Physics 502: Problem Set 2 (*DUE ON MONDAY 2/13*)

1) Show explicitly that adding Shankar (17.3.12) and (17.3.21), one obtains (17.3.22), the formula for the 1st-order fine-structure energy shift that depends on j and not ℓ.

2) In class we did a spin 1/2 system in a B field: $H = -\gamma \vec{S}\cdot\vec{B}$, with $\vec{B}(t) = \frac{B_0}{\tau}(t\hat{x}+(\tau-t)\hat{z})$ for $0 < t < \tau$ and $\vec{B}(t) = B_0\hat{z}$ for $t < 0$ and $\vec{B}(t) = B_0\hat{x}$ for $t > \tau$. Solve the full TDSE *numerically* and verify that the sudden and adiabatic approximations are reproduced in the appropriate regimes.

3) SHO in Heisenberg representation:

a) Show that Shankar Eqs. (18.3.34) can be solved by writing $X_H(t)$ and $P_H(t)$ explicitly in terms of the Schrodinger operators and t, i.e.,

$$X_H(t) = X_S \cos\omega t + \frac{P_S}{m\omega}\sin\omega t \tag{1}$$

and find the similar equation for $P_H(t)$.

b) Show by example that a Heisenberg operator does not generally commute with itself at different times, i.e., $[A_H(t_1), A_H(t_2)] \neq 0$.

c) Use the result of (a) to prove that, no matter what initial state the system is prepared in, the coordinate expectation value has a simple oscillatory behavior, $\langle X\rangle_t = A\cos(\omega t + \phi)$.

4) In class we derived the integral version of the Schrodinger equation for the time-evolution operator in the interaction picture:

$$\mathcal{U}_I(t,t_0) = 1 - \frac{i}{\hbar}\epsilon\int_{t_0}^{t} dt'\, V_I(t')\mathcal{U}_I(t',t_0) \tag{2}$$

a) Using the iterative method developed in class, solve this to *third* order in ϵ.

b) Show explicitly by expanding in ϵ that your result in part (a) agrees with the general all-orders solution we obtained in class:

$$\mathcal{U}_I(t,t_0) = Te^{-\frac{i}{\hbar}\epsilon\int_{t_0}^{t} dt' V_I(t')} \tag{3}$$

5) At time $t = -\infty$, a particle of mass m and charge q is in the ground state of a 1D SHO potential and an electric field is turned on:

$$E(t) = E_0\frac{1+\tanh(t/\tau)}{2} \tag{4}$$

Find the probability of finding the particle in the nth SHO eigenstate of the final Hamiltonian $H = H_0 - E_0 q x$ in the following regimes:

a) In the adiabatic limit, $\tau \to \infty$.

b) In the sudden limit, $\tau \to 0$

c) Arbitrary τ but small E_0 – obtain the answer to first order in time-dependent perturbation theory and compare with your results in part (a) and (b).