Rutgers University
Department of Physics & Astronomy

01:750:271 Honors Physics I
Fall 2015

Lecture 19

12. Equilibrium and Elasticity

• How do objects behave under applied external forces? Under what conditions can they remain static or stationary?

• Under what conditions do objects deform and what are the effects of their deformations?

• Equilibrium

An object is in **equilibrium** if:

- The linear momentum $\vec{P}$ of its center of mass is constant.

- Its angular momentum about its center of mass, or about any other point, is also constant.

$$\vec{P}, \quad \vec{L} \quad \textbf{constant}$$

An object is in

static equilibrium if

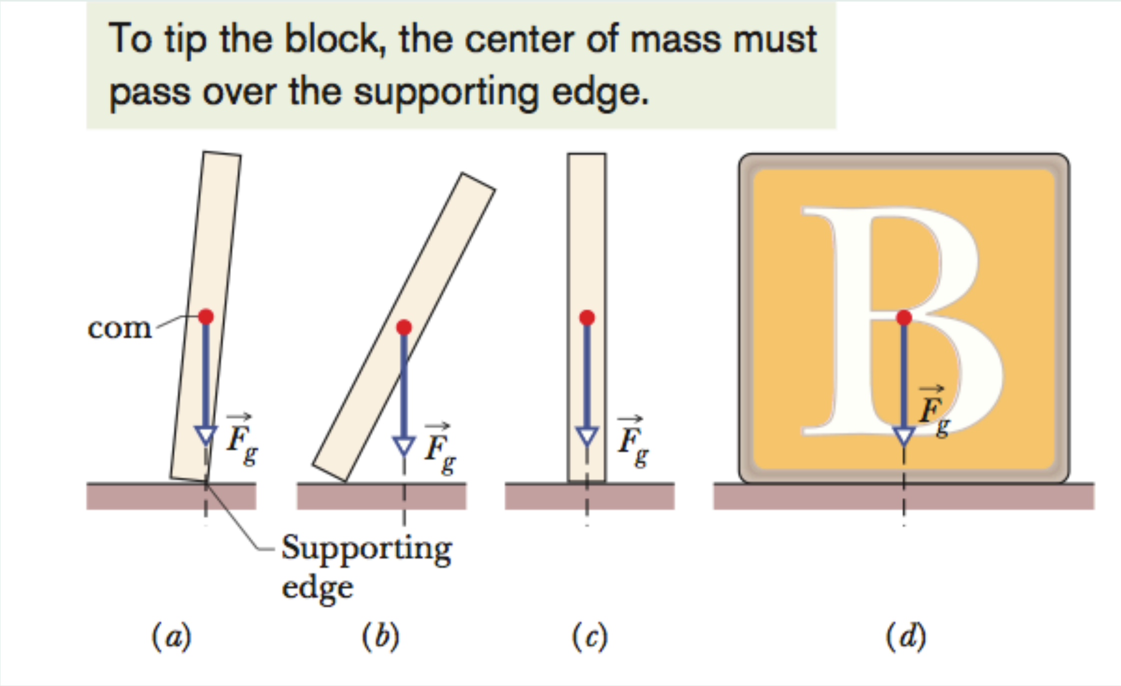
$$\vec{P} = 0, \quad \vec{L} = 0$$

No translation, no rotation.

Static equilibrium is:

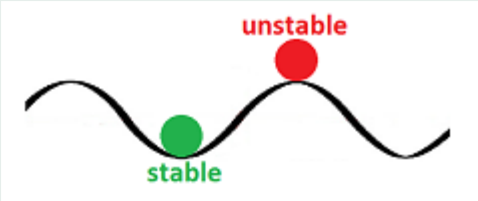
- **Stable** if the body returns to the state of static equilibrium after having been displaced from that state by a **small** force.

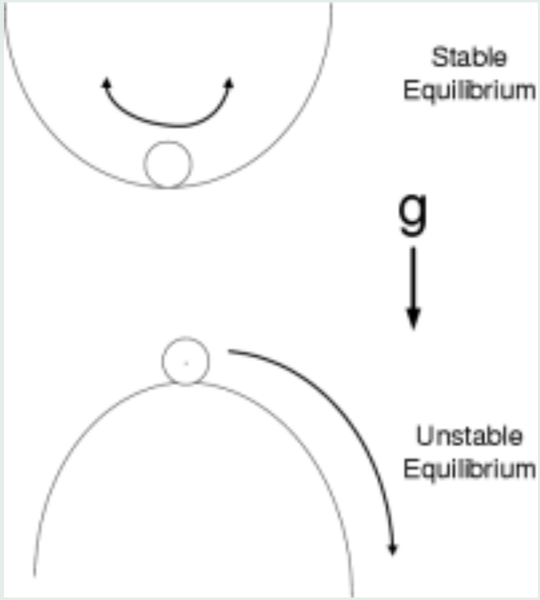
- **Unstable** if any small force can displace the body and end the equilibrium.



(a) **unstable** static equilibrium

$(c), (d)$ **stable** static equilibrium


unstable
stable


Stable
Equilibrium
g
Unstable
Equilibrium

- **Conditions for equilibrium**

$$\frac{d\vec{P}}{dt} = \vec{F}_{\text{net}} \qquad \vec{P}\ \textbf{constant}\ \Rightarrow \vec{F}_{\text{net}} = 0$$

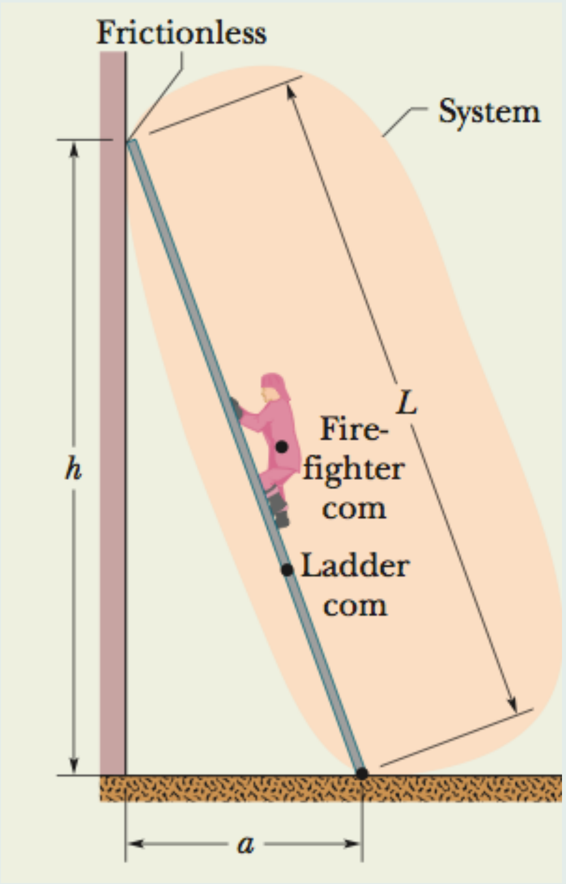
$$\frac{d\vec{L}}{dt} = \vec{\tau}_{\text{net}} \qquad \vec{L}\ \textbf{constant}\ \Rightarrow \vec{\tau}_{\text{net}} = 0$$

1. The vector sum of all the external forces that act on the body must be zero.

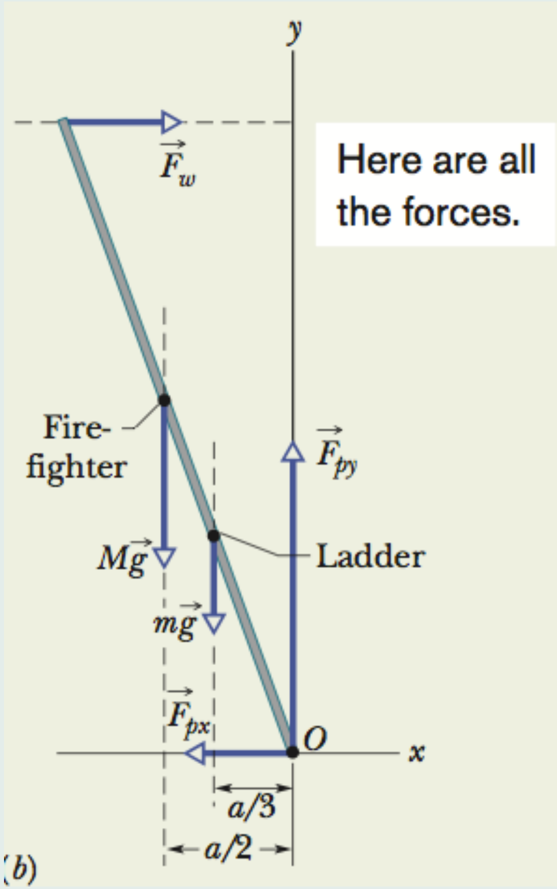
2. The vector sum of all external torques that act on the body, measured about any possible point, must also be zero.

- **Conditions for equilibrium**

$$F_{\text{net},x} = 0 \qquad \tau_{\text{net},x} = 0$$
$$F_{\text{net},y} = 0 \qquad \tau_{\text{net},y} = 0$$
$$F_{\text{net},z} = 0 \qquad \tau_{\text{net},z} = 0$$



- A ladder of length $L = 12\text{m}$ and mass $m = 45\text{kg}$ leans against a frictionless wall. Its upper end is at height $h = 9.3\text{m}$ above the pavement.
- The ladder's center of mass is $L/3$ from the lower end.
- A firefighter of mass $M = 72\text{kg}$ climbs the ladder until her center of mass is $L/2$ from the lower end.
- What then are the magnitudes of the forces on the ladder from the wall and the pavement?



Forces on the ladder:

- Ladder's weight:

$$m\vec{g} = -mg\hat{j}$$

- Firefighter's weight:

$$M\vec{g} = -Mg\hat{j}$$

- Normal to wall:

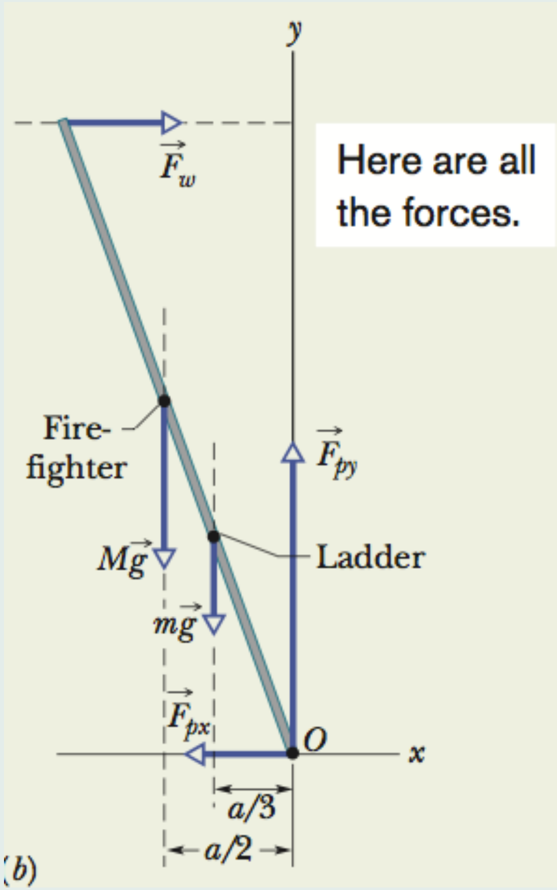
$$\vec{N}_w = N_w\hat{i}$$

- Normal to pavement:

$$\vec{N}_p = N_p\hat{j}$$

- Static friction:

$$\vec{f}_s = -f_s\hat{i}$$



Force balance:

$$x - \text{axis}: \quad N_w - f_s = 0$$

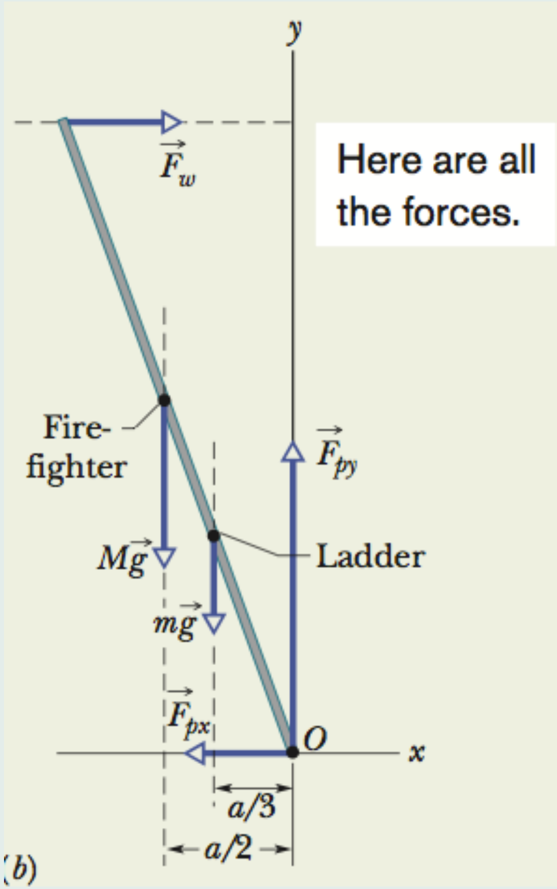
$$y - \text{axis}: \quad N_p - Mg - mg = 0$$

Torque balance about O:

$$Mg(a/2) + mg(a/3) - N_w h = 0$$

a = length of projection of ladder onto pavement

$$a = \sqrt{L^2 - h^2}$$

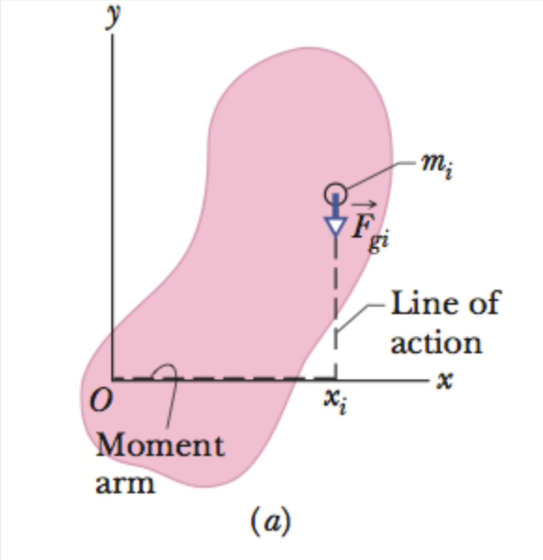


$$N_w = \frac{ag}{h}\left(\frac{M}{2} + \frac{m}{3}\right)$$

$$f_s = N_w$$

$$N_p = (M + m)g$$

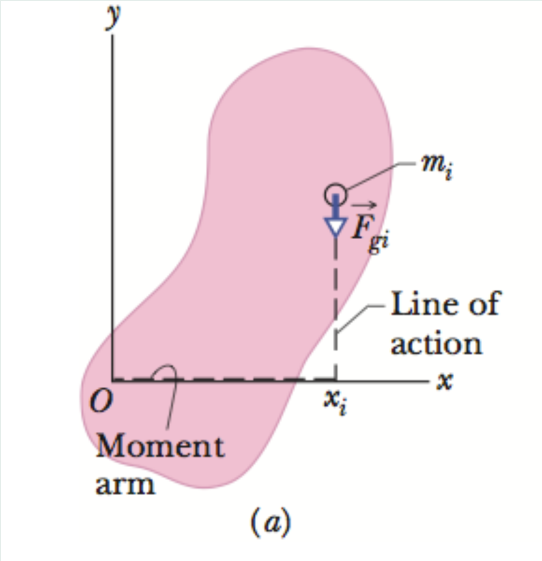
- **Center of gravity**



- Gravitational force acting on a rigid body:

$$\vec{F}_g = \sum_i (\Delta m_i)\vec{g} = M\vec{g}$$

provided that the gravitational field is **uniform** i.e. $\vec{g}$ is the same for **all** mass elements Δm_i

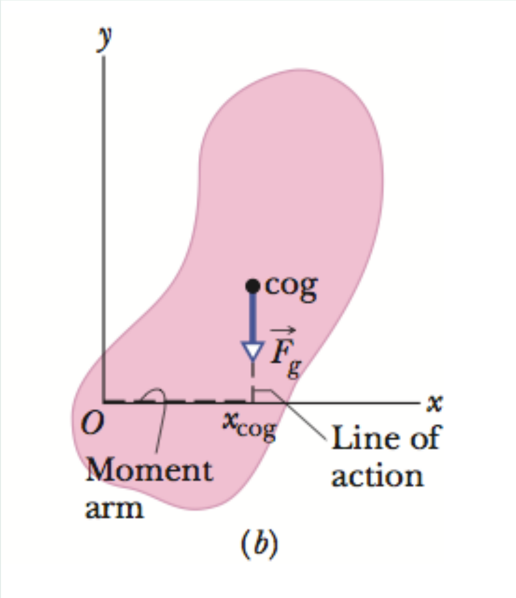


- Torque of gravitational force acting on a rigid body:

$$\vec{\tau}_{F_g} = \sum_i (\Delta m_i)\vec{r}_i \times \vec{g}$$
$$= \vec{r}_{\text{com}} \times (M\vec{g})$$

provided that the gravitational field is **uniform** i.e. $\vec{g}$ is the same for **all** mass elements Δm_i

Conclusions:

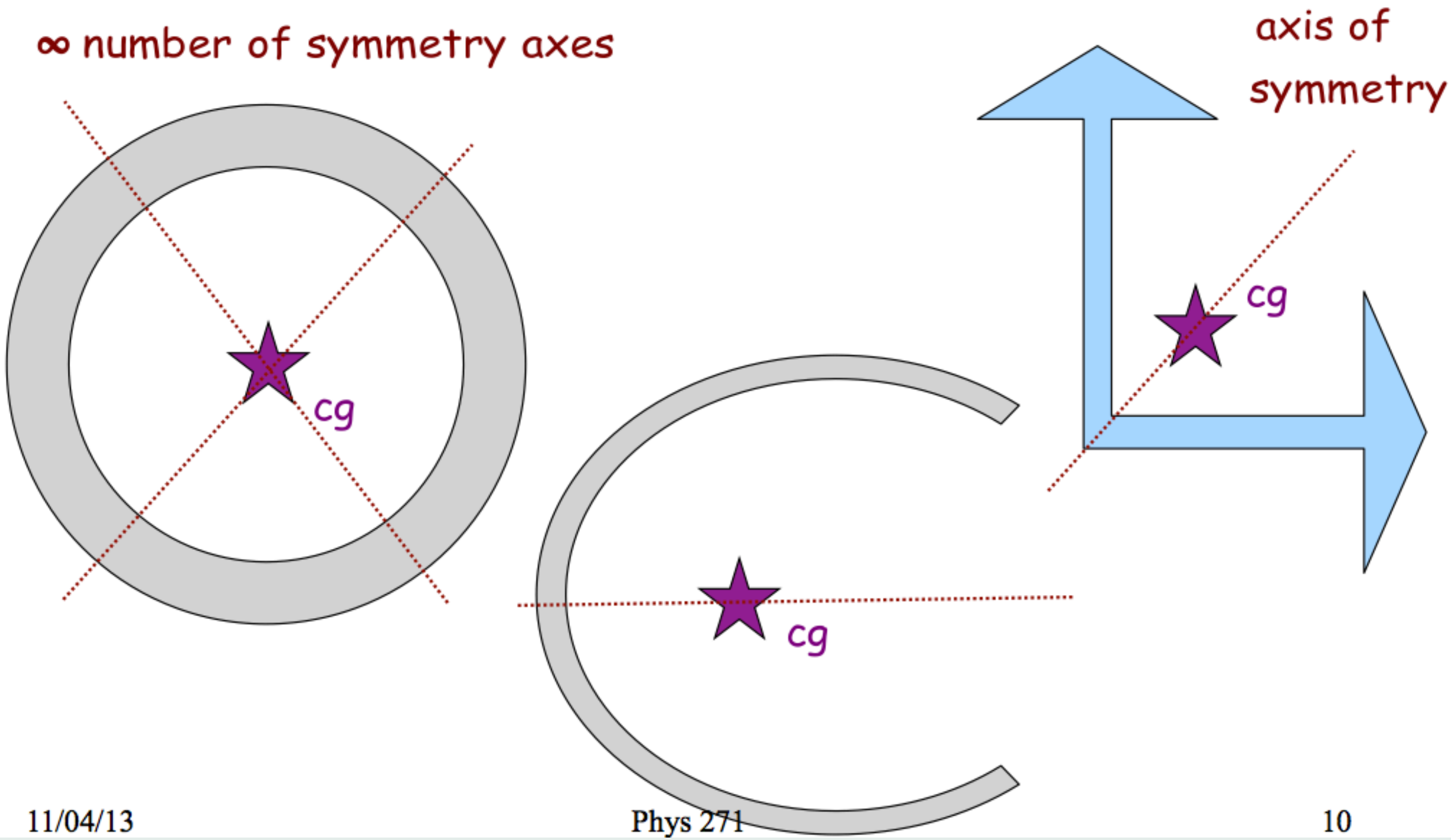


1. The gravitational force $\vec{F}_g$ on a body effectively acts at a single point, called the **center of gravity (cog)** of the body.

2. If $\vec{g}$ is the same for all elements of a body, then the body's **center of gravity (cog)** is coincident with the body's **center of mass (com)**.

3. If $\vec{g}$ is **not** the same for all mass elements

$$\text{COG} \neq \text{COM}$$



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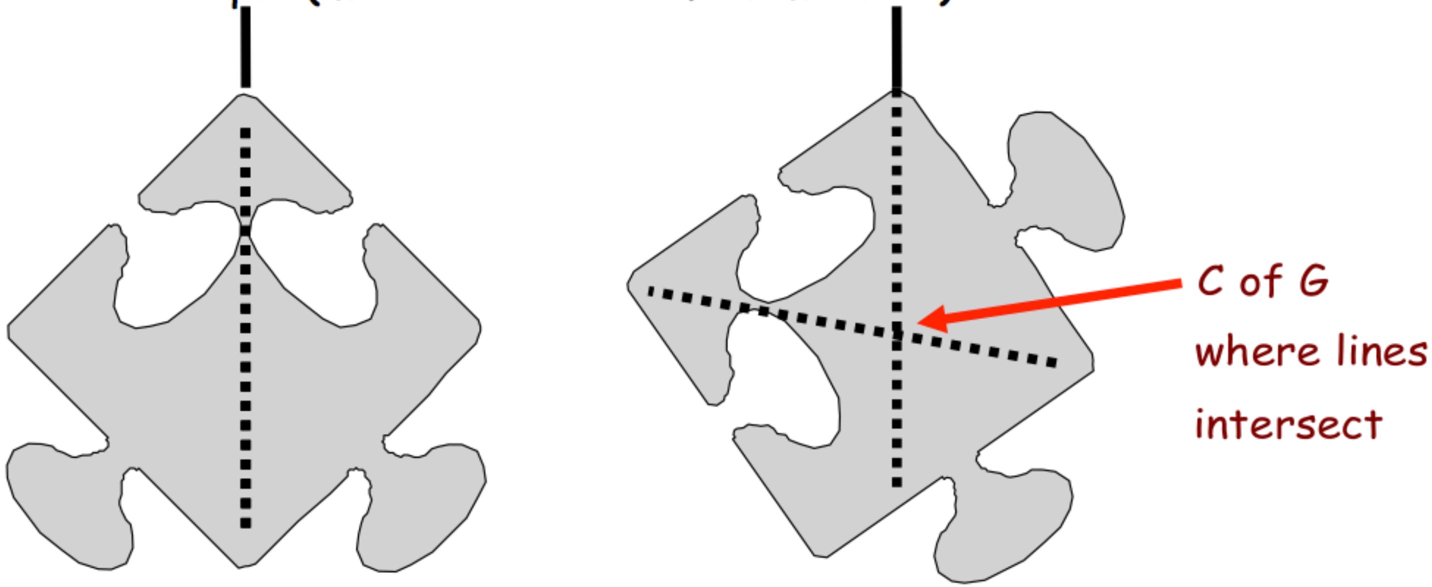
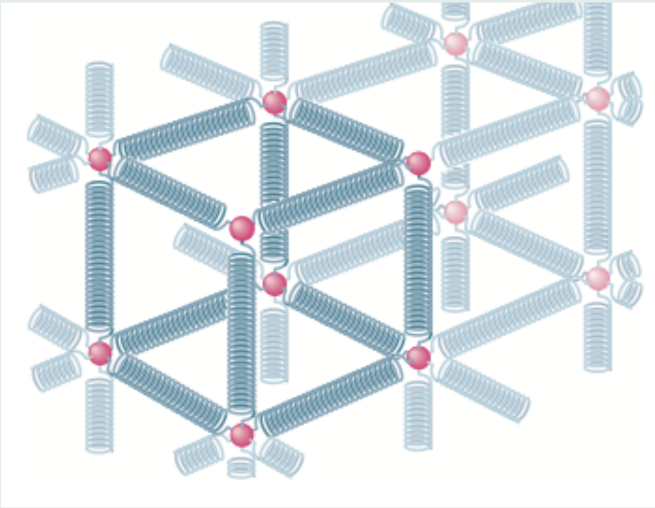
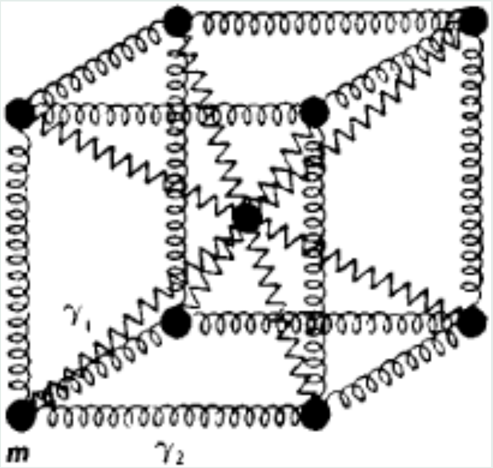
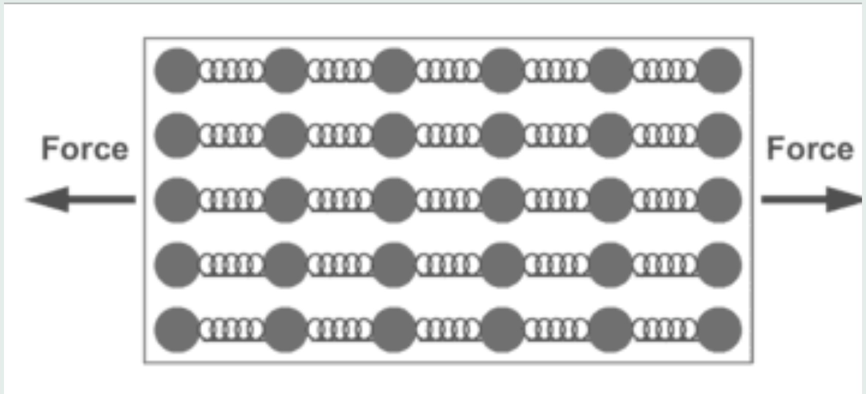
- Hang it, twice, from different points (if 2 dimensional object)
  - C of G must be under each pivot point, for it to be in static equilibrium - other $F_{gravity}$ and $F_{support}$ do not line up, and there is a torque (more on this in a few minutes)



- ‘Rigid’ objects actually deform under applied external forces.
- **Elastic deformations:** the object returns to its original shape when the force is removed.





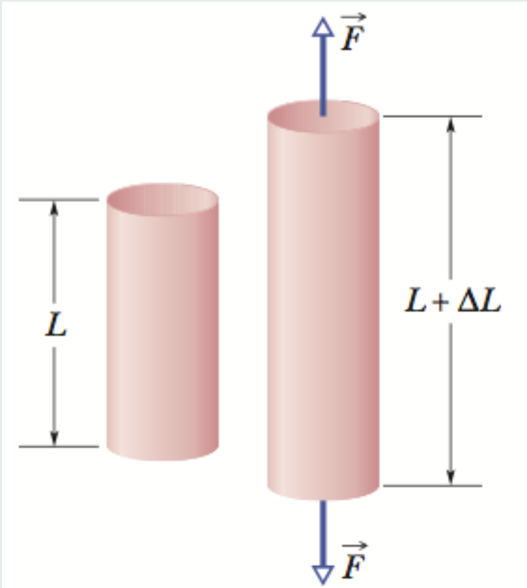


- **Stress:** force per unit area
- **Strain:** deformation per unit length

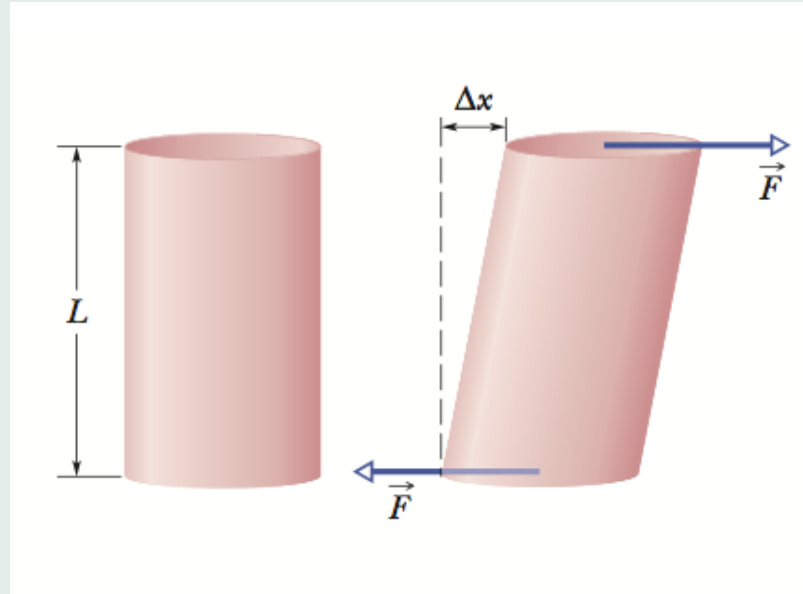
- **Elastic deformations:**

$$\mathbf{stress} = \mathrm{modulus} \times \mathbf{strain}$$

The modulus is an intrinsic property of the material.

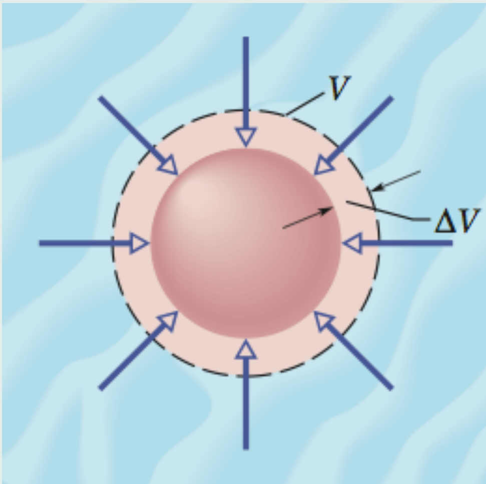


(a)



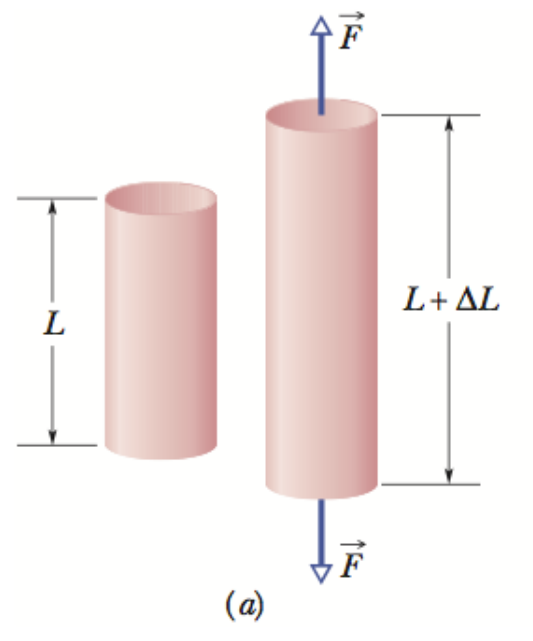
(b)

- **Tensile stress:** the force stretches the cylinder.
- **Shearing stress:** the deformation is perpendicular to main axis.



- **Hydraulic stress:** uniform applied force from all sides

- **Tension and compression**



- Suppose the applied force is $\perp$ to the face of the object. It can stretch or compress the object.
- **Strain:**

$$\frac{\Delta L}{L}$$

- **Young's modulus:**

$$\frac{F}{A} = E\frac{\Delta L}{L}$$

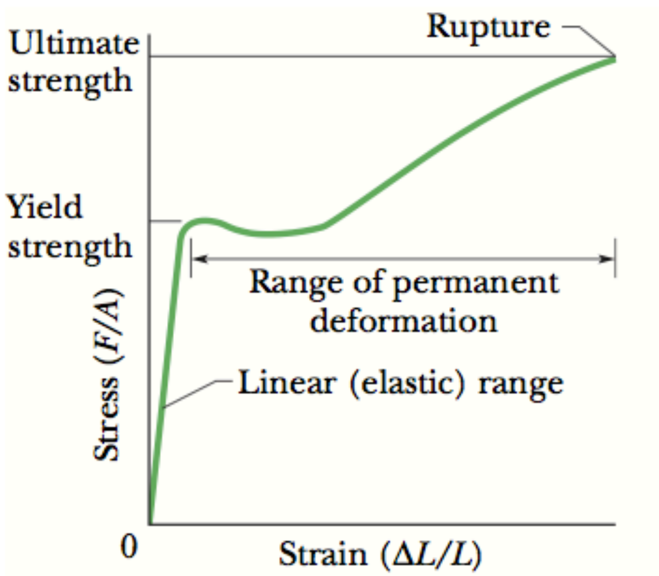
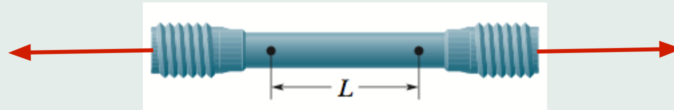
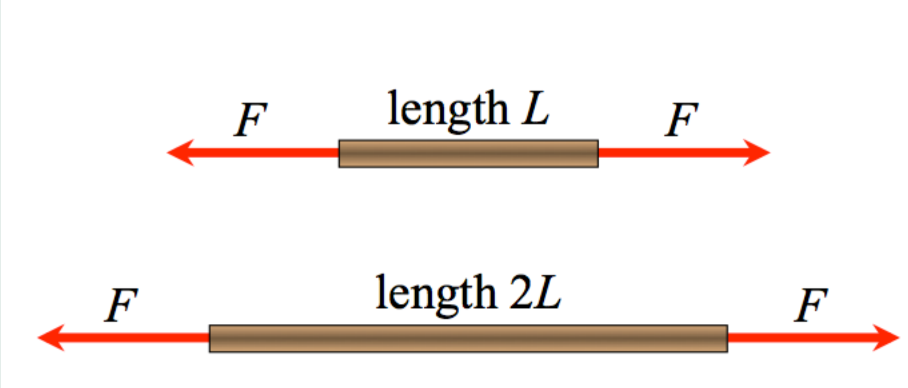


Fig. 12-12 A stress–strain curve for a steel test specimen such as that of Fig. 12-11. The specimen deforms permanently when the stress is equal to the *yield strength* of the specimen's material. It ruptures when the stress is equal to the *ultimate strength* of the material.



- **Small stress:** elastic deformations.
- **Yield strength:** magnitude of stress causing **permanent** deformations.
- **Ultimate strength:** magnitude of stress tearing the object apart.



i-Clicker

- same steel, same diameter, same applied force.

Compared to the first rod the second rod has:

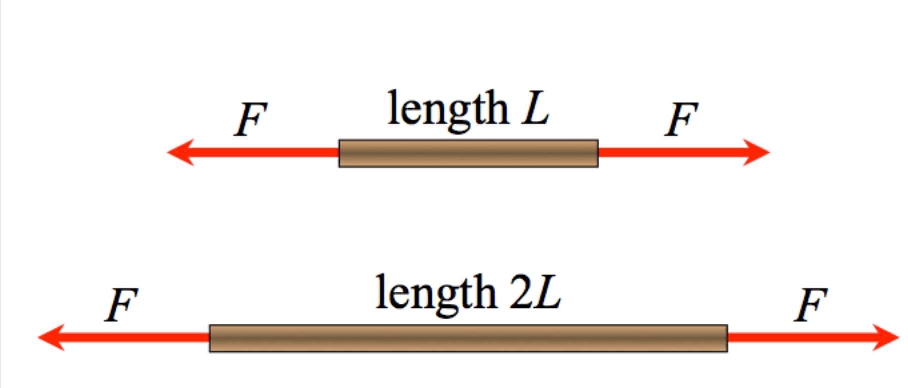
$A)$ more stress and more strain.

$B)$ the same stress and more strain.

$C)$ the same stress and less strain.

$D)$ less stress and less strain.

$E)$ the same stress and the same strain.



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A) more stress and more strain.

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C) the same stress and less strain.

D) less stress and less strain.

E) the same stress and the same strain.

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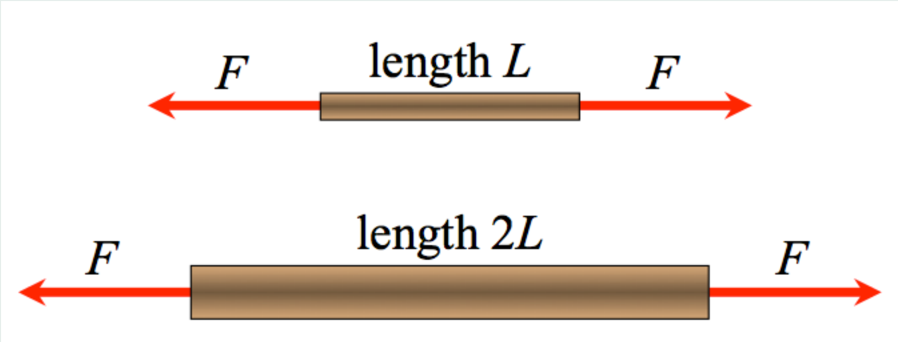


- same steel, same applied force.
- longer rod has greater diameter

Compared to the first rod the second rod has:

A) more stress and more strain.

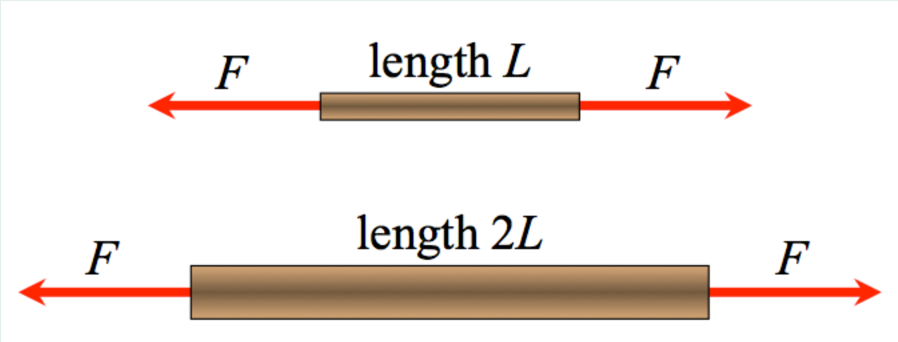
B) the same stress and more strain.

C) the same stress and less strain.

D) less stress and less strain.

E) the same stress and the same strain.

i-Clicker



- same steel, same applied force.
- longer rod has greater diameter

Compared to the first rod the second rod has:

A) more stress and more strain.

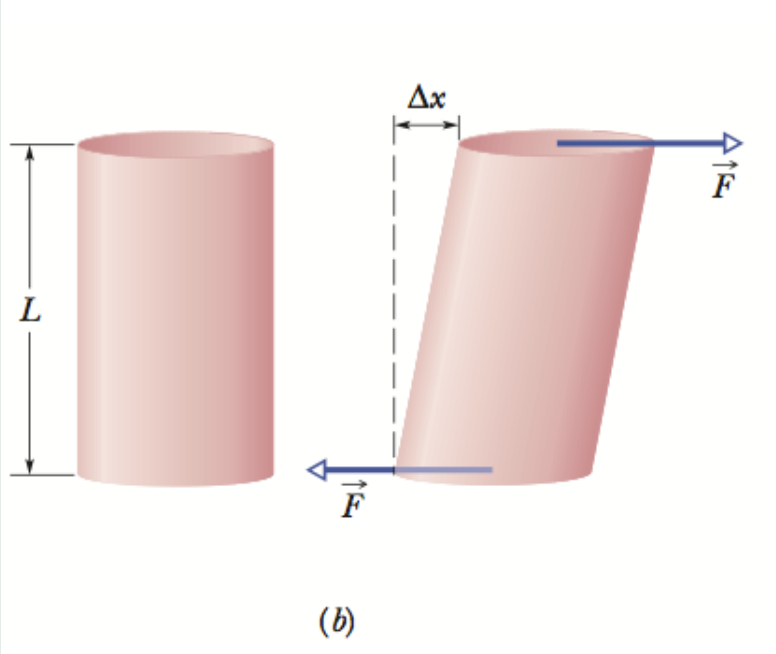
B) the same stress and more strain.

C) the same stress and less strain.

D) less stress and less strain.

E) the same stress and the same strain.

- **Shearing**



- Applied force vector lies in the same plane as the face of the object.

- **Stress:** force per unit area

$$\frac{F}{A}$$

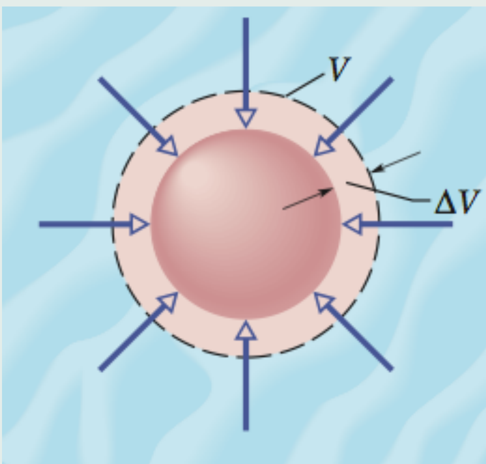
- **Strain:**

$$\frac{\Delta x}{L}$$

- **Shear modulus:**

$$\frac{F}{A} = G\frac{\Delta x}{L}$$

- **Hydraulic Stress**



- Force applied uniformly from all sides.
- **Stress = pressure =** force per unit area p
- **Strain:** change in volume per unit volume

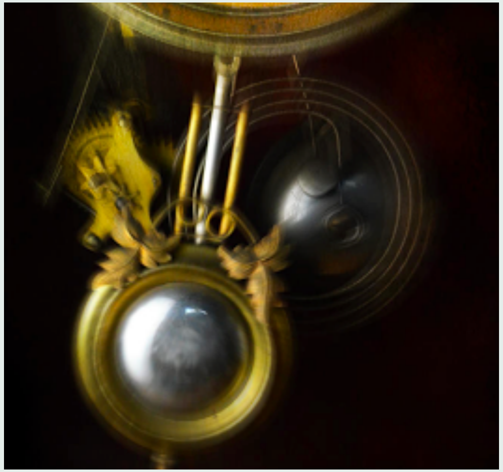
$$\frac{\Delta V}{V}$$

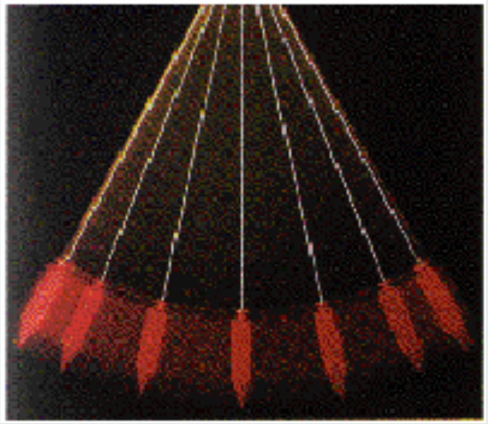
- **Bulk modulus:**

$$p = B\frac{\Delta V}{V}$$

15. Oscillations

- **Periodic or harmonic motion:** periodic in time that is motion that repeats itself in time.





- **Frequency:**

$f =$ number of oscillations per unit time.

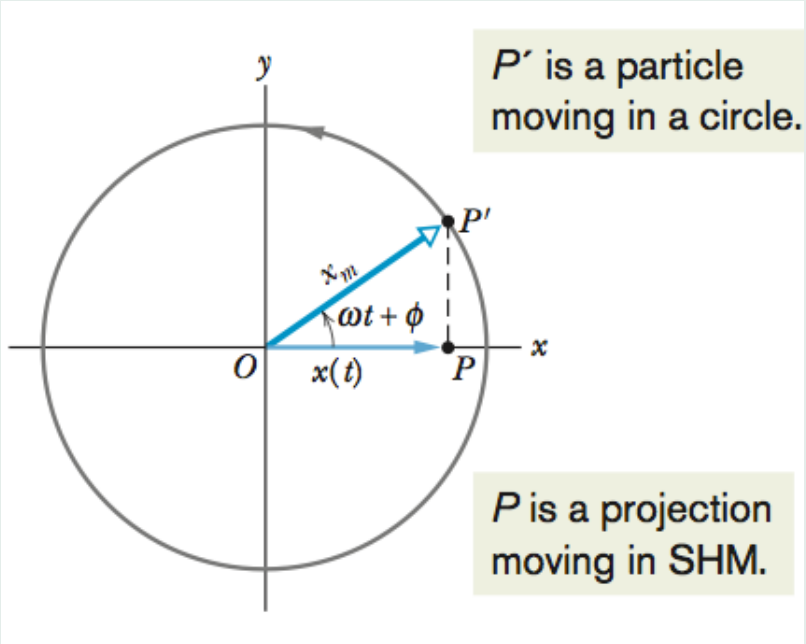
$$\text{Units}: 1 \text{ hertz} = 1\,\text{Hz} = 1 \text{ oscillation per second} = 1\,\text{s}^{-1}$$

- **Period:**

$T =$ time needed to complete one oscillation

$$T = \frac{1}{f}.$$

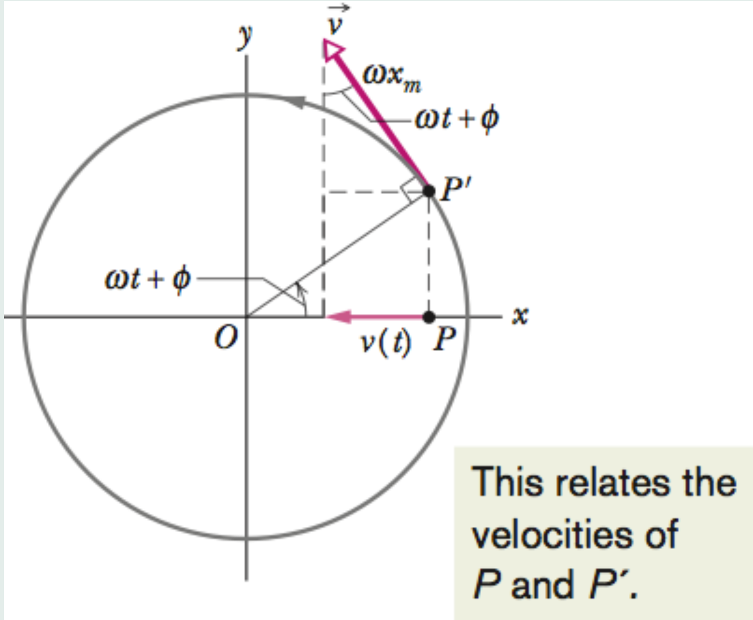
- **Projection of uniform circular motion**

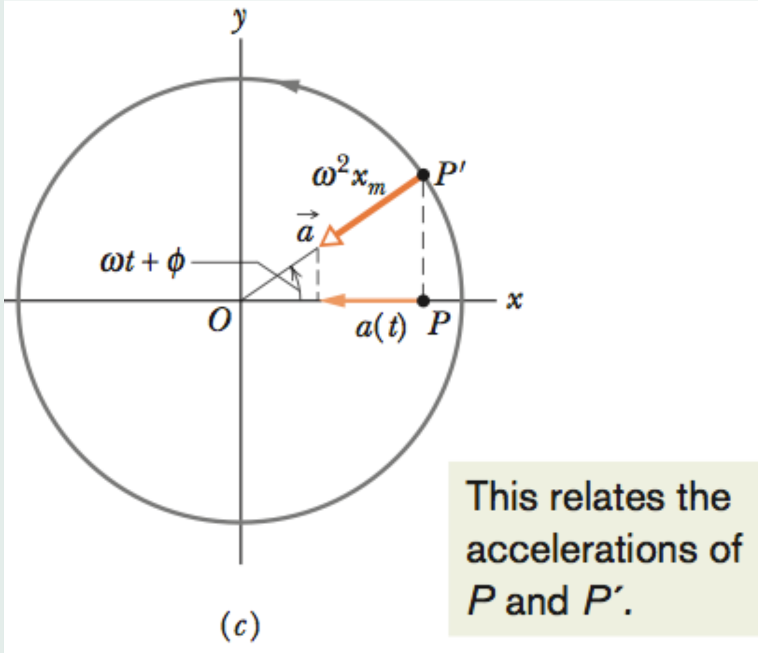


- Consider a particle P' on a circular trajectory of radius x_m with constant angular speed ω.

- The **projection** P of the particle on the x-axis moves according to the law:

$$x(t) = x_m \cos(\omega t + \phi)$$





$$v_{Px} = v_{P'x}$$

$$v_{Px} = -\omega x_m \sin(\omega t + \phi).$$

$$a_{Px} = a_{P'x}$$

$$a_{Px} = -\omega^2 x_m \cos(\omega t + \phi).$$

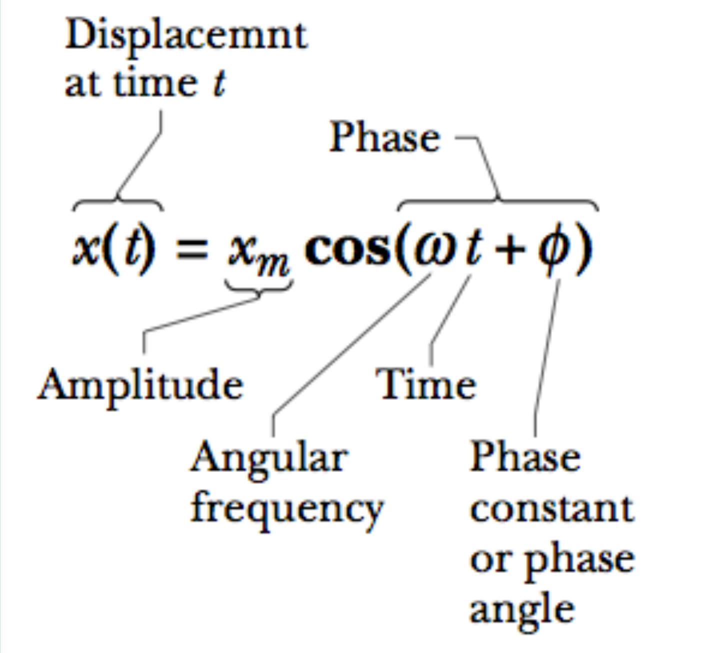
- **Simple harmonic motion (SHM)**

SHM is the projection of uniform circular motion on a diameter of the circular trajectory.

$$\Downarrow$$

One dimensional motion of a point particle given by

$$x(t) = x_m \cos(\omega t + \phi).$$



- x_m **amplitude** = maximum displacement
- t time
- ω **angular frequency**
- ϕ **phase constant** or **phase angle**