Rutgers University
Department of Physics & Astronomy

01:750:271 Honors Physics I
Fall 2015

Lecture 13

9. Center of Mass. Linear Momentum II

- **Linear momentum for a system of particles**

$$\vec{P} = \sum_i \vec{p}_i = \sum_i m_i \vec{v}_i$$

$$\vec{P} = M \vec{v}_{\text{com}}, \qquad M = \sum_i m_i$$

$$\frac{d\vec{P}}{dt} = \sum_i m_i \frac{d\vec{v}_i}{dt} = \sum_i m_i \vec{a}_i = \sum_i \vec{F}_i$$

$$\boxed{\frac{d\vec{P}}{dt} = \vec{F}_{\text{ext}}}$$

• Collision and Impulse

The collision of a ball with a bat collapses part of the ball. *(Photo by Harold E. Edgerton. ©The Harold and Esther Edgerton Family Trust, courtesy of Palm Press, Inc.)*

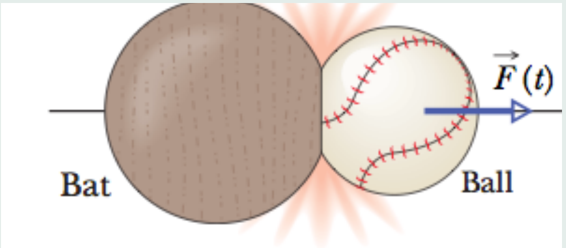

Fig. 9-8 Force $\vec{F}(t)$ acts on a ba as the ball and a bat collide.

The figure depicts the collision at one instant. The ball experiences a force $F(t)$ that varies during the collision and changes the linear momentum of the ball.

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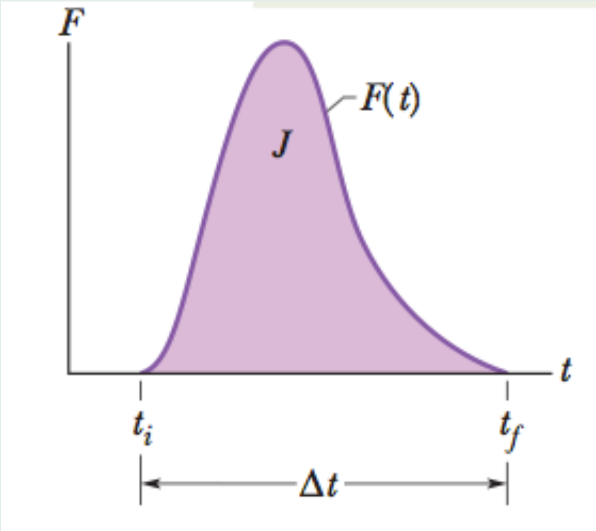
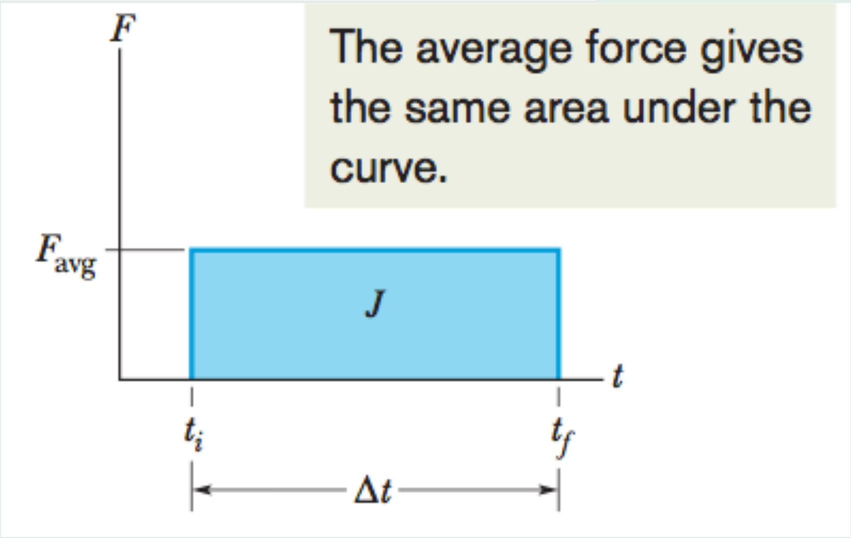
The change in linear momentum of the ball is related to the force by Newtons second law:

$$\vec{F} = \frac{d\vec{p}}{dt} \Rightarrow \Delta\vec{p} = \int_{t_i}^{t_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F} dt$$

- **Impulse:**

$$\boxed{\vec{J} = \int_{t_i}^{t_f} \vec{F} dt}$$





- The magnitude of $\vec{J}$ equals the area under the curve $F(t)$.
- Average force

$$\vec{F}_{\text{average}} = \frac{\vec{J}}{\Delta t}$$

- Newton's 3rd law:

$$\vec{F}_{\text{ball}}(t) + \vec{F}_{\text{bat}}(t) = 0$$

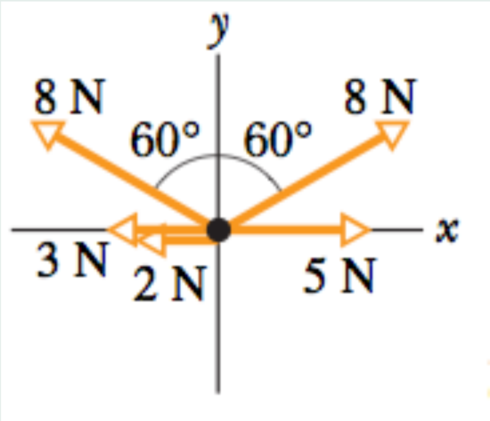
at all times. Hence:

$$\vec{J}_{\text{bat}} = -\vec{J}_{\text{ball}}$$

$$|\vec{J}_{\text{bat}}| = |\vec{J}_{\text{ball}}|$$

i-Clicker

A particle constrained to move on a horizontal smooth surface is subject to five forces as shown below. Which components of its momentum are conserved?



$A)$ p_x, p_y are conserved

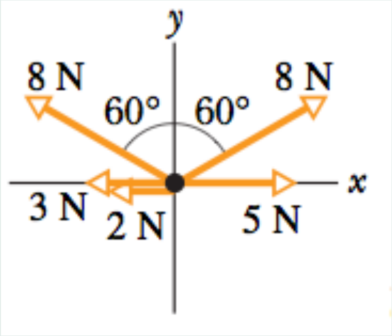
$B)$ p_x conserved, p_y not conserved

$C)$ p_x not conserved, p_y conserved

$D)$ p_x, p_y not conserved.

Answer

A particle constrained to move on a horizontal smooth surface is subject to five forces as shown below. Which components of its momentum are conserved?



$$F_x = dp_x/dt = 0$$
$$F_y = dp_y/dt = 8\text{N}$$

$A)$ p_x, p_y are conserved

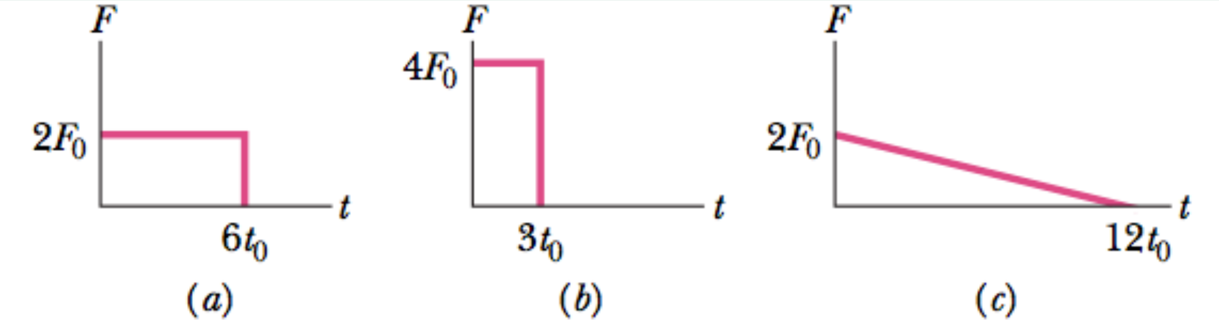
$B)$ p_x conserved, p_y not conserved

$C)$ p_x not conserved, p_y conserved

$D)$ p_x, p_y not conserved.

i-Clicker

The graphs below encode the time dependence of the force magnitude for a body involved in a collision. How are the impulse magnitudes ordered?

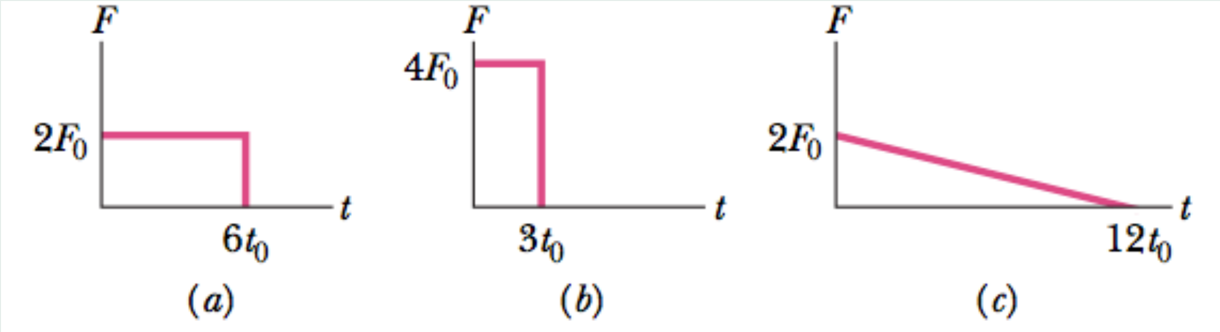


A) $J_a > J_b > J_c$ B) $J_a < J_b < J_c$

C) $J_a = J_b > J_c$ D) $J_a = J_b = J_c$

i-Clicker

The graphs below encode the time dependence of the force magnitude for a body involved in a collision. How are the impulse magnitudes ordered?



$A)\ J_a > J_b > J_c$ $\quad$ $B)\ J_a < J_b < J_c$

$C)\ J_a > J_c < J_b$ $\quad$ $D)\ J_a = J_b = J_c$

- **Conservation of linear momentum**

A system is **closed** if no particles leave or enter the system.

A system is **isolated** if no external forces act on the system.

Isolated closed system:

$$\frac{d\vec{P}}{dt} = \vec{F}_{\text{ext}} = 0 \;\Rightarrow\; \vec{P} \text{ conserved}$$

If no net external force acts on a closed system of particles, the total linear momentum P of the system cannot change.

Closed system:

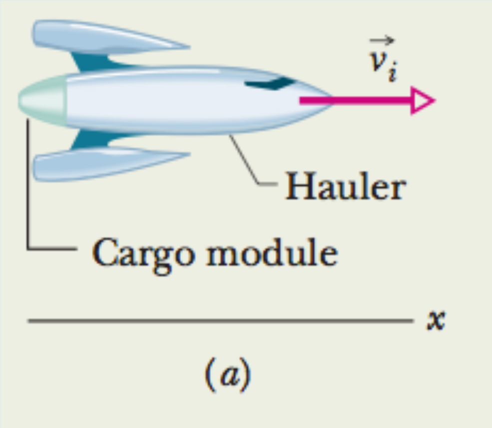
$$(F_{\text{ext}})_x = 0 \;\Rightarrow\; \frac{dP_x}{dt} = 0 \Rightarrow P_x \text{ conserved}$$

$$(F_{\text{ext}})_y = 0 \;\Rightarrow\; \frac{dP_y}{dt} = 0 \Rightarrow P_y \text{ conserved}$$

$$(F_{\text{ext}})_z = 0 \;\Rightarrow\; \frac{dP_z}{dt} = 0 \Rightarrow P_z \text{ conserved}$$

If the component of the net external force on a closed system is zero along an axis, then the component of the linear momentum of the system along that axis cannot change.

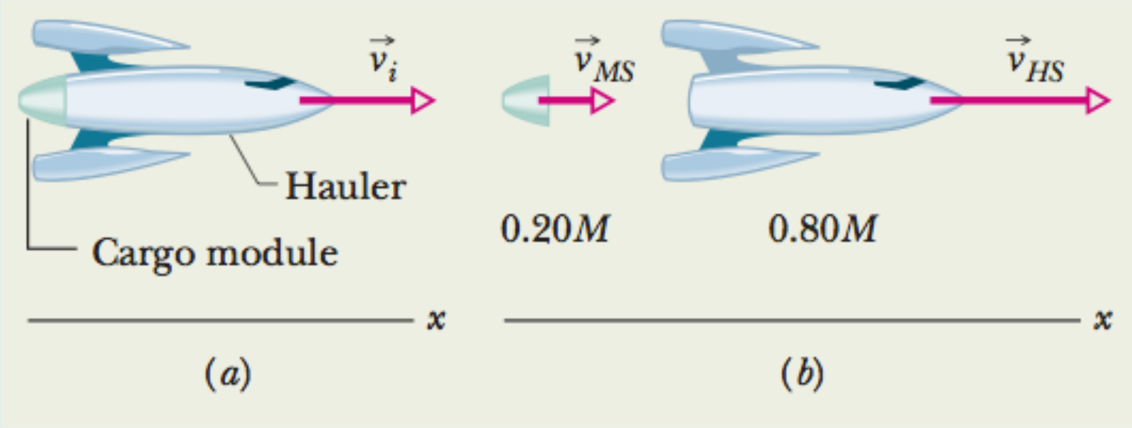
- Example: one dimensional explosion



A space hauler coupled to a space module moves with velocity $\vec{v}_i = v_i \hat{i}$ relative to the Sun. The total mass of the system is M and the mass of the module is $m < M$.

The module is ejected by a small explosion such that the relative velocity of the hauler with respect to the module is $\vec{v}_{\text{rel}} = v_{\text{rel}} \hat{i}$.

Find the velocity of the hauler $\vec{v}_{HS}$ relative to the Sun after the explosion.

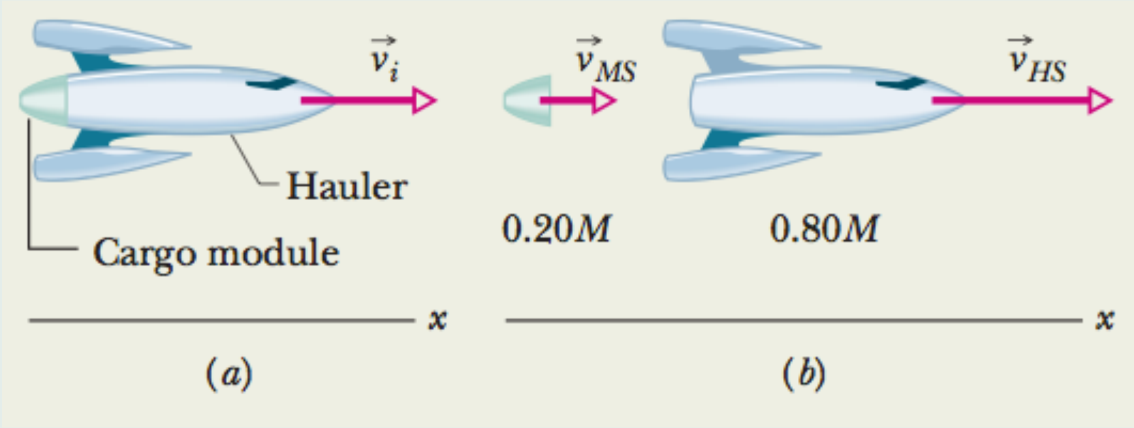


Isolated closed system: $\vec{P}$ **conserved.**

$$\vec{P}_i = \vec{P}_f \qquad P_{ix} = P_{fx}$$

$$P_{ix} = Mv_{ix} \qquad P_{fx} = (M-m)v_{HS,x} + mv_{MS,x}$$

$$\vec{v}_{HS} = \vec{v}_{MS} + \vec{v}_{rel} \qquad v_{HS,x} = v_{MS,x} + v_{rel,x}$$

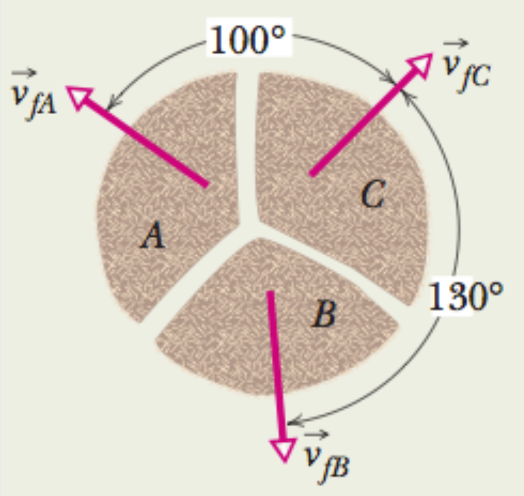


$$(M-m)v_{HS,x} + m v_{MS,x} = M v_{ix} \qquad v_{MS,x} = v_{HS,x} - v_{rel,x}$$

$$(M-m)v_{HS,x} + m(v_{HS,x} - v_{rel,x}) = M v_{ix}$$

$$v_{HS} = v_{ix} + \frac{m}{M} v_{rel,x}$$

- **Example:** two dimensional explosion



A firecracker placed inside a coconut of mass M, initially at rest on a frictionless floor, blows the coconut into three pieces that slide across the floor.

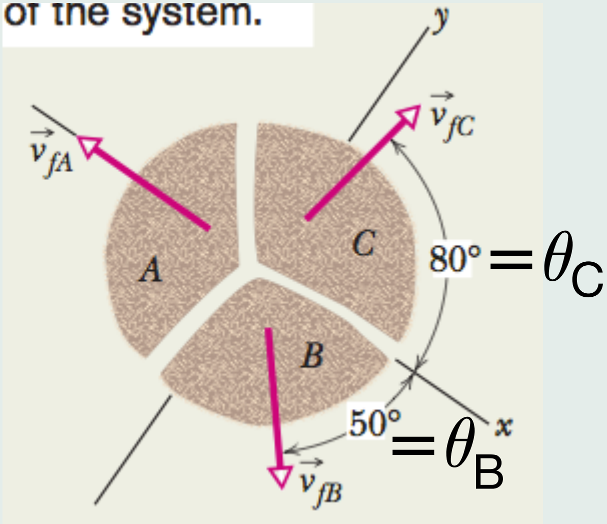
Piece C, with mass $M_C = 0.3M$, has final speed $v_{fC} = 5.0$ m/s.

(a) What is the speed of piece B, which has mass $M_B = .20M$?

(b) What is the speed of A ?

Isolated closed system:

$$\vec{P}_i = \vec{P}_f$$



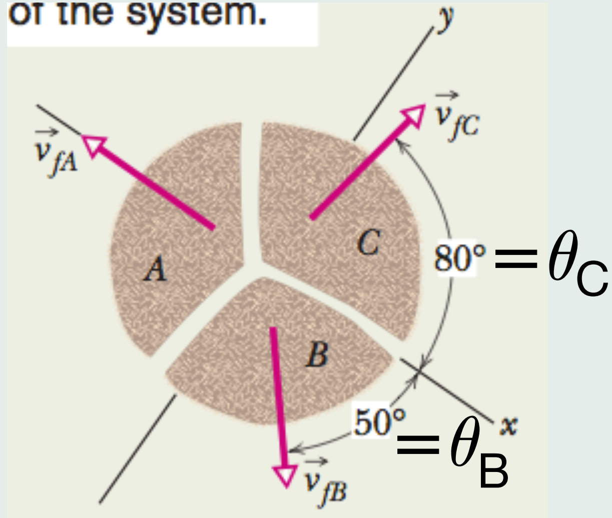
$$0 = M_A\vec{v}_{fA} + M_B\vec{v}_{fB} + M_C\vec{v}_{fC}$$

$$0 = M_A v_{fA,x} + M_B v_{fB,x} + M_C v_{fC,x}$$

$$0 = M_A v_{fA,y} + M_B v_{fB,y} + M_C v_{fC,y}$$

$$\mathrm{x-axis:}\quad M_A v_{fA} = M_B v_{fB} \cos\theta_B + M_C v_{fC} \cos\theta_C$$

$$\mathrm{y-axis:}\quad 0 = M_B v_{fB} \sin\theta_B - M_C v_{fC} \sin\theta_C$$



$$M_A v_{fA} = M_B v_{fB} \cos\theta_B + M_C v_{fC} \cos\theta_C$$

$$M_B v_{fB} \sin\theta_B = M_C v_{fC} \sin\theta_C$$

$$v_{fB} = \frac{M_C v_{fC} \sin\theta_C}{M_B \sin\theta_B} \qquad v_{fA} = \frac{M_B}{M_A} v_{fB} \cos\theta_B + \frac{M_C}{M_A} v_{fC} \cos\theta_C$$

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A golf ball of mass m moving with speed v_0 hits a bowling ball initially at rest. The golf ball bounces back with speed $0.9v_0$. Let p_B denote the magnitude of the momentum of the bowling ball after collision. Which of the following statements is true?

$A)\ p_B = mv_0$ $\qquad B)\ p_B = 0.9mv_0$

$C)\ p_B = 0.1mv_0$ $\qquad D)\ p_B = 1.9mv_0$

Answer

A golf ball of mass m moving with speed v_0 hits a bowling ball initially at rest. The golf ball bounces back with speed $0.9v_0$. Let p_B denote the magnitude of the momentum of the bowling ball after collision. Which of the following statements is true?

$A)\ p_B = mv_0$ $\qquad B)\ p_B = 0.9mv_0$

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$$\vec{P}_i = \vec{P}_f \;\Rightarrow\; mv_0 = -0.9mv_0 + p_{B,x}$$

$$p_{B,x} = 1.9mv_0$$

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- **Momentum and kinetic energy in collisions**

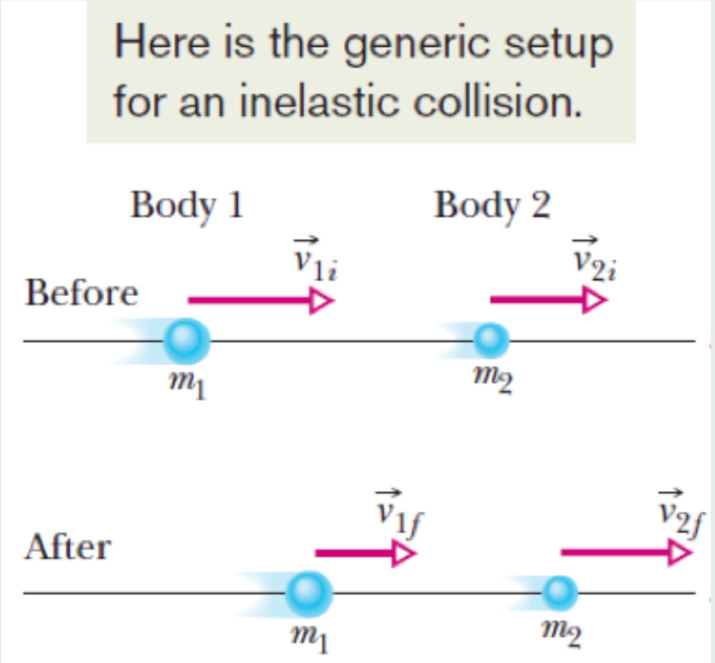
Collisions in closed isolated system

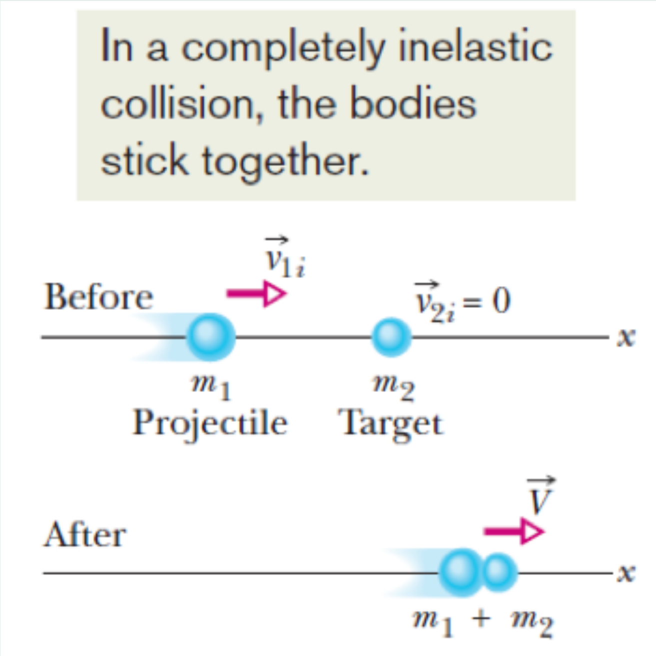
- kinetic energy conserved $\Rightarrow$ **elastic collisions**

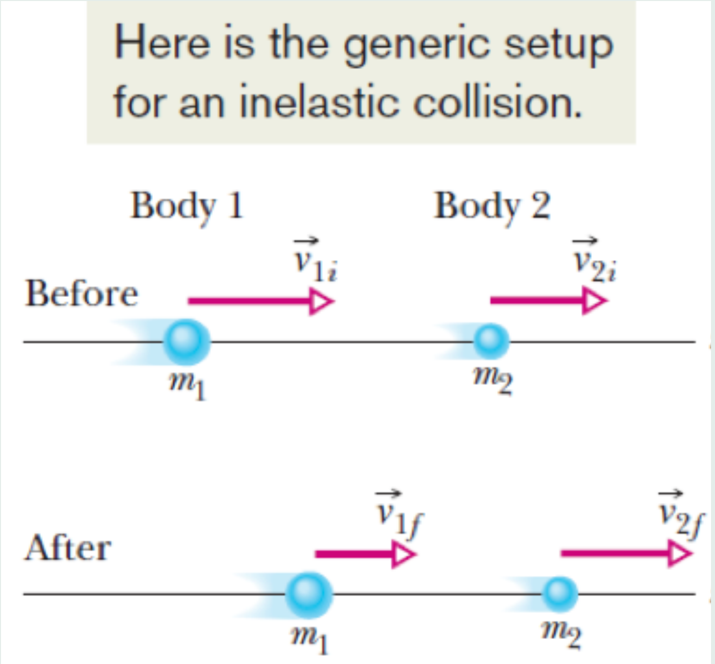
- kinetic energy not conserved, transferred to other forms of energy such as thermal energy $\Rightarrow$ **inelastic collisions**

- **Completely inelastic collisions in 1D**

Completely inelastic: the objects stick together after collision.

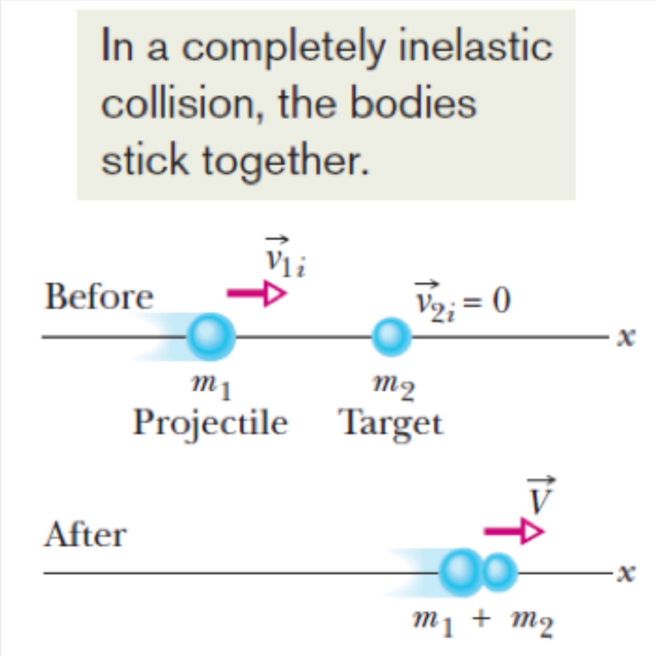






$$\vec{P}_i = \vec{P}_f$$

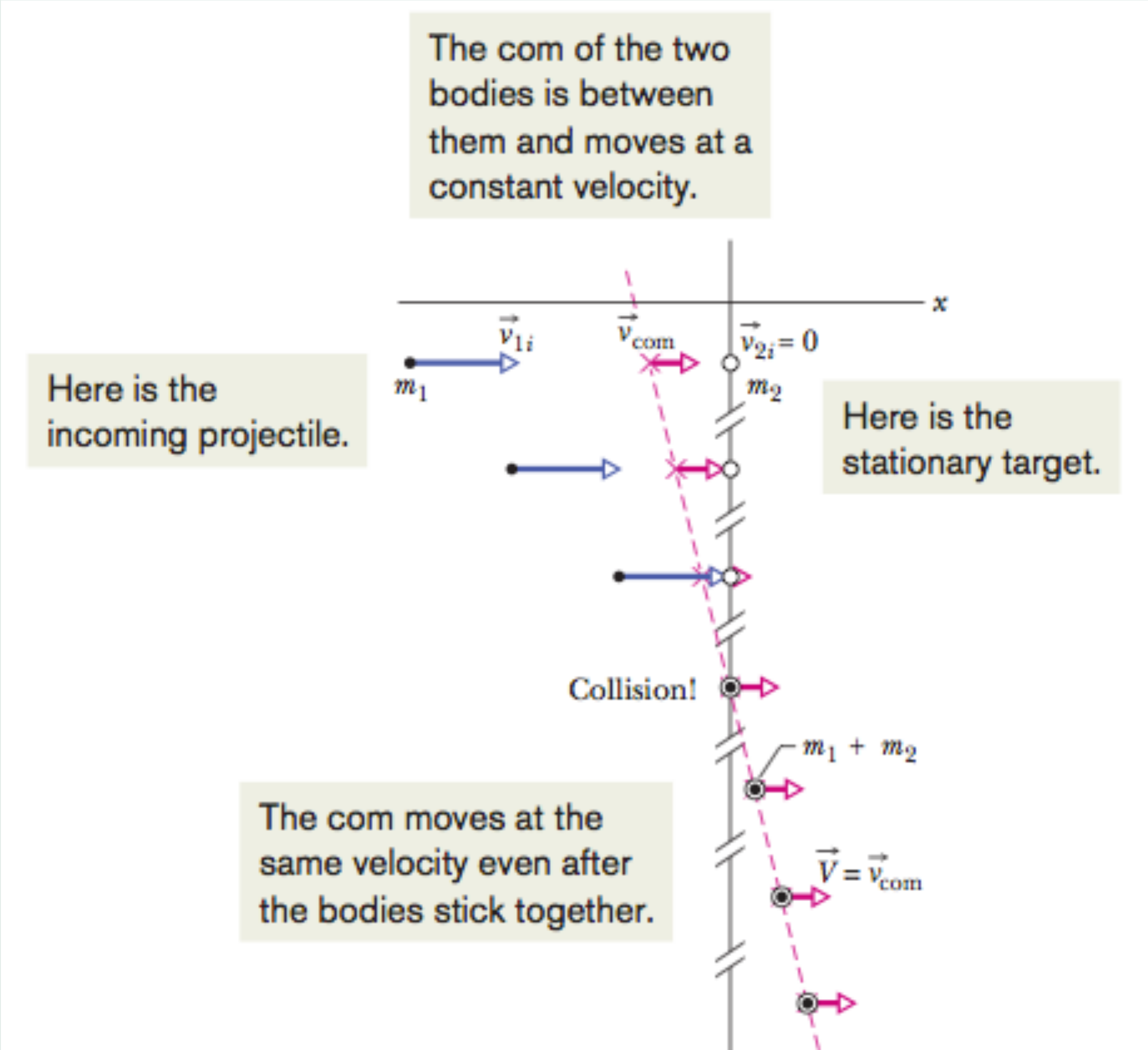
$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$



$$\vec{P}_i = \vec{P}_f$$

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{V}$$

$$\boxed{\vec{V} = \frac{m_1\vec{v}_{1i} + m_2\vec{v}_{2i}}{m_1 + m_2} \qquad V_x = \frac{m_1 v_{1i,x} + m_2 v_{2i,x}}{m_1 + m_2}}$$



$$\vec{V} = \vec{v}_{\text{com}}$$

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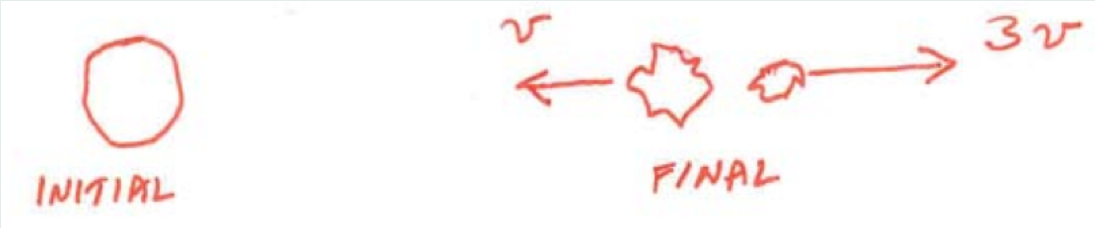
An object at rest explodes into two pieces of unequal mass. One piece flies west at a speed v and the second flies east at a speed $3v$. What is the velocity of the center of mass?



$A)$ 0

$B)$ $2v$ west

$C)$ $2v$ east

$D)$ cannot be determined

i-Clicker

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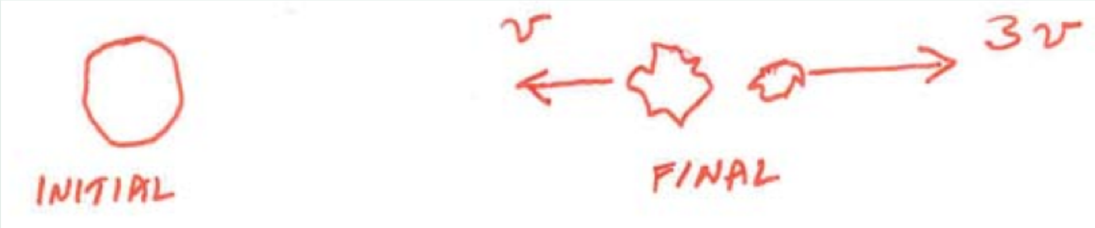


$A)$ 0

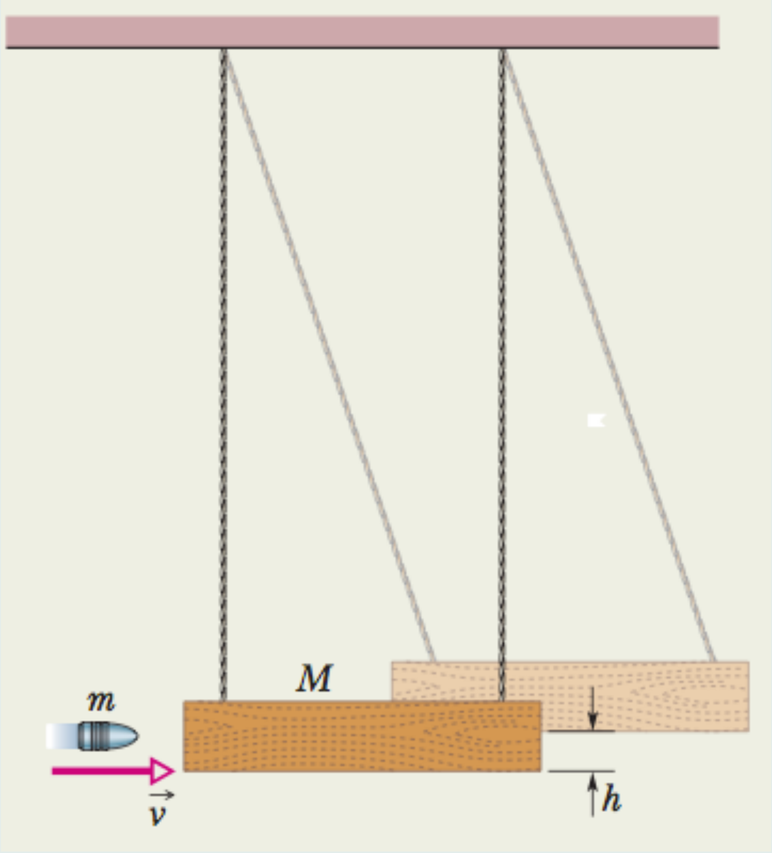
$B)$ $2v$ west

$C)$ $2v$ east

$D)$ cannot be determined

$$\vec{P}_i = \vec{P}_f = M\vec{v}_{\text{com}} = 0$$

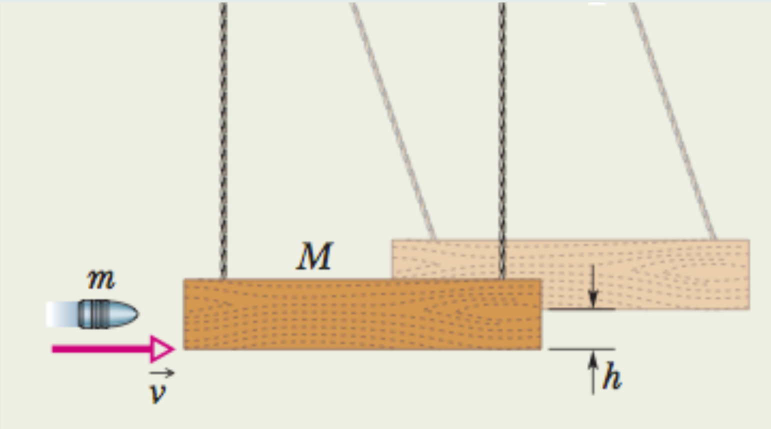
- **Example: ballistic pendulum**



- large block of wood of mass $M = 5.4\,\mathrm{kg}$ suspended from two long cords.
- bullet of mass $m = 9.5\,\mathrm{g}$ fired into the block.
- system block + bullet swings upward a vertical distance $h = 6.3\,\mathrm{cm}$.
- v_{bullet} ?

Step 1:

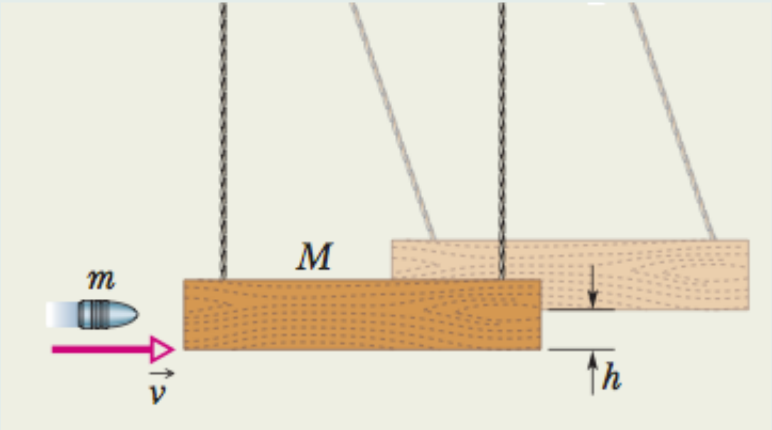
- completely inelastic collision
- linear momentum conserved
- kinetic energy not conserved



$$mv = (m + M)V$$

Step 2:

- upward swing
- linear momentum not conserved
- mechanical energy conserved



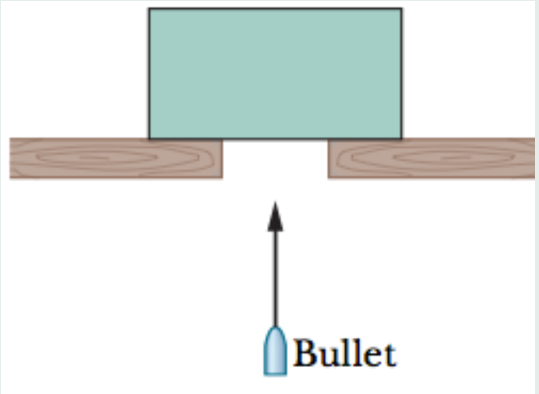
$$E_{\mathsf{mec}} = K + U = \text{constant}$$

$$\frac{1}{2}(M+m)V^2 = (M+m)gh$$

$$V = \sqrt{2gh}$$

$$v = \frac{m+M}{m}\sqrt{2gh}$$

- **Example:** generic inelastic collision



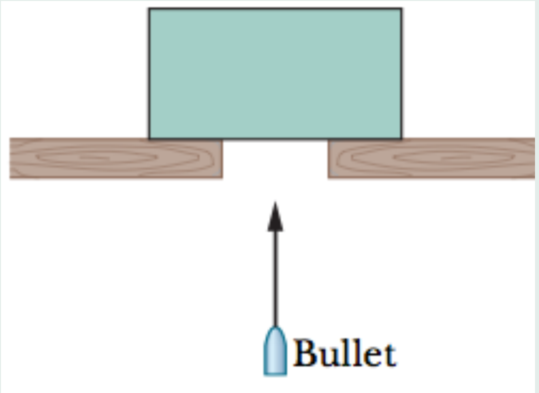
A bullet of mass $m = 10\,\mathrm{g}$ moving directly upward at $v = 1000\,\mathrm{m/s}$ strikes and passes through the center of mass of a $M = 5.0\,\mathrm{kg}$ block initially at rest.

The bullet emerges from the block moving directly upward at $v_1 = 400\,\mathrm{m/s}$.

To what maximum height does the block then rise above its initial position?

Step 1:

- inelastic collision
- linear momentum conserved
- kinetic energy not conserved



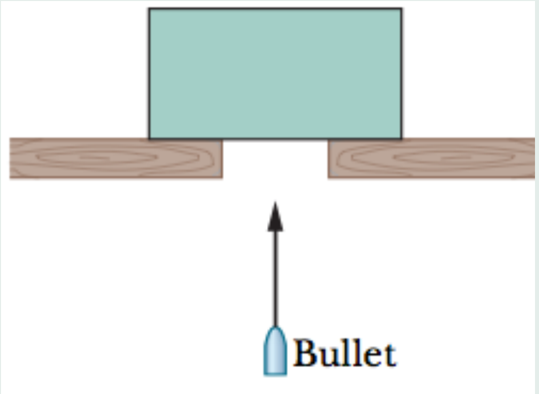
$$\vec{P}_i = \vec{P}_f$$

$$mv = mv_1 + Mv_2$$

$$v_2 = \frac{m}{M}(v - v_1)$$

Step 2:

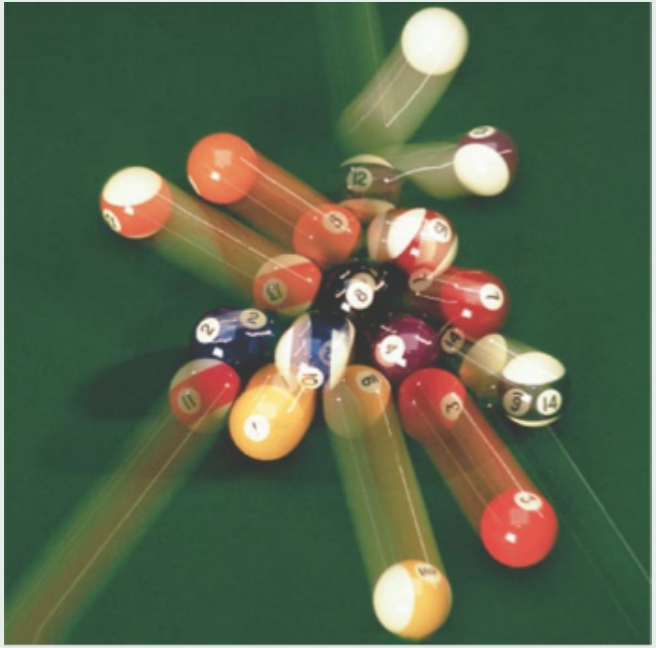
- upward motion
- linear momentum not conserved
- mechanical energy conserved
- assume the block does not move much during the collision

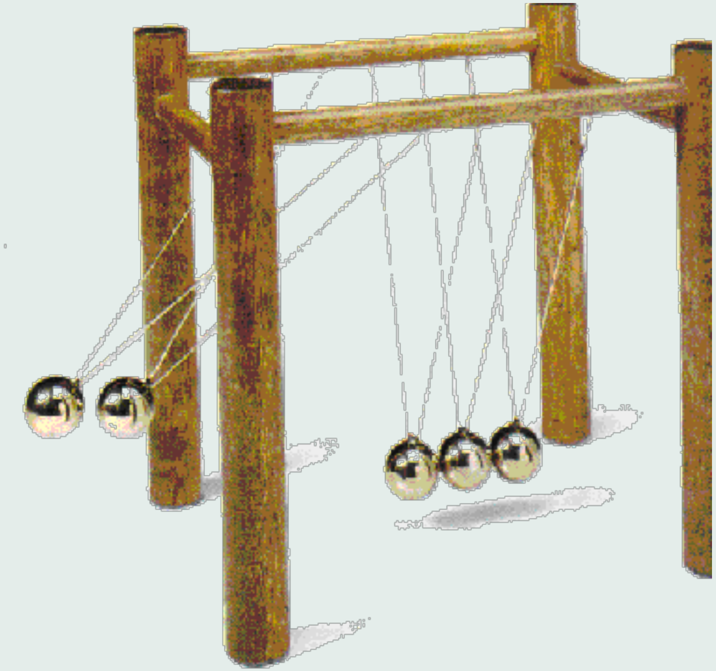


$$\frac{1}{2}Mv_2^2 = Mgh$$

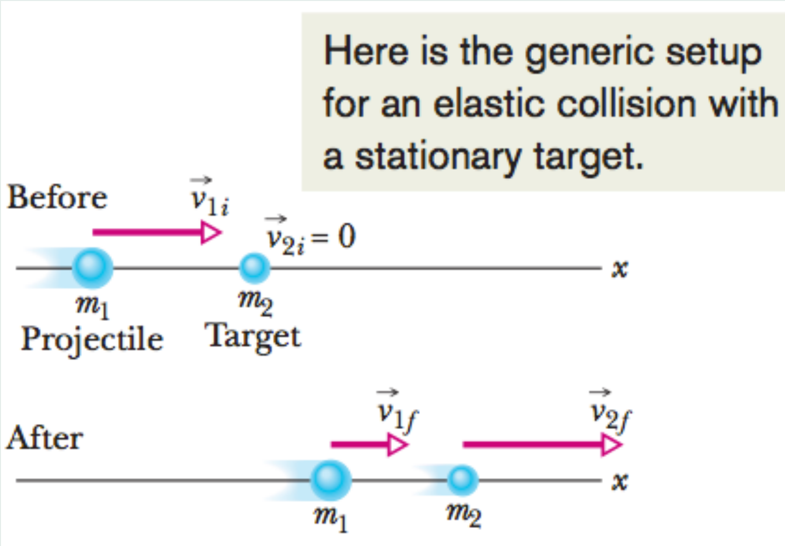
$$h = \frac{v_2^2}{2g} = \frac{m^2}{2M^2g}(v - v_1)^2$$

• **Ellastic collisions in 1D: both** linear momentum and kinetic energy are conserved





- **Generic setup – stationary target**



- **The linear momentum of the systems is conserved:**

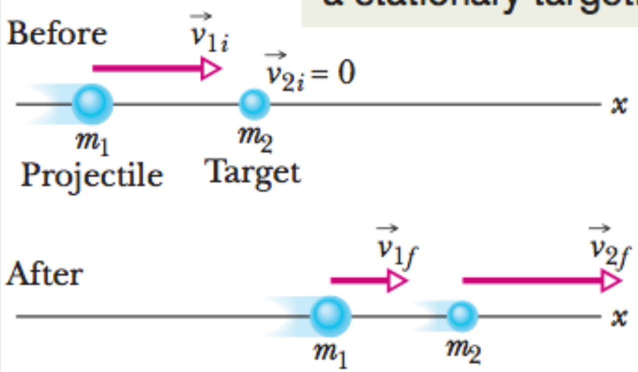
$$\vec{P}_i = \vec{P}_f$$

- **The total kinetic energy of the system is conserved:**

$$K_i = K_f$$

Note: the kinetic energy of each colliding body may change, but the total kinetic energy of the system does not change.

Here is the generic setup for an elastic collision with a stationary target.



$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$m_1 (v_{1i} - v_{1f}) = m_2 v_{2f}$$

$$m_1 (v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2 v_{2f}^2$$

$$\Downarrow$$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \qquad v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$