Physics 502: Problem Set 1 (*DUE ON WEDNESDAY 2/1*)

1) Some simple warm-up problems.

a) Consider a 2-state system with $H^{(0)} = \begin{pmatrix} -E & 0 \\ 0 & E \end{pmatrix}$ and $V = \begin{pmatrix} 0 & \Delta \\ \Delta & 0 \end{pmatrix}$. Solve for the exact eigenenergies and eigenstates. Compare against perturbation theory (2nd order in the energies and 1st order in the states).

b) Consider a 3-state system with $H^{(0)} = \begin{pmatrix} E & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & 3E \end{pmatrix}$ and $V = \begin{pmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 0 \end{pmatrix}$. Find the first and second order perturbed energies. Check that they satisfy the characteristic polynomial to the appropriate order in perturbation theory.

2) Consider the simple harmonic oscillator $H^{(0)} = \frac{1}{2m}p^2 + \frac{1}{2}m\omega^2 x^2$ with perturbation $V = \frac{1}{2}\epsilon m\omega^2 x^2$ where ϵ is small.

a) Solve exactly for the perturbed energy levels and position-space energy eigenfunctions.

b) Solve for the first- and second-order shifts in the energy levels using perturbation theory (using operator algebra, i.e., writing x in terms of a and $a^\dagger$) and compare with the exact results.

c) Repeat (b) for the first-order eigenstates.

3) Shankar Ex. 17.2.1 (anharmonic oscillator).

4) Shankar Ex. 17.3.2 (spin-1 degenerate perturbation theory).

5) In class we derived the master equations for perturbation theory:

$$\begin{aligned} \langle m^{(0)}|n\rangle &= \frac{\epsilon\langle m^{(0)}|V|n\rangle}{E_n^{(0)} - E_m^{(0)} + \epsilon\Delta_n} \\ \Delta_n &= \langle n^{(0)}|V|n\rangle \end{aligned} \tag{1}$$

a) In the non-degenerate case, iterate these equations to derive a formula for the second-order perturbation to the states, $|n^{(2)}\rangle$.

b) In class we showed that in the degenerate case, the first order states were given by $\langle m^{(0)}|n^{(1)}\rangle = \sum_{m'\notin D} \frac{V_{m'n}V_{mm'}}{(E_n^{(0)} - E_{m'}^{(0)})(V_{nn} - V_{mm})}$ for $m \in D$. Show how this follows from your *second-order* result in part (a), after you redefine the problem to absorb $V\Big|_D$ into the definition of $H^{(0)}$.