Physics 502: Problem Set 5 (*due Wednesday 3/22*)

1) Shankar 19.5.3 p. 554 (hard sphere in limits of small and large r_0).

2) Shankar 19.5.4 p. 554 (s-wave phase shift for square well)

3) Shankar 19.5.6 p. 555 (optical theorem).

4) Let's fill in some of the steps and flesh out some of the results in our study of quasi-bound states and resonance scattering in the spherical square well+barrier at $\ell = 0$. As in class, the s-wave potential is given by:

$$V(r) = \begin{cases} -V_0 & r < R_0 \\ +V_1 & R_0 < r < R_1 \\ 0 & r > R_1 \end{cases} \tag{1}$$

where both V_0 and V_1 are positive. **In this problem you will be expected to make use of a program like Mathematica for symbolic algebra, numeric calculation and plotting. Make sure to include a print out of at least some of the explicit steps in your calculations. Don't hesitate to ask me or your fellow students for help.**

a) Consider a quasi-bound state given by

$$u(r) = \begin{cases} A \sin k'r & r < R_0 \\ Be^{-\kappa r} + Ce^{\kappa r} & R_0 < r < R_1 \\ e^{ikr} & r > R_0 \end{cases} \tag{2}$$

where $k'^2 = E + V_0$, $k^2 = E$ and $\kappa^2 = V_1 - E$. (As in class, we are setting $2m/\hbar^2 = 1$ WLOG.) Show that the continuity conditions imply

$$\frac{\kappa - k' \cot k'R_0}{\kappa + k' \cot k'R_0} = e^{2\Delta R \kappa} \frac{\kappa - ik}{\kappa + ik} \tag{3}$$

where $\Delta R = R_1 - R_0$.

b) Find values of V_0, V_1, R_0 and R_1 such that a solution to this equation $E = E_r - i\Gamma/2$ exists in the complex E plane with E_r, $\Gamma > 0$. Make a 2d plot to demonstrate your solution.

c) Make a plot of Γ vs. increasing ΔR and verify that Γ indeed decreases to zero exponentially, while E_r converges to a bound state of the well with depth $V_0 + V_1$.

d) Now consider the analogous scattering problem:

$$u(r) = \begin{cases} A \sin k'r & r < R_0 \\ Be^{-\kappa r} + Ce^{\kappa r} & R_0 < r < R_1 \\ De^{ikr} + e^{-ikr} & r > R_0 \end{cases} \tag{4}$$

(E is real now!) Compute the s-wave cross section σ_0. For a choice of V_0, V_1, R_0 and R_1 found above that leads to a quasi bound state with small Γ, make a plot of σ_0 vs. E. Verify that at $E = E_r$ there is indeed a sharp resonance and show that it is well-fit by a Breit-Wigner line shape with half-width $\Gamma/2$.