Physics 502: Problem Set 8 (*due Friday 4/28*)

1) In class we introduced the coherent states of the SHO as eigenstates of the annihilation operator, $a|\alpha\rangle = \alpha|\alpha\rangle$. We showed that these are minimum uncertainty states and that their uncertainties in X and P matched those of the ground state for all time t: $\Delta X = \frac{\hbar}{2m\omega}$ and $\Delta P = \frac{m\omega\hbar}{2}$. In this problem, we will prove that the coherent states are the unique states of the SHO with this property.

(a) Show that for any state, $\langle a^m \rangle_t = e^{-im\omega t}\langle a^m \rangle_{t=0}$ where m is a positive integer.

(b) Use your result in part (a) to express $(\Delta X)_t = \langle X^2 \rangle_t - \langle X \rangle_t^2$ in terms of information at $t = 0$.

(c) Now suppose the state satisfies $(\Delta X)_t = \frac{\hbar}{2m\omega}$ for all t. Prove that it must be an eigenstate of a. (Hint: you will probably need to use the Cauchy-Schwartz inequality at some point.)

2) Consider a free Dirac particle moving along the z direction with momentum p.

(a) Find all four normalized solutions to the Dirac equation and classify them in terms of eigenvalues of the *helicity operator* $h = \hat{p} \cdot \vec{S}$.

(b) Take the nonrelativistic limit $p \to 0$ and interpret your results in terms of spin and particle vs antiparticles.

(c) Explain how the solutions you found in part (a) would transform under a boost in the z direction. (Hint: use the fact that the Dirac equation is Lorentz covariant to avoid doing any calculation.) What happens to the helicity under a general boost?

3) Shankar 20.1.1 on p. 566. (Derivation of continuity equation)

4) Shankar 12.3.8 on p. 317.

5) Shankar 20.2.2 on p. 569. (Dirac particle in uniform magnetic field)