchburn's complex index parameters and approximate expressions for the two incident polarizations.


Metal reflection coefficients vs. incidence angle,
for normal incidence
Reflectivity for normal incidence
Angle of incidence i
1.2
1
0.8
0.6
0.4
0.2
-20
0
20
40
60
80
100
Cu(normal)
Au(normal)
Ni(normal)
Pt(normal)
Ag(normal)
Steel(normal)



welcome datacomp

From Ditchburn: Light (1953 edition) Metals, p 449, Table 15.1:

KaleidaGraph normal polarization expressions from Ditchburn's approximate Eq. 15(19a) and 15(19b) (approximation condition: n^2+n^2k^2 >>1): c0 (KG column 0) = angle of incidence

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-

Copper (normal): n = 0.64, k = 4.08; n^2+k^2 = 7.23

c1 = 1*(0.64^ 2*(1+4.08^ 2)*(cos(c0))^ 2-2*0.64*cos(c0)+1)

/(0.64^ 2*(1+4.08^ 2)*(cos(c0))^ 2+2*0.64*cos(c0)+1)

Gold (normal): n = 0.366, k = 7.70; n^2+k^2 = 8.076

c2 = 1*(0.366^ 2*(1+7.70^ 2)*(cos(c0))^ 2-2*0.366*cos(c0)+1)

/(0.366^ 2*(1+7.70^ 2)*(cos(c0))^ 2+2*0.366*cos(c0)+1)

Nickel (normal): n = 1.79, k = 1.86; n^2+k^2 = 14.289

c3 = 1*(1.79^ 2*(1+1.86^ 2)*(cos(c0))^ 2-2*1.79*cos(c0)+1)

/(1.79^ 2*(1+1.86^ 2)*(cos(c0))^ 2+2*1.79*cos(c0)+1)

-

Platinum (normal): n = 2.06, 2.06; n^2+k^2 = 22.251

c4 = 1*(2.06^ 2*(1+2.06^ 2)*(cos(c0))^ 2-2*2.06*cos(c0)+1)

/(2.06^ 2*(1+2.06^ 2)*(cos(c0))^ 2+2*2.06*cos(c0)+1)

-

Silver (normal): n = 0.181, k = 20.2; n^2+k^2 = 13.400

c5 = 1*(0.181^ 2*(1+20.2^ 2)*(cos(c0))^ 2-2*0.181*cos(c0)+1)

/(0.181^ 2*(1+20.2^ 2)*(cos(c0))^ 2+2*0.181*cos(c0)+1)

-

Steel (normal): n = 2.41, k = 1.38; n^2+k^2 = 16.87

c6 = 1*(2.41^ 2*(1+1.38^ 2)*(cos(c0))^ 2-2*2.41*cos(c0)+1)

/(2.41^ 2*(1+1.38^ 2)*(cos(c0))^ 2+2*2.41*cos(c0)+1)

==

KaleidaGraph parallel polarization expressions from Ditchburn's approximate Eq. 15(19a) and 15(19b) (approximation condition: n^2+n^2k^2 >>1): c0 (KG column 0) = angle of incidence i.

-

Copper (parallel): n = 0.64, k = 4.08; n^2+k^2 = 7.23

c8 = 1*(0.64^ 2*(1+4.08^ 2)-2*0.64*cos(c0)+(cos(c0))^2)

/(0.64^ 2*(1+4.08^ 2)+2*0.64*cos(c0)+(cos(c0))^2)

Gold (parallel): n = 0.366, k = 7.70; n^2+k^2 = 8.076

c9 = 1*(0.366^ 2*(1+7.70^ 2)-2*0.366*cos(c0)+(cos(c0))^2)

/(0.366^ 2*(1+7.70^ 2)+2*0.366*cos(c0)+(cos(c0))^2)

Nickel (parallel): n = 1.79, k = 1.86; n^2+k^2 = 14.289

c10 = 1*(1.79^ 2*(1+1.86^ 2)-2*1.79*cos(c0)+(cos(c0))^2)

/(1.79^ 2*(1+1.86^ 2)+2*1.79*cos(c0)+(cos(c0))^2)

Platinum (parallel): n = 2.06, 2.06; n^2+k^2 = 22.251

c11 = 1*(2.06^ 2*(1+2.06^ 2)-2*2.06*cos(c0)+(cos(c0))^2)

/(2.06^ 2*(1+2.06^ 2)+2*2.06*cos(c0)+(cos(c0))^2)

Silver (parallel): n = 0.181, k = 20.2; n^2+k^2 = 13.400

c12 = 1*(0.181^ 2*(1+20.2^ 2)-2*0.181*cos(c0)+(cos(c0))^2)

/(0.181^ 2*(1+20.2^ 2)+2*0.181*cos(c0)+(cos(c0))^2)

Steel (parallel): n = 2.41, k = 1.38; n^2+k^2 = 16.87

c13 = 1*(2.41^ 2*(1+1.38^ 2)-2*2.41*cos(c0)+(cos(c0))^2)

/(2.41^ 2*(1+1.38^ 2)+2*2.41*cos(c0)+(cos(c0))^2)

==

λ– A.U	**Light Crown**	**Dense crown**	**Light Flint**	**Dense Flint**	**Heavy Flint**	**Fused Quartz**	**Fluorite**

4000	1.5238	1.5854	1.5932	1.6912	1.8059	1.4699	1.4421
4600	1.5180	1.5801	1.5853	1.6771	1.7843	1.4655	1.4390
5000	1.5139	1.5751	1.5796	1.6770	1.7706	1.4624	1.4366
5600	1.5108	1.5732	1.5757	1.6951	1.7611	1.4599	1.4350
6000	1.5085	1.5679	1.5728	I.6542	1.7539	1.4581	1.4336
6500	1.5067	1.5651	1.5703	I.6503	1.7485	1.4566	1.4324
7000	1.5051	1.5640	1.5684	1.6473	1.7435	1.4553	1.4318
7500	1.5040	1.5625	1.5668	1.6450	1.7389	1.4542	1.4311