Rutgers University
Department of Physics & Astronomy

01:750:271 Honors Physics I
Fall 2015

Lecture 8

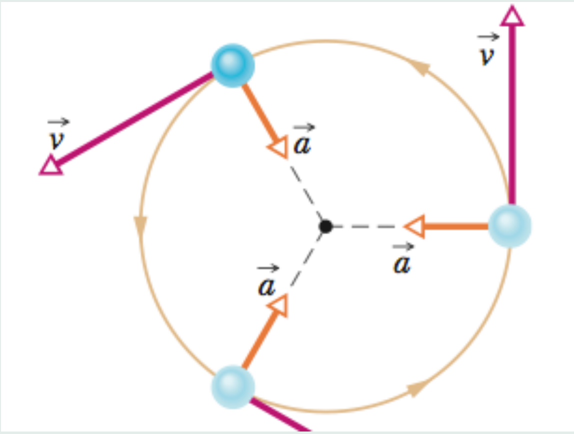
Midterm 1: Monday October 5th 2014

- Motion in one, two and three dimensions
- Forces and Motion I (no friction)
- No energy and work.

6. Forces and Motion II

- **Dynamics of uniform circular motion**

Centripetal acceleration:



$$\vec{a} = -\frac{v^2}{r}\widehat{r}$$

$r =$ radius of the circle

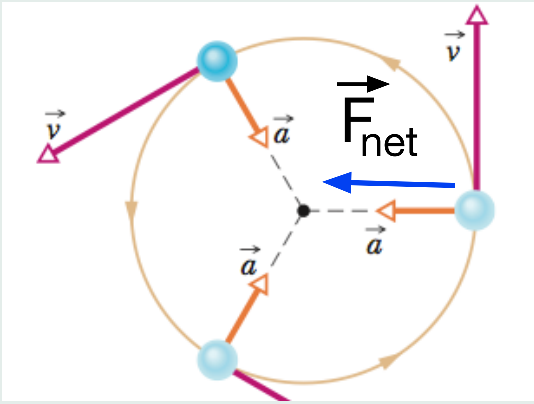
$v =$ speed

$\widehat{r} = \frac{\vec{r}}{r}$ **unit** radial vector

Newton's 2nd law:

$$\vec{F}_{\text{net}} = m\vec{a}$$

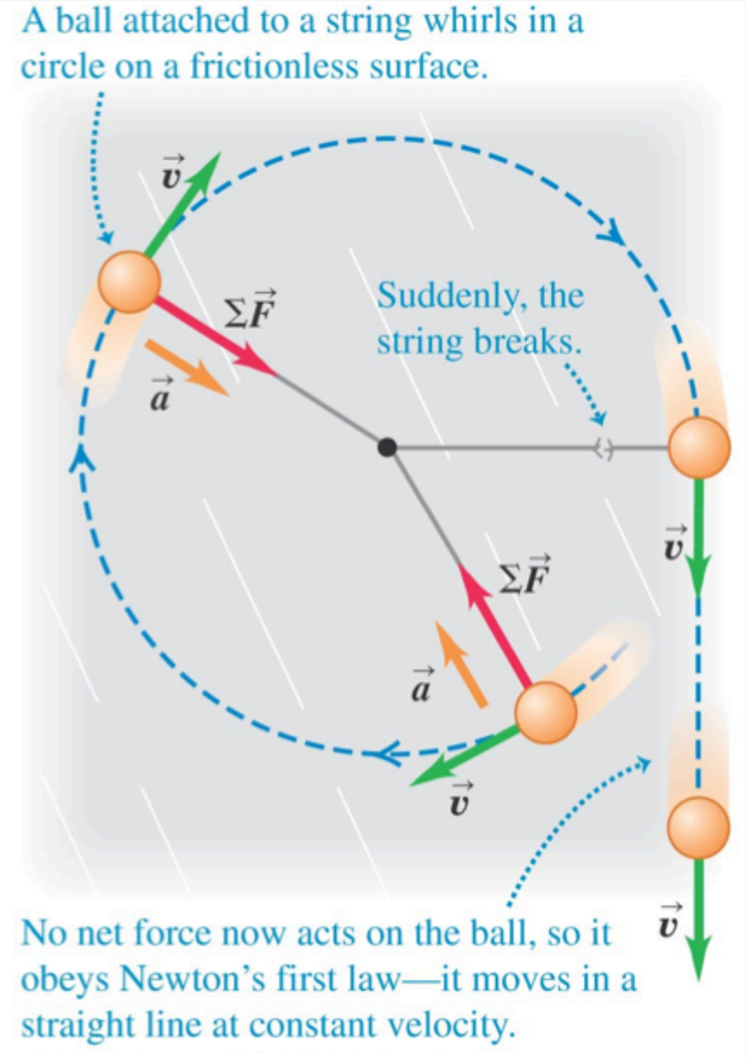
$$\Downarrow$$



$$\boxed{\vec{F}_{\text{net}} = -\frac{mv^2}{r}\hat{r}}$$

Centripetal Force

A centripetal force accelerates a body by changing the direction of the body's velocity without changing the body's speed.



- ball attached to a string in uniform circular motion on a horizontal frictionless plane

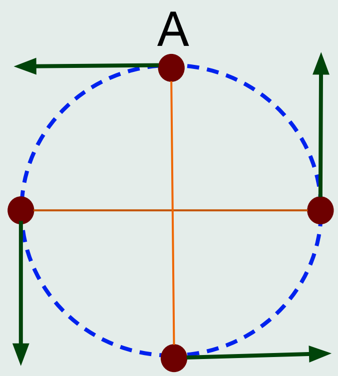
$$\vec{F}_{\text{net}} = \vec{T} \quad \text{(tension)}$$

- the string breaks $\Rightarrow$ uniform **linear** motion, according to Newton's 1st law.

- no $\vec{T}$ $\Rightarrow$ no centripetal force $\Rightarrow$ $\vec{a} = 0$

i-Clicker

A ball attached to a string rotates in a vertical plane near Earth's surface such that the string is stretched taut all points on the circular path. What is the minimum value of its speed at the highest point A of the trajectory.



$A)$ $v_{\text{min}} = 0$

$B)$ $v_{\text{min}} = \sqrt{rg}$

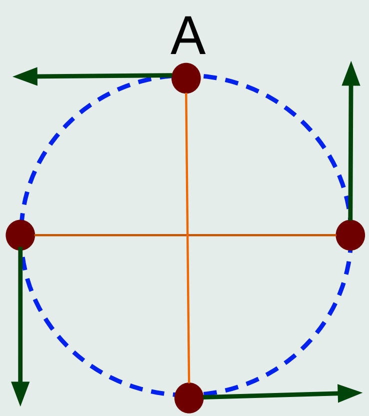
$C)$ $v_{\text{min}} = \sqrt{rg/2}$

$D)$ $v_{\text{min}} = \sqrt{2gr}$

$E)$ none of the above

Answer

A ball attached to a string rotates in a vertical plane near Earth's surface. For a point P on the trajectory, let T_P denote the magnitude of the tension in the string. Which of the following statements is true?



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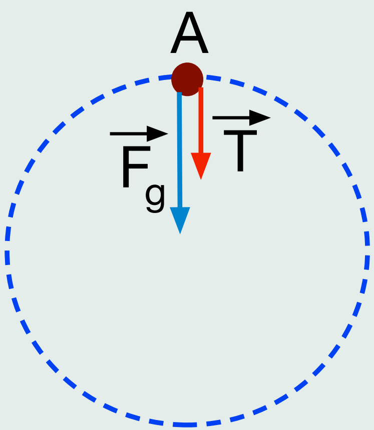


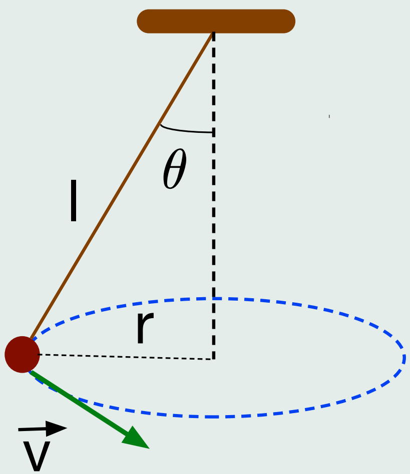
$$\vec{F}_{\text{net}} = -\frac{mv^2}{r}\widehat{r} \qquad \vec{F}_{\text{net}} = \vec{F}_g + \vec{T}$$

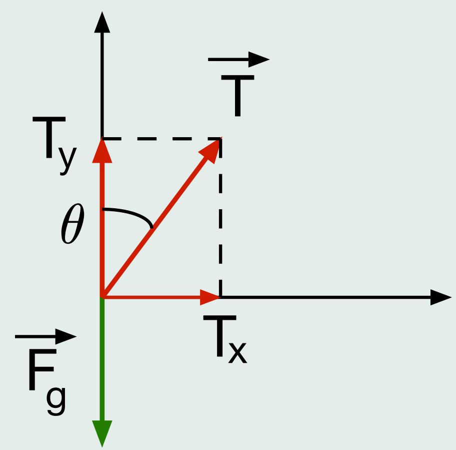
$$(\mathsf{F}_{net})_y = -(mg + T) = -\frac{mv^2}{r}$$

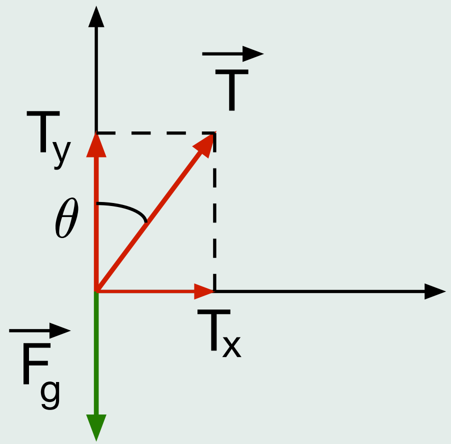
$$T = \frac{mv^2}{r} - mg \geq 0$$

$$v^2 \geq rg \;\Rightarrow\; v_{\text{min}} = \sqrt{rg}$$

• **Example:** A ball attached to a string of length l, which makes an angle θ with the vertical, rotates uniformly in a horizontal plane as shown below. Find the speed v.







$$\vec{F}_{\text{net}} = m\vec{a} \qquad \vec{F}_{\text{net}} = \vec{F}_g + \vec{T}$$

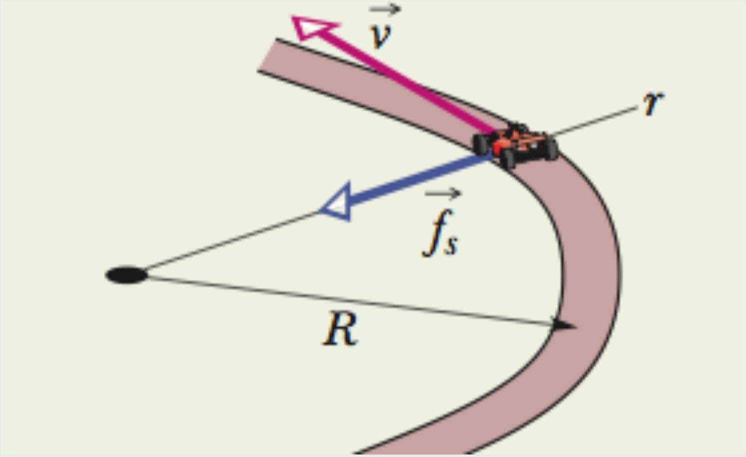
$$(F_{\text{net}})_x = T\sin\theta$$
$$(F_{\text{net}})_y = -mg + T\cos\theta$$

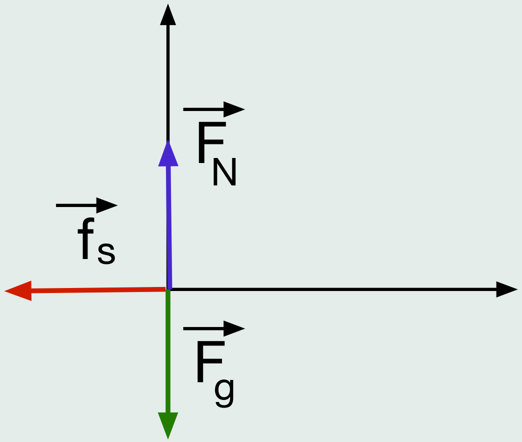
$$a_x = v^2/r \qquad a_y = 0$$

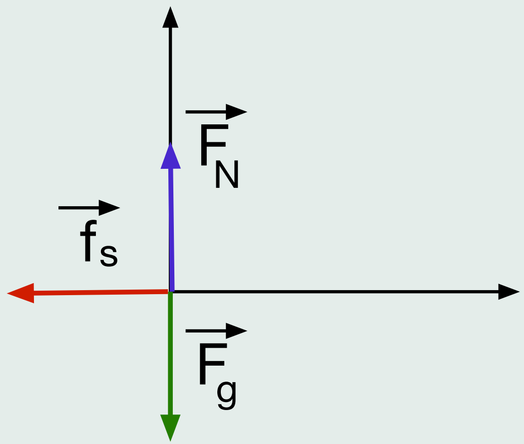
$$T\sin\theta = \frac{mv^2}{r} = \frac{mv^2}{l\sin\theta}$$
$$T\cos\theta - mg = 0$$

$$\frac{v^2}{lg\sin\theta} = \tan\theta \;\Rightarrow v = \sin\theta\sqrt{\frac{lg}{\cos\theta}}$$

• **Example:** a car of mass m moving with constant speed makes a turn of radius r. Suppose the static friction coefficient between the wheels and the road is μ_s. Find the maximum value of the speed v such that the car will remain on the road.







$$\vec{F}_{\text{net}} = m\vec{a} \qquad \vec{a} = -\frac{mv^2}{r}\hat{i}$$

$$F_N - mg = ma_y = 0$$

$$-f_s = ma_x = -\frac{mv^2}{r}$$

$$f_s \le \mu_s F_N \quad \Rightarrow \quad \frac{mv^2}{r} \le \mu_s mg$$

$$v \le \sqrt{\mu_s r g}$$

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7. Kinetic energy and work

- **What is energy?**

  - Basic Idea: scalar quantity with quantifies the state of motion and/or the capacity for motion of a system

  - Conserved as the state of motion of the system changes.

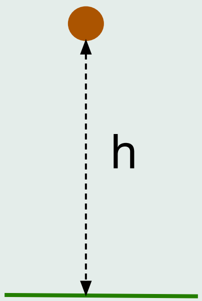
Speed of train at the lowest point?

Speed of arrow in flight?

Energy

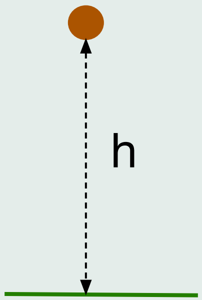
- Kinetic energy : associated to motion
- Potential energy : associated to the capacity of generating motion

- **Example:** freely falling ball from height h.



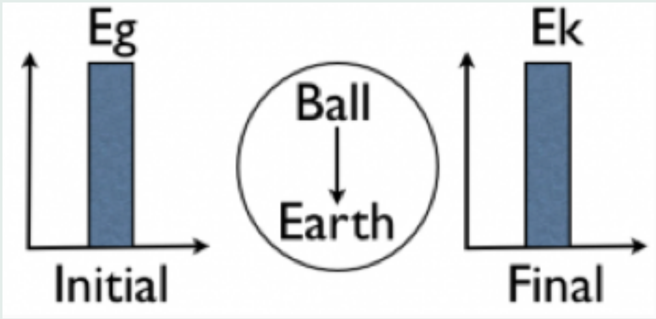
- initial speed: $v_0 = 0$
- final speed:

$$v^2 = v_0^2 + 2gh = 2gh$$



- initially: **potential energy:** mgh
- finally: **kinetic energy:** $mv^2/2$
- **Energy conservation:**

$$mgh = \frac{mv^2}{2}$$



- **Kinetic Energy:** energy associated to the motion of an object

$$K = \frac{1}{2}mv^2$$

Unit: Joule

$$1\mathrm{J} = 1\mathrm{kg} \times \mathrm{m}^2/\mathrm{s}^2$$

Non-zero force

$$\Downarrow$$

Acceleration: change in velocity

$$\Downarrow$$

Change in kinetic energy

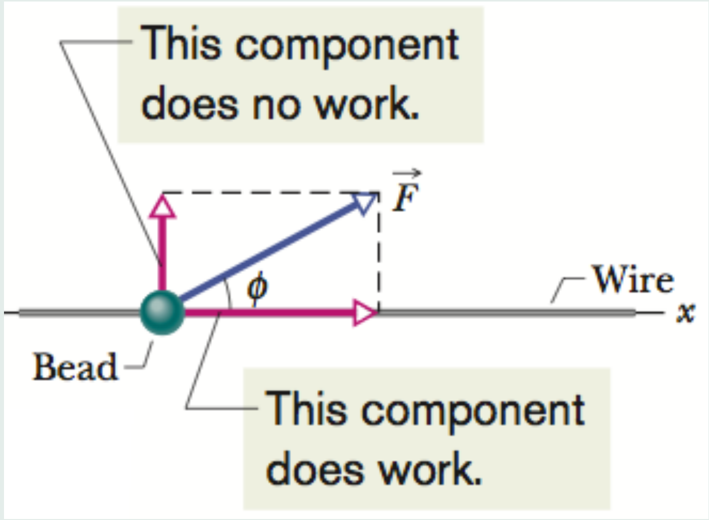
How does K change under an applied force?

- **Work**: energy transferred to or from an object by means of a force acting on the object.

Energy transferred to the object is positive work while energy transferred from the object is negative work.

How do we calculate the work of a force?

- **Example:** bead on frictionless rod subject to constant force $\vec{F}$



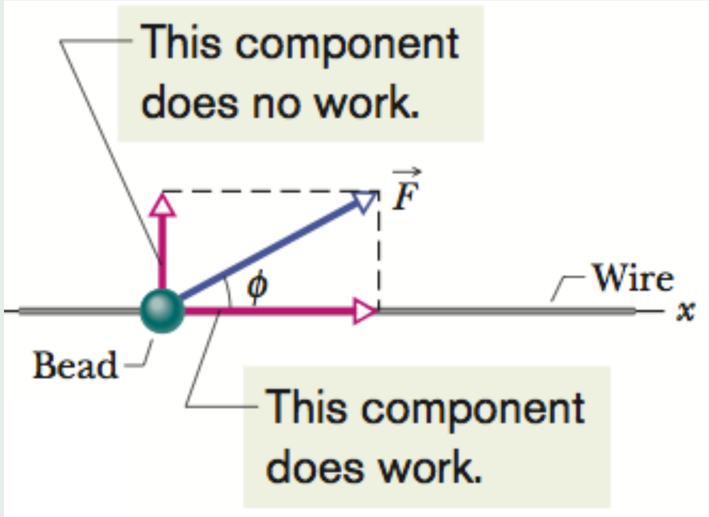
Newton's 2nd law:

$$F_x = ma_x$$

Constant acceleration model:

$$v_x^2 = v_{0x}^2 + 2a_x\,\Delta x$$

$$\frac{mv^2}{2} - \frac{mv_0^2}{2} = F_x\,\Delta x$$



- Work done by the force $\vec{F}$

$$F d \cos\phi = \vec{F} \cdot \vec{d}$$

$$\vec{d} = (\Delta x)\hat{i}$$

displacement vector

$$F d \cos\phi \begin{cases} > 0, \text{ for } 0 \leq \phi < \pi/2 & \text{positive work}, \; K_f > K_i \\ = 0, \text{ for } \phi = \pi/2 & \text{zero work}, \; K_f = K_i \\ < 0, \text{ for } \pi/2 < \phi \leq \pi & \text{negative work}, \; K_f < K_i \end{cases}$$

- **Net work:**

When two or more forces act on an object, the **net work** done on the object is the sum of the works done by the individual forces.

$$W_{\text{net}} = \sum W = \sum \vec{F} \cdot \vec{d} = \left(\sum \vec{F}\right) \cdot \vec{d} = \vec{F}_{\text{net}} \cdot \vec{d}$$

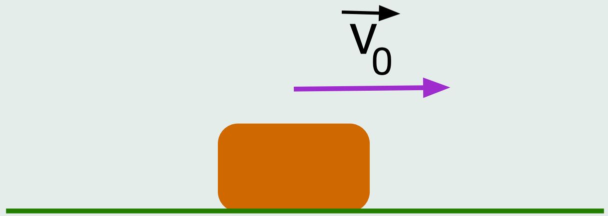
- **Work-Kinetic Energy Theorem**

$$\begin{pmatrix}\text{change in the kinetic} \\ \text{energy of a particle}\end{pmatrix} = \begin{pmatrix}\text{net work done on} \\ \text{the particle}\end{pmatrix}.$$

$$\Delta K = W \qquad K_f = K_i + W$$

i-Clicker

An object of mass $m = 1\,\text{kg}$ is launched with initial speed $v_0 = 2\,\text{m/s}$ along a rough horizontal surface. What is the total work done by the frictional force until it stops?



$A)\ 2\,\text{J}$

$B)\ 1\,\text{J}$

$C)\ -1\,\text{J}$

$D)\ -2\,\text{J}$

$E)\ 0\,\text{J}.$

Answer

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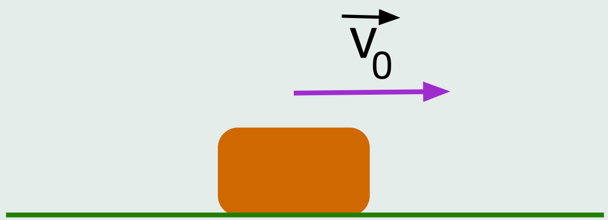


$A)\ 2\,\mathrm{J}$

$B)\ 1\,\mathrm{J}$

$C)\ -1\,\mathrm{J}$

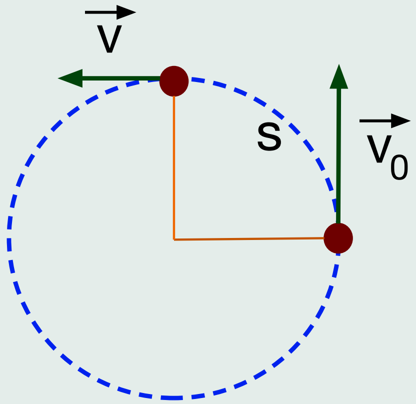
$D)\ -2\,\mathrm{J}$

$E)\ 0\,\mathrm{J}.$

$$W = \Delta K = K_f - K_i = -\frac{m v_0^2}{2}$$

i-Clicker

A ball of mass m attached to string is launched with initial speed v_0 on a circular trajectory on horizontal plane. The friction force between the ball and the surface has constant magnitude f_k. What is the speed of the ball after travelling a distance s along the circle.



$A)\ v = v_0$

$B)\ v = \sqrt{v_0^2 - \frac{2 f_k s}{m}}$

$C)\ v = v_0 - \frac{f_k s}{m v_0}$

$D)$ cannot be determined from the data.

Answer

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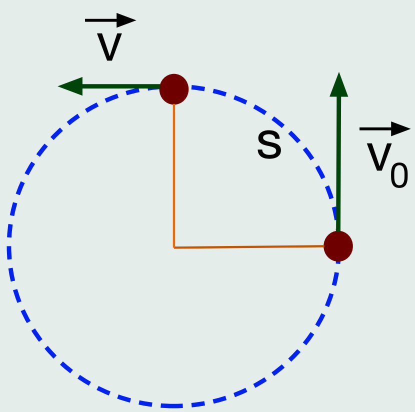

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$B)\ v = \sqrt{v_0^2 - \frac{2 f_k s}{m}}$

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$D)$ cannot be determined from the data.

Infinitesimal displacement

$$\vec{ds} = \vec{v}dt$$

Note that $\vec{T} \perp \vec{ds}$ while $\vec{f_k}$ makes an angle $\theta = 180^\circ$ with $\vec{ds}$.

Infinitesimal work:

$$dW_{f_k} = \vec{f_k} \cdot \vec{ds} = -f_k ds$$

Total work:

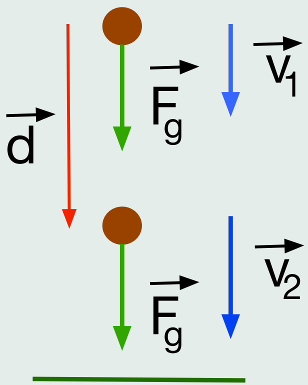
$$W = \int dW = -f_k s$$

Work-energy theorem:

$$\frac{mv^2}{2} - \frac{mv_0^2}{2} = -f_k s$$

$$v = \sqrt{v_0^2 - \frac{2f_k s}{m}}$$

- **Work done by the gravitational force**

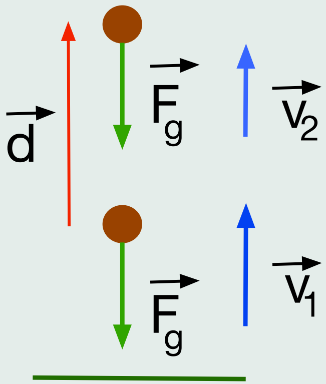


Freely falling object moving downwards:

$$W = \vec{F}_g \cdot \vec{d} = mgd\cos 0^\circ = mgd > 0$$

$$K_1 = \frac{mv_1^2}{2} \qquad K_2 = \frac{mv_2^2}{2}$$

$$K_2 - K_1 = mgd > 0$$



Freely falling object moving upwards:

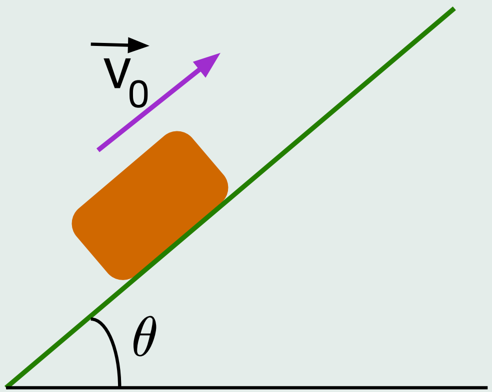
$$W = \vec{F}_g \cdot \vec{d} = mgd\cos 180^\circ = -mgd > 0$$

$$K_1 = \frac{mv_1^2}{2} \qquad K_2 = \frac{mv_2^2}{2}$$

$$K_2 - K_1 = -mgd < 0$$

i-Clicker

An object of mass m is launched with initial speed v_0 along an inclined plane making an angle $\theta = 45^\circ$ with the horizontal. The kinetic friction coefficient between the object and the plane is $\mu_k = 0.5$. Let W_{f_k} be the total work done by the friction force until it stops. Which of the following statements is false?



A) $W_{f_k} < 0$

B) $W_{f_k} = -mv_0^2/2$

C) $|W_{f_k}| < mv_0^2/2$

Answer

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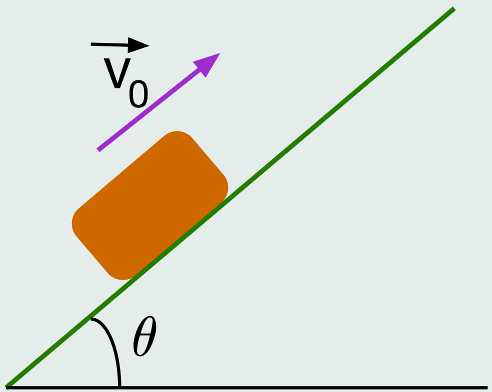

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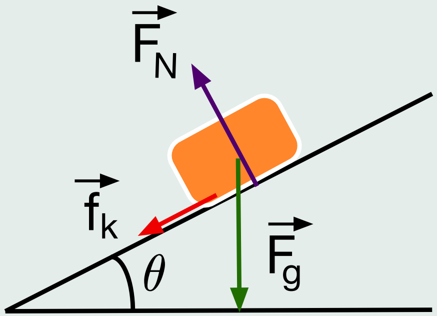
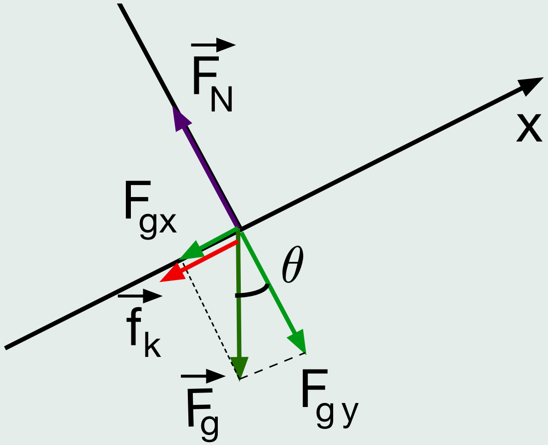


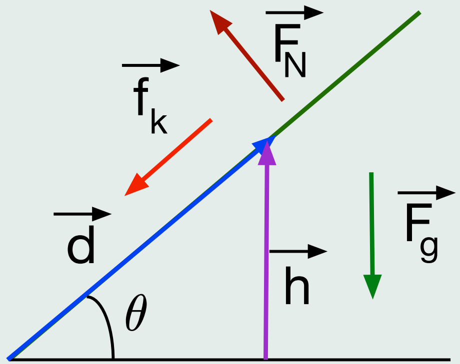


$$\vec{F}_{\text{net}} = m\vec{a} \qquad (F_{\text{net}})_x = ma_x \qquad (F_{\text{net}})_y = ma_y = 0$$

$$\vec{F}_{\text{net}} = \vec{F}_g + \vec{F}_N + \vec{f}_k$$

$$(F_{\text{net}})_x = -mg\sin\theta - f_k \qquad (F_{\text{nety}})_y = F_N - mg\cos\theta = 0$$

$$F_N = mg\cos\theta \qquad f_k = \mu_k mg\cos\theta$$



$$f_k = \mu_k mg\cos\theta$$

$$h = d\sin\theta$$

Work done by kinetic friction:

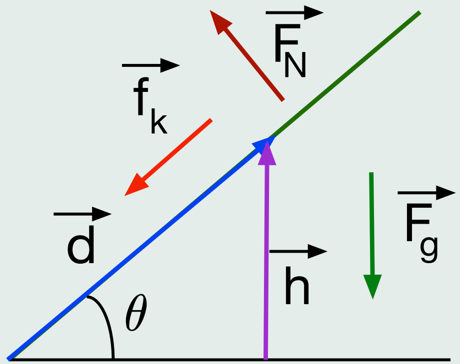
$$W_{f_k} = \vec{f}_k \cdot \vec{d} = -f_k d = -\mu_k mgd\cos\theta$$

Work done by gravitational force:

$$W_{F_g} = \vec{F}_g \cdot \vec{d} = -mgh = -mgd\sin\theta$$

Work done by normal force

$$\vec{F}_N \cdot \vec{d} = 0$$



Total work:

$$W = W_{f_k} + W_{F_g} = -mgd\left(\sin\theta + \mu_k \cos\theta\right)$$

Work-Kinetic energy theorem:

$$W = \Delta K = 0 - K_i = -mv_0^2/2$$

Work done by kinetic friction:

$$\frac{W_{f_k}}{|W|} = -\frac{\mu_k \cos\theta}{\sin\theta + \mu_k \cos\theta} = \frac{1}{3}$$

$$W_{f_k} = -\frac{mv_0^2}{6}$$