1 Physics 613: Problem Set 3 (*due Monday March 4*)

1.1 Dirac Matrix Identities

Prove the following identities involving Dirac matrices:

1. $\mathrm{Tr}(\gamma^\mu) = 0$

2. $\mathrm{Tr}(\gamma^\mu \gamma^\nu) = 4\eta^{\mu\nu}$

3. $\mathrm{Tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma) = 4\eta^{\mu\nu}\eta^{\rho\sigma} - 4\eta^{\mu\rho}\eta^{\nu\sigma} + 4\eta^{\mu\sigma}\eta^{\nu\rho}$

4. $(\gamma^\mu)^\dagger = \gamma^0 \gamma^\mu \gamma^0$

5. $(\bar{f}\gamma^{\mu_1} \dots \gamma^{\mu_n} f')^* = \bar{f}'\gamma^{\mu_n} \dots \gamma^{\mu_1} f$ where f and f' can be any Dirac spinor (i.e. u_s or v_s).

1.2 Rutherford Scattering

In class we used crossing symmetry to transform $e^+e^- \to \mu^+\mu^-$ into $e^-\mu^- \to e^-\mu^-$ (in class we called it p instead of μ^- but it doesn't matter); the answer for the squared and summed/averaged matrix element is

$$\frac{1}{4}|\mathcal{M}|^2 = \frac{2e^4}{t^2}(u^2 + s^2 + 4t(m_e^2 + m_\mu^2) - 2(m_e^2 + m_\mu^2)^2) \tag{1}$$

1. Rederive this directly from the t-channel Feynman diagram for $e^-\mu^- \to e^-\mu^-$ scattering (thereby verifying explicitly the validity of crossing symmetry in this example).

2. Carefully take the $m_\mu \to \infty$ limit and derive the Mott formula:

$$\left.\frac{d\sigma}{d\Omega}\right|_{m_\mu \to \infty} = \frac{e^4}{64\pi^2 v^2 p^2 \sin^4 \frac{\theta}{2}}(1 - v^2 \sin^2 \frac{\theta}{2}) \tag{2}$$

   where $v = p/E$ and p and E are the 3-momentum and energy of the incoming electron respectively. [Be careful! I think the treatment in Matt Schwartz's book may not be completely correct!]

1.3 Yukawa Theory

Consider a theory of a massive scalar ϕ with mass m and an electron with mass M (described by a massive Dirac fermion field Ψ) coupled together via the interaction term:

$$H_{int} = g \int d^3x \phi \bar{\Psi} \Psi \tag{3}$$

This is known as the Yukawa theory.

1. List all possible $2 \to 2$ scattering processes allowed by the theory, and draw all tree-level Feynman diagrams for each (don't calculate them or worry about relative minus signs). Is there any process that is allowed but not present at tree-level?

2. Use the momentum space Feynman rules to calculate $\frac{d\sigma_{CM}}{d\Omega}$ for $e^-\phi \to e^-\phi$ scattering, summed over final state spins and averaged over initial state spins.

3. Assuming $m > 2M$, ϕ can decay to e^+e^-. Compute the total decay rate $\Gamma(\phi \to e^+e^-)$, in the ϕ rest frame, summed over final state spins.