Rutgers University
Department of Physics & Astronomy

01:750:271 Honors Physics I
Fall 2015

Lecture 24

19. Kinetic Theory of Gasses

Macrosopic quantities

$$p, V, T, \ldots$$

$$\updownarrow$$

Microscopic quantities

velocity vectors of atoms $\vec{v}_1, \vec{v}_2, \ldots$

Avogadro's number

Avogadro's principle: all gases occupying the same volume under the same conditions of temperature and pressure contain the same number of atoms or molecules.

How many such atoms or molecules?

One mole: amount of any substance which contains the same number of atoms or molecules as a 12 g sample of carbon-12.

Number of atoms/molecules per mole:

$$N_A = 6.02 \times 10^{23} \mathrm{mol}^{-1}$$

- The number of moles n contained in any sample of substance is

$$n = \frac{N}{N_A} = \frac{M_{\text{sam}}}{M} = \frac{M_{\text{sam}}}{mN_A}$$

- $N =$ total number of atoms/molecules in sample
- $M_{\text{sam}} =$ mass of the sample
- $m =$ mass of one atom/molecule of substance
- $M = mN_A$ molar mass

Ideal Gases

Equation of state : $$pV = nRT$$

- $p, V =$ pressure, volume
- $n =$ number of moles of gas
- $T =$ temperature (kelvins)
- $R = 8.31\,\mathrm{J/mol \cdot K}$ for all gases.

$$pV = NkT$$

- $N =$ number of atoms/molecules

Boltzmann constant : $k = \frac{R}{N_A} = 1.38 \times 10^{-23}\, J/K$

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An ideal gas is enclosed within a container by a movable piston. If the final temperature is two times the initial temperature and the volume is reduced to one-fourth of its initial value, what will the final pressure of the gas be relative to its initial pressure, p_1?

$A)$ $8p_1$

$B)$ $4p_1$

$C)$ $2p_1$

$D)$ $p_1/2$

$E)$ $p_1/4$

i-Clicker

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E) $p_1/4$

$$p_1 V_1 = NkT_1$$

$$p_2 V_2 = NkT_2$$

$$\frac{p_2}{p_1} = \frac{T_2}{T_1}\frac{V_1}{V_2} = 2 \times 4 = 8$$

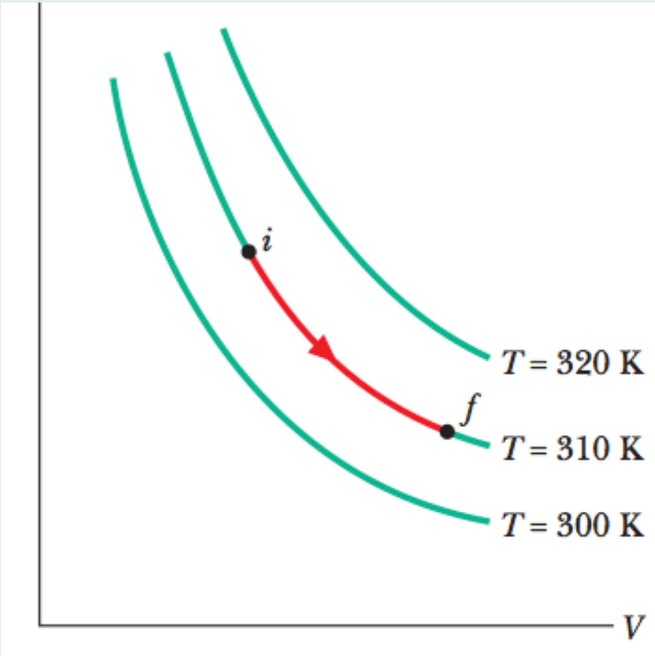
- **Work done by ideal gas at constant volume**:

$$dV = 0 \;\Rightarrow\; W = \int p dV = 0$$

- **Work done by ideal gas at constant pressure**:

$$dp = 0 \;\Rightarrow\; p = \text{constant} \;\Rightarrow\; W = \int p dV = p\Delta V$$

- **Work done by ideal gas at constant T**



$$pV = nRT \;\Rightarrow p = \frac{nRT}{V}$$

$$W = \int p dV = nRT \int \frac{dV}{V}$$

(since $T =$ constant)

$$W = nRT \ln\frac{V_2}{V_1}$$

isothermal (hyperbola)

$pV =$ constant

Pressure, Temperature, RMS speed

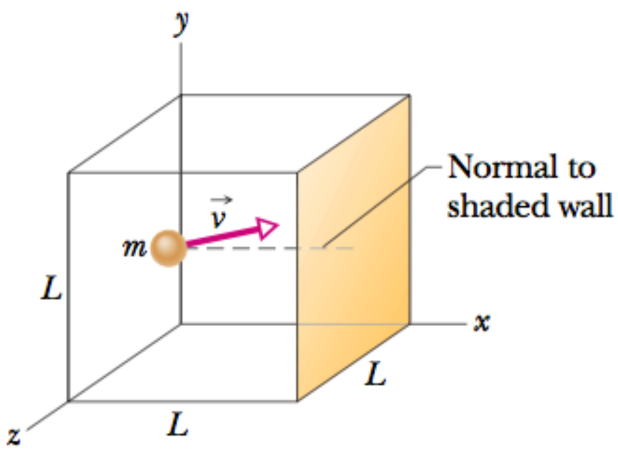


Fig. 19-4 A cubical box of edge length L, containing n moles of an ideal gas. A molecule of mass m and velocity $\vec{v}$ is about to collide with the shaded wall of area L^2. A normal to that wall is shown.

- a gass molecule of mass m with velocity $\vec{v}$ is about to collide with a wall
- Assume the collision **elastic**

$$\Delta p_x = (v_x)_{\text{after}} - (v_x)_{\text{before}}$$
$$= -mv_x - mv_x = -2mv_x$$

- Time between 2 successive collisions with the **same** wall:

$$\Delta t = \frac{2L}{v_x}$$

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- Average force on the wall:

$$F_x = \frac{\Delta p_x}{\Delta t} = \frac{2mv_x}{2L/v_x} = \frac{mv_x^2}{L}$$

- Average pressure:

$$p = \frac{F_x}{L^2} = \frac{mv_x^2}{L^3}$$

- N molecules:

$$p = \frac{m}{L^3}(v_{1,x}^2 + \cdots + v_{N,x}^2) = \frac{mN}{L^3}(v_x^2)_{\text{avg}}$$

$$(v_x^2)_{\text{avg}} = \frac{v_{1,x}^2 + \cdots + v_{N,x}^2}{N}$$

average value of v_x^2.

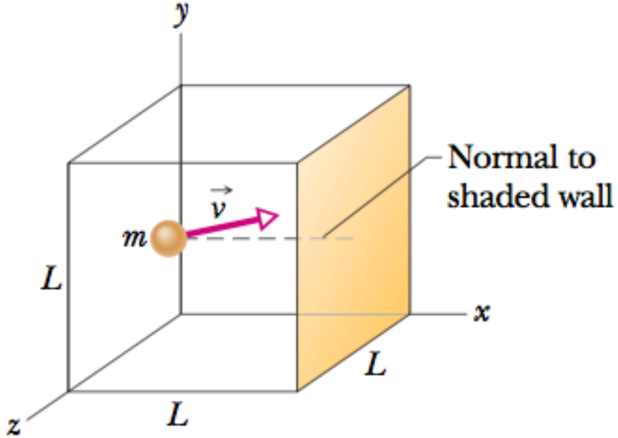


Fig. 19-4 A cubical box of edge length L, containing n moles of an ideal gas. A molecule of mass m and velocity $\vec{v}$ is about to collide with the shaded wall of area L^2. A normal to that wall is shown.

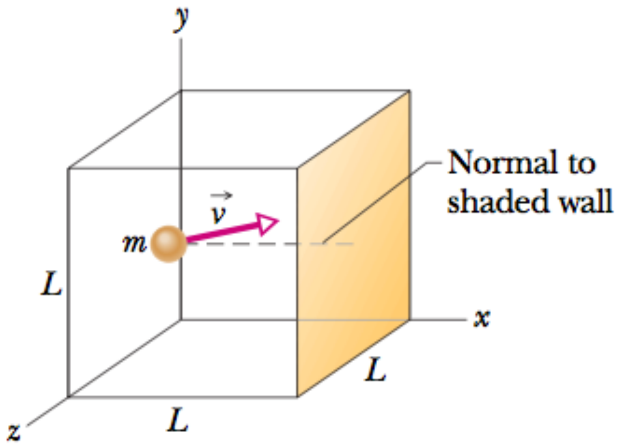


Fig. 19-4 A cubical box of edge length L, containing n moles of an ideal gas. A molecule of mass m and velocity $\vec{v}$ is about to collide with the shaded wall of area L^2. A normal to that wall is shown.

- $M = mN_A$ molar mass

$$p = \frac{nmN_A}{V}(v_x^2)_{\text{avg}}$$

$$v^2 = v_x^2 + v_y^2 + v_z^2 \Rightarrow (v_x^2)_{\text{avg}} = \frac{1}{3}v_{\text{avg}}^2$$

- **Root-mean-square** speed

$$v_{\text{rms}} = \sqrt{v_{\text{avg}}^2}$$

$$p = \frac{nM}{3V}v_{\text{rms}}^2$$

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

Distribution of Molecular Speeds

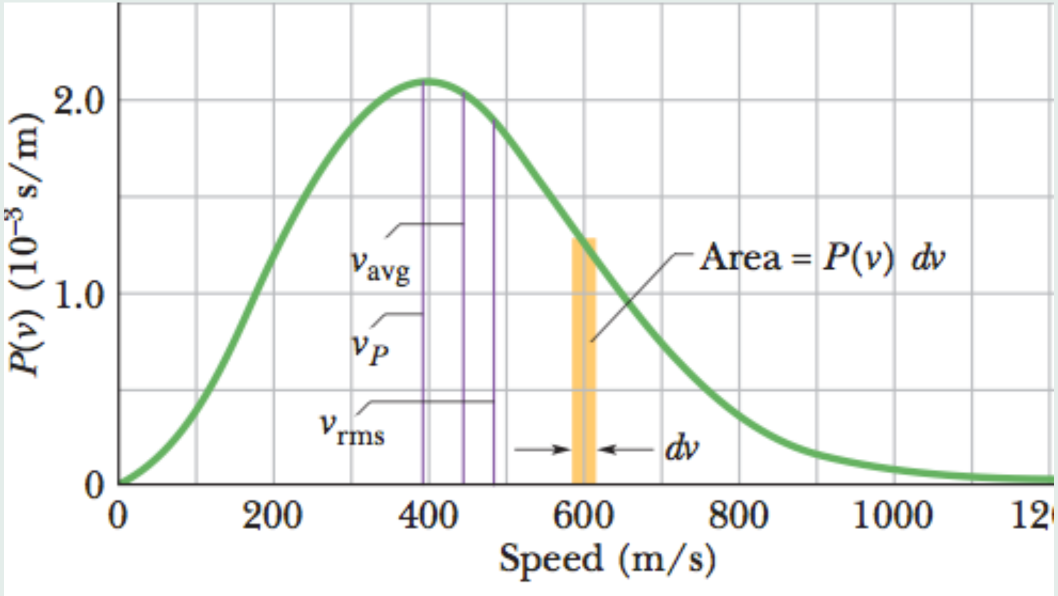
$$P(v) = 4\pi \left(\frac{M}{2\pi RT} \right)^{3/2} v^2 e^{-Mv^2/2RT}$$

Maxwell's distribution law

- M = molar mass; T = temperature; R= gas constant
- $P(v)$ **probability distribution function**

$P(v)dv$ = fraction of molecules with speeds in the interval

$$(v - dv/2\,,\, v + dv/2)$$



$$0 < P(v)dv < 1$$

$$\int_0^{+\infty} P(v)dv = 1$$

$$P(v)dv = \text{fraction of molecules with speeds in the interval } (v - dv/2\,,\, v + dv/2)$$

The fraction of molecules with speeds in the interval $(v_1\,,\, v_2)$ is $\int_{v_1}^{v_2} P(v)dv$.

- **Average speed**

$$v_{\text{avg}} = \int_0^\infty vP(v)dv = \sqrt{\frac{8RT}{\pi M}}$$

- **RMS**

$$v_{\text{avg}}^2 = \int_0^\infty v^2 P(v)dv = \frac{3RT}{M}$$

- **Most probable speed:** $P(v_P)$ maximum

$$\frac{dP}{dv}(v_P) = 0 \;\Rightarrow\; v_P = \sqrt{\frac{2RT}{M}}$$

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An ideal gas is stored in a container with a constant volume. When the temperature of the gas is T, the RMS speed of the gas molecules is v_{rms}. What is the RMS speed when the gas temperature is increased to $3T$?

$A)$ $v_{\text{rms}}/9$

$B)$ $v_{\text{rms}}/\sqrt{3}$

$C)$ $3v_{\text{rms}}$

$D)$ $\sqrt{3}v_{\text{rms}}$

$E)$ $9v_{\text{rms}}$

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$D)$ $\sqrt{3}v_{\text{rms}}$

$E)$ $9v_{\text{rms}}$

Note: ideal gasses are classified as

- **monatomic**: each molecule consists of a single atom
- **diatomic**: each molecule consists of two atoms bound together
- **polyatomic**: each molecule consists of three or more atoms bound together

Kinetic energy of translation

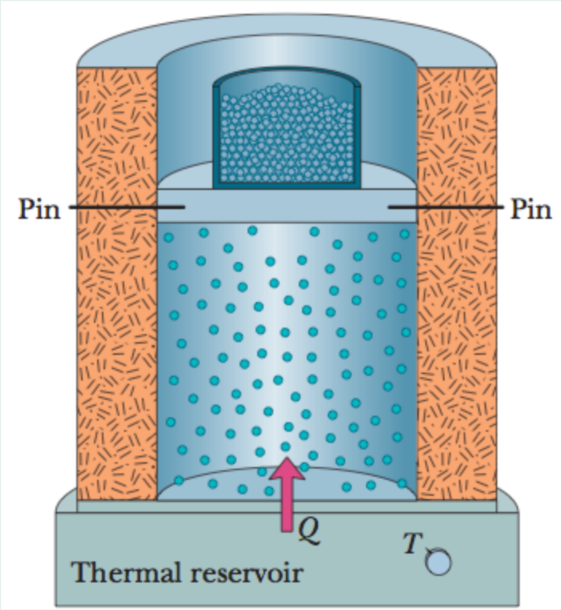
$$K_{\text{avg}} = \frac{1}{2}m(v^2)_{\text{avg}} = \frac{1}{2}mv^2_{\text{rms}}$$

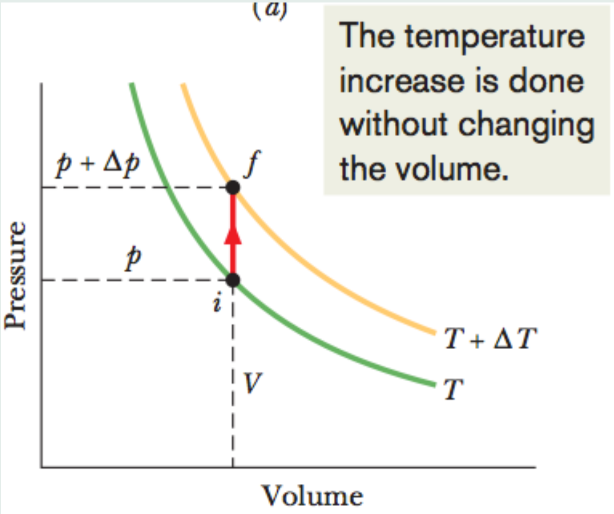
$$\boxed{K_{\text{avg}} = \frac{3kT}{2}} \qquad k = \text{Boltzmann's constant}$$

At a given temperature T, all ideal gas molecules no matter what their mass have the same average translational kinetic energy, namely $3kT/2$.

Internal energy and molar specific heat

- **Constant volume**

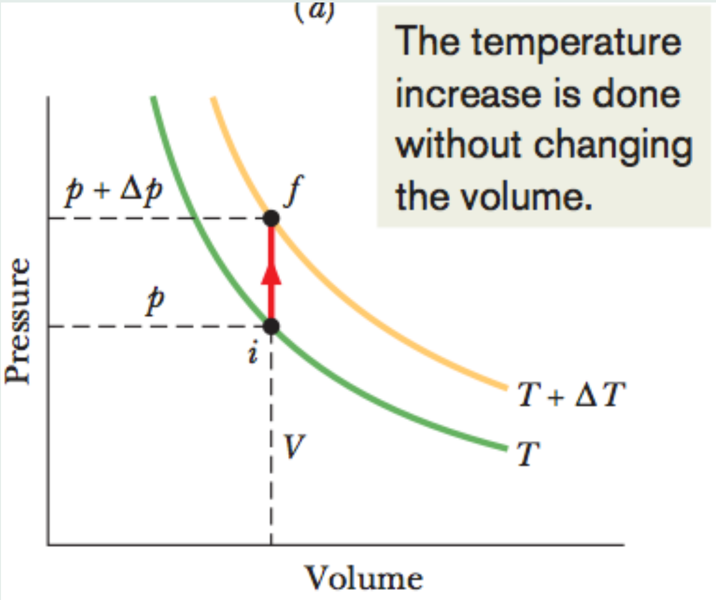




$$T \nearrow, \; V = \text{constant}$$

$$Q = nC_V \Delta T$$

C_V = molar specific heat at constant volume



$$Q = nC_V \Delta T$$

$$V = \text{constant} \quad \Rightarrow \quad W = 0$$

$$\Delta E_{\text{int}} = nC_V \Delta T$$

$T \nearrow$, $V = \text{constant}$

A change in the internal energy E_{int} of a confined ideal gas depends only on the change in the temperature, not on what type of process produces the change.

- **Internal energy for n moles of ideal gas**

$$E_{int} = nC_V T$$

- **Internal energy** for n moles of **monatomic** ideal gas:

$$E_{\text{int}} = (nN_A)K_{\text{avg}} = (nN_A) \times \frac{3}{2}kt$$

$$E_{\text{int}} = \frac{3}{2}nRT \qquad C_V = \frac{3R}{2}$$

Degrees of freedom and Molar specific heats

Molar Specific Heats at Constant Volume

Molecule	Example		C_V (J/mol·K)
Monatomic	Ideal		$\frac{3}{2}R = 12.5$
	Real	He	12.5
		Ar	12.6
Diatomic	Ideal		$\frac{5}{2}R = 20.8$
	Real	N_2	20.7
		O_2	20.8
Polyatomic	Ideal		$3R = 24.9$
	Real	NH_4	29.0
		CO_2	29.7

- **Monatomic** ideal gas

$$C_V = \frac{3}{2}R$$

- **Diatomic** ideal gases:

$$C_V = \frac{5}{2}R$$

- **Polyatomic** ideal gases:

$$C_V = 3R$$

Why?

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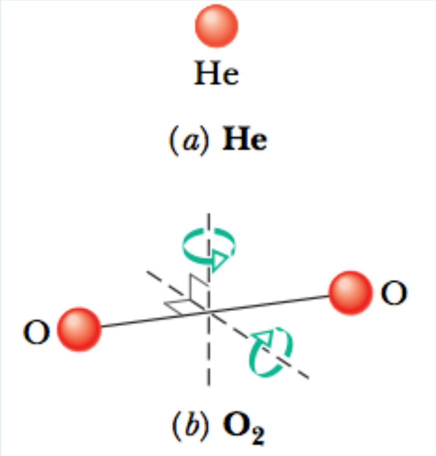
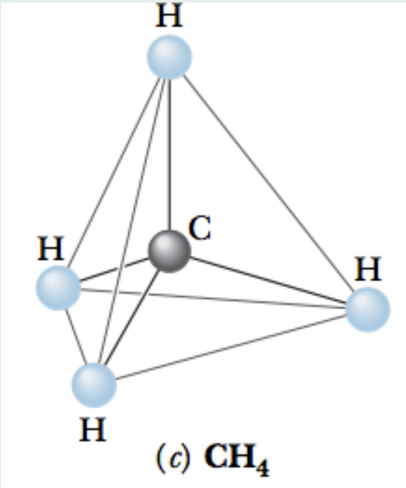




Equipartition of energy

Every kind of molecule has a certain number f of degrees of freedom, which are independent ways in which the molecule can store energy.

Each such degree of freedom has associated with it on average an energy of $\frac{1}{2}kT$ per molecule (or $\frac{1}{2}RT$ per mole).

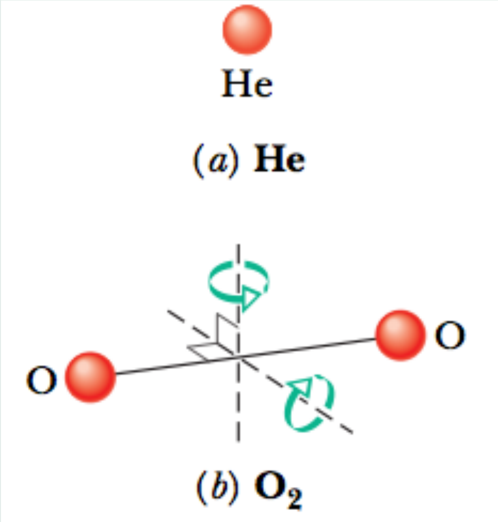
$$C_V = \left(\frac{f}{2}\right) R$$

- **Monatomic**:

translation along x, y, z axes,

no rotation

$$f = 3 \;\Rightarrow\; C_V = \frac{3}{2}R$$

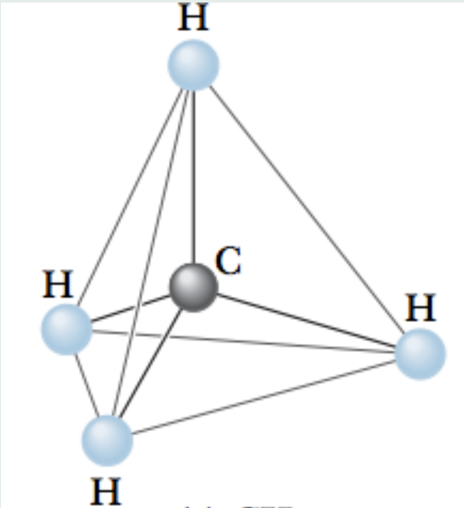


- **Diatomic**:

translation along x, y, z axes,

rotation about the two directions $\perp$ to molecular axis

$$f = 5 \;\Rightarrow\; C_V = \frac{5}{2}R$$



(*c*) CH_4

- **Polyatomic**:

translation along x, y, z axes,

rotation about the three axes

$$f = 6 \;\Rightarrow\; C_V = \frac{6}{2}R = 3R$$