Physics 502: Problem Set 3 (*DUE ON FRIDAY 2/24*)

1) A hydrogen atom is in the ground state at $t = -\infty$. A weak electric field $E(t) = E_0 \hat{z} e^{-|t/\tau|}$ is applied starting at time $t = -\infty$.

a) Using first order TDPT, find the probabilities for the atom to end up in each of the $|21m\rangle$ states (i.e., for each of the three values of m). (You may use that $\langle 210|z|100\rangle = \frac{2^{7.5}}{3^5} a_0$.)

b) Discuss the limit $\tau \to \infty$; does the result agree with what you would expect based on the adiabatic approximation?

2) Consider a composite system made of two spin 1/2 particles with Hamiltonian:

$$H = \begin{cases} 0 & t < 0 \\ \frac{4\Delta}{\hbar^2} \vec{S}_1 \cdot \vec{S}_2 & t > 0 \end{cases} \tag{1}$$

Suppose the system is in $|-+\rangle$ for $t \leq 0$. Find as a function of time the probability of being found in each of the following states, $|++\rangle$, $|+-\rangle$, $|-+\rangle$ and $|--\rangle$ by:

a) Solving the problem exactly.

b) Using first order TDPT. Under what conditions does (b) give the correct results?

3) A perturbation consisting of a series of N oscillatory pulses

$$V(t) = \sum_{j=1}^{N} A e^{-i\omega_0 t} f(t - t_j) \tag{2}$$

is applied to a system in which two discrete levels s and n are separated in energy by $\hbar\omega_0$. Here A is an operator, $f(t)$ is an envelope function describing the shape of each pulse, and t_j is the time of arrival of the jth pulse.

a) Show that, in first-order time-dependent perturbation theory, the probability of excitation from s to n is proportional to N^2.

b) This peculiar result is a consequence of the fact that each pulse is applied in phase, i.e., the excitation is coherent. How does the probability scale with N if the excitation is instead incoherent, i.e., each term in the expression for $V(t)$ is multiplied by a random phase $e^{i\phi_j}$? (If you think of the photons emerging from a light bulb or a laser as being the pulses, you can appreciate the fact that coherent sources such as lasers may have very different effects on matter than incoherent ones.)

4) A hydrogen atom initially in its $1s$ ground state is subject to an electric field $E(t) = E_0 \cos(\omega t)\hat{z}$ whose frequency is large enough that $\hbar\omega$ exceeds the ionization energy of 1 Ry. Assuming a plane-wave final state, what is the rate for transitions to an ionized state, and what is the angular distribution of emitted electrons? (*Note: you will need to evaluate the matrix element* $\langle \vec{k}|z|1s\rangle$*. To proceed with the rest of the problem, it is sufficient to evaluate this to leading nonzero order in the small* k *limit. For bonus points, figure out how to evaluate the matrix element exactly.*)

5) In class we derived the $\omega = 0$ version of the transition probability as a function of time for transitions of $s \to n \neq s$:

$$\mathcal{P}(s \to n) = \frac{4}{\hbar^2}|V_{ns}|^2 \frac{\sin^2 \frac{\omega_{ns} t}{2}}{\omega_{ns}^2} \tag{3}$$

We did this starting from the result for $\omega \neq 0$ obtained using TDPT, and then taking $\omega \to 0$. Rederive this result more directly using time-*independent* perturbation theory.