Consider free field first

- What is L?

- Recall: Maxwell:

$$\begin{cases} \vec{\nabla}\cdot B = 0 \\ \vec{\nabla}\times E + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0 \end{cases} \rightarrow (\phi, \vec{A}) \quad \vec{B} = \vec{\nabla}\times\vec{A}, \quad E = -\vec{\nabla}\Phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$$

$$\begin{cases} \vec{\nabla}\cdot\vec{E} = 0 \\ \vec{\nabla}\times\vec{B} - \frac{1}{c}\frac{\partial E}{\partial t} = 0 \end{cases} \rightarrow \text{Coulomb gauge} \quad \begin{cases} \phi = 0 \\ \vec{\nabla}\cdot\vec{A} = 0 \end{cases} \quad \vec{\nabla}\cdot\vec{E} = 0 \text{ autom.}$$

$$\downarrow$$

$$\boxed{\nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{A} = 0} \quad \text{EOM}$$

plane wave sol'ns!
(sign it is free thy, canonical quantizati,

- Lorentz covariant formulation: (set $c=1$)

$$A_\mu = (\Phi, \vec{A}) \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

~~EOM~~

"field strength"
$\rightarrow$ gauge inv't!
$A_\mu \rightarrow A_\mu - \partial_\mu \phi$.

$$\begin{cases} F_{0i} = \dot{\vec{A}} + \vec{\nabla}\Phi \rightarrow -\vec{E} \\ F_{ij} = \partial_i A_j - \partial_j A_i = \varepsilon_{ijk}(\vec{\nabla}\times\vec{A})_k \rightarrow \vec{B} \end{cases}$$

{note: $\mathcal{L}$ doesn't depend on $\dot{\Phi}$!
$\downarrow$
not dynamical.

Simplest {Lorentz inv't, gauge inv't} : $S = \int d^4x\,\mathcal{L} = -\frac{1}{16\pi}\int d^4x F_{\mu\nu}F^{\mu\nu}$

$= \frac{1}{8\pi}\int (E^2 - B^2)$

Check: EOM $\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)} = 0 \quad \left(\frac{\partial \mathcal{L}}{\partial A} = 0\right)$

$$\parallel$$

$$\partial_\mu F^{\mu\nu} = \underbrace{\partial_\mu \partial^\mu A^\nu}_{\text{free wave eqn!}} - \underbrace{\partial^\nu \cancel{(\partial_\mu A^\mu)}}_{\substack{= 0 \text{ in C. gauge} \\ \phi = 0 \\ \vec\nabla\cdot\vec A = 0.}}$$

So $L = -\frac{1}{16\pi}\int d^3x \, F^2$ reproduces Maxwell!

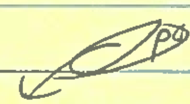
Hamiltonian

Conj mom: $\vec{\Pi} = \frac{\partial \mathcal{L}}{\partial \dot{\vec{A}}} = \frac{\partial \mathcal{L}}{\partial(\partial_0 A_i)} = \frac{1}{4\pi}\dot{\vec{A}} = -\frac{1}{4\pi}\vec{E}$

$$\mathcal{H} = \vec{\Pi}\cdot\dot{\vec{A}} - \mathcal{L} = \frac{1}{8\pi}\left(\vec{E}^2 + \vec{B}^2\right)$$

$$= 2\pi\,\vec{\Pi}^2 + \frac{1}{8\pi}(\vec{\nabla}\times\vec{A})^2 .$$

Mode expansion:



Let $\vec{A} = \sqrt{\frac{4\pi}{V}} \sum_{k\lambda} Q_{k\lambda}(t)\, \hat{e}_{k\lambda}\, e^{i\vec{k}\cdot\vec{r}}$

polarization vectors

Coulomb gauge: $\vec{k}\cdot\hat{e}_{k\lambda} = 0$

$\lambda = 1,2$ transverse pols.

Similarly: $\vec{\Pi} = \frac{1}{\sqrt{4\pi V}} \sum_{k\lambda} P_{-k\lambda}(t)\hat{e}_{k\lambda} e^{i\vec{k}\cdot\vec{r}}$

$\vec{\Pi} = \frac{1}{4\pi}\dot{\vec{A}} \Longrightarrow \boxed{P_{k\lambda}(t) = \dot{Q}_{k\lambda}(t)}$

$\vec{\Pi} = \frac{\partial \mathcal{L}}{\partial \dot{\vec{A}}} \quad \Big| \quad P_{k\lambda} = \frac{\partial L}{\partial \dot{Q}_{k\lambda}}$

Check: consistent w/ defn above!

- So $P_{k\lambda}(t) = \dot{Q}_{-k\lambda}(t)$ is conj momentum!
- Repeat scalar field

$$\Longrightarrow \boxed{H = \int d^3x\, \mathcal{H} = \frac{1}{2}\sum_{k\lambda}\left(P^*_{k\lambda}P_{k\lambda} + k^2 Q^*_{k\lambda}Q_{k\lambda}\right)}$$

Again, decoupled oscillators w/ $\omega_k = |\vec{k}|$!

As for scalar: $[Q_{k\lambda}, P_{k'\lambda'}] = i\hbar\, \delta_{kk'}\delta_{\lambda\lambda'}$

$$\begin{cases} a_{k\lambda} = \sqrt{\frac{\omega_k}{2\hbar}}\left(Q_{k\lambda} + \frac{i}{\omega_k} P^{+}_{k\lambda}\right) \\ a^{+}_{k\lambda} = \text{c.c.} \end{cases}$$

$$\rightarrow \begin{cases} Q_{k\lambda} = \sqrt{\frac{\hbar}{2\omega_k}}\left(a_{k\lambda} + a^{+}_{-k\lambda}\right) \\ P_{k\lambda} = -i\sqrt{\frac{\hbar\omega_k}{2}}\left(a_{-k\lambda} - a^{+}_{k\lambda}\right) \end{cases}$$

$$\boxed{H = \sum_{k\lambda} \hbar\omega_k \left(a^{+}_{k\lambda} a_{k\lambda} + \frac{1}{2}\right)}$$

- each mode of EM field independent oscillator w/ energy $\hbar\omega_k = \hbar|\vec{k}|$ $\rightarrow$ photons!!
- Hilbert space $|n_{k_1\lambda_1}, n_{k_2\lambda_2}, \cdots\rangle = |\{n_{k\lambda}\}\rangle$

∞ is a UV divergence
can cut off @ short distance
"effective field theory"

$a^{+}_{k\lambda}$ creates one photon of energy ω_k, momentum $\vec{k}$, polarization $\epsilon_{k\lambda}$

$$E_{\{n_{k\lambda}\}} = \sum_k \hbar\omega_k\left(n_k + \tfrac{1}{2}\right) = \infty + \hbar\sum k\, n_k$$

- Vacuum $|0\rangle$ ann. by all $a_{k\lambda}$, $n_{k\lambda} = 0 \ \forall k, \lambda$
  "darkness"

Note $\langle 0|\vec{E}|0\rangle = 0$ but $\langle 0|\vec{E}^2|0\rangle \neq 0$! as in SHO

Vacuum full of quantum fluctuations ($\vec{E} \sim \vec{\Pi}$ $\sim a_{\vec{p}} a^{+}_{\vec{p}}$)

(∞ zero pt energy)

Coupling to matter

Recall we'd like to understand spontaneous decay of 2s state of H.

- $\langle\vec{A}\rangle = 0$ (no B field)
  but qtm fluctuations of $\vec{A}$ destabilize 2s.
- Subtlety: what about $\Phi(\vec{r})$? We've been setting it to zero for free rad ($\rho=0$) but in atom, have backgrnd. Φ of proton!
  $\longrightarrow$ glossed over in textbook!

saved phy ptcls

$$\begin{cases} L_{part} = \sum_i \frac{1}{2} m v_i^2 \\ L_{int} = \int d^3x\, \mathcal{L}_{int} = \int d^3x \left(-\rho\Phi + \vec{J}\cdot\vec{A}\right) \end{cases}$$

$$= \sum_i \left(-q\Phi_i + q\,\vec{v}_i\cdot\vec{A}_i\right)$$

$$L_{rad} = \frac{1}{8\pi}\int d^3x \left(E^2 - B^2\right) \qquad \begin{cases} \rho(\vec{r}) = \sum q\,\delta(r-r_i) \\ \vec{J}(\vec{r}) = \sum q\,\vec{v}_i\,\delta(r-r_i) \end{cases}$$

Hamiltonian: $\vec{p}_i = \frac{\partial L}{\partial v_i} = m\vec{v}_i + q\vec{A}_i$

Recall: Φ is not a dynamical dof! Can solve for it eventually!

$\Pi_\Phi = 0$

$$\vec{\Pi}_A = \frac{\partial\mathcal{L}}{\partial\dot{A}} = \frac{1}{4\pi}\left(\vec{\nabla}\Phi + \dot{\vec{A}}\right) = -\frac{1}{4\pi}\vec{E}$$

F_{0i}

$$H = \sum \vec{p}_i \cdot \vec{v}_i + \int d^3r\, \vec{\Pi} \cdot \dot{\vec{A}} - L$$

$$\cancel{= \sum \vec{p}_i \cdot \frac{(\vec{p}_i - q\vec{A}_i)}{m} + \int d^3r\, \vec{\Pi} \cdot 4\pi(\vec{\Pi} - \vec{\nabla}\Phi)}$$

$$\cancel{- \left(\sum_i \frac{1}{2m} p_i^2 + \sum (-q\Phi_i + q p_i \cdot A \right.}$$

$$= \sum (m\vec{v}_i + \cancel{q\vec{A}_i}) \cdot \vec{v}_i + \int d^3x\, \frac{\vec{E}}{4\pi} \cdot (\vec{E} + \nabla\vec{\Phi})$$

$$- \sum_i \frac{1}{2} m v_i^2 + \sum (q_i \Phi_i - \cancel{q v_i A_i})$$

$$- \frac{1}{8\pi} \int d^3x\, (E^2 - B^2)$$

$$= \sum \left(\frac{1}{2} m v_i^2 + q_i \Phi_i \right) + \frac{1}{8\pi} \int \left[(E^2 + B^2) + 2\vec{E} \cdot \vec{\nabla}\Phi \right]$$

- Must be written in terms of conj mom!

$$\begin{cases} \frac{1}{2} m v_i^2 = \frac{1}{2m} (p_i - qA_i)^2 \\ \vec{E} = -4\pi \vec{\Pi} \\ \vec{B} = \vec{\nabla} \times \vec{A} \end{cases}$$

- Impose Coulomb gauge: $\vec{\nabla} \cdot \vec{A} = 0 \Rightarrow \nabla^2 \Phi = -4\pi\rho$

$$\Rightarrow \Phi(r) = \int d^3r' \frac{\rho(r')}{|r - r'|}$$

$$= \sum_i \frac{q_i}{|r - r_i|}$$

Useful to decompose fields into long. & transv.

$$\vec{E} = \vec{E}_L + \vec{E}_T \qquad \vec{\nabla}\cdot\vec{E}_T = 0$$
$$\vec{\nabla}\times\vec{E}_L = 0$$

· So $\vec{A} = \vec{A}_L + \vec{A}_T$ but $\vec{A}_L = 0$ by gauge choice.

· note: $\nabla\Phi$ purely L b/c $\nabla\times\nabla\Phi = 0$.

$$\begin{cases} A_L = 0 & A_T = A \\ \Pi_L = \frac{1}{4\pi}\nabla\Phi & \Pi_T = \frac{1}{4\pi}\dot{A} \\ E_L = -\nabla\Phi & E_T = -\dot{A} \\ B_L = 0 & B_T = \cancel{\nabla\times A}\; B = \nabla\times A \end{cases}$$

note: $\int E_T\cdot\nabla\Phi = 0$ → integration by parts!

~~→ cross terms in E^2+B^2 → 0.~~

$$\frac{1}{8\pi}\int E^2 + B^2 = \frac{1}{8\pi}\int (\nabla\Phi)^2 + E_T^2 + \cancel{2(-\nabla\Phi)E_T} + B_T^2$$

Combine $\frac{1}{8\pi}\int(\nabla\Phi)^2 + 2\vec{E}_L\cdot(\nabla\Phi) = -\frac{1}{8\pi}\int(\nabla\Phi)^2 = +\frac{1}{8\pi}\int\Phi\nabla^2\Phi$

$$= -\frac{4\pi}{8\pi}\int\Phi\rho$$
$$= -\frac{1}{2}\sum q_i\Phi_i$$

So

$$H = \sum\left(\frac{1}{2}mv_i^2 + \frac{1}{2}q_i\Phi_i\right) + \frac{1}{8\pi}\int\left(E_T^2 + B_T^2\right)$$

long. dof dropout!!

$$\begin{cases} v_i = \frac{p_i - qA_i}{m} \\ E_T = -4\pi\Pi_T \\ B_T = \nabla\times A \end{cases}$$

Final:

$$H = H_{rad} + H_{part} + H_{int}$$

H_{rad}: $\frac{1}{8\pi}\int\left(E_T^2 + B_1^2\right)$ just solved

H_{part}: $\sum\frac{p_i^2}{2m} + \frac{1}{2}\sum\frac{q^2}{|r_i - r_j|}$ hydrogen

H_{int}: $\sum\left(-\frac{q}{m}p_i\cdot A_i + \frac{q^2}{2m}A_i^2\right)$ pert'n.

don't have to worry about Φ of proton in hydrogen atom! already taken care of!

Atomic transitions (nucleus mass $\to\infty$, $H \ni$ electrons only)

$$H_0 = H_{part} + H_{rad} \qquad V = H_{int}^{(I)}$$

$$H_{part} = \sum\frac{p_i^2}{2m} - \sum\frac{Ze^2}{|r_i|} + \frac{1}{2}\sum_{i\neq j}\frac{e^2}{|r_i - r_j|}$$

$$H_{rad} = \sum\hbar\omega_k\left(a_{k\lambda}^\dagger a_{k\lambda} + \frac{1}{2}\right)$$

What is H_{int}? Focus on linear term $A^2 \to 2$ photon trans

$(q = -e)$

$$H_{int} = \frac{e}{m} \sum p_i A_i = \frac{e}{m}\sqrt{\frac{4\pi}{V}} \sum_i \vec{p}_i \cdot \hat{e}_{k\lambda}\, Q_{k\lambda}\, e^{i\vec{k}\cdot\vec{r}_i}$$

$$= \frac{e}{m}\sqrt{\frac{2\pi\hbar}{V}} \sum_i \sum_{k\lambda} \frac{1}{\sqrt{\omega_k}}\, \hat{e}_{k\lambda}\cdot\vec{p}_i\, e^{i\vec{k}\cdot\vec{r}_i} \, (a_{k\lambda} + a^\dagger_{k\lambda})$$

(note: $\hat{e}_{k\lambda} \perp \vec{k}$ $\Rightarrow [\vec{p}\cdot\hat{e}, e^{i\vec{k}\cdot\vec{r}}] = 0$.)

In 1st Born approx / FGR, H_{int} will mediate 1-photon transitions (abs or emitted)

+ : em

− : abs

· Initial: $|\beta\rangle = |i; \cdots n_{k\lambda} \cdots\rangle$ $\quad E_\beta = E_i + \hbar\omega_k n_k + \cdots$

state of atom

· Final: $|\alpha\rangle = |f; \cdots (n_{k\lambda} \pm 1) \cdots\rangle$ $\quad E_\alpha = E_f + \hbar\omega_k (n_k \pm 1) + \cdots$

state of free radiation

· Rate (FGR): $\frac{2\pi}{\hbar} |V_{\alpha\beta}|^2 \delta(E_\alpha - E_\beta) \leftarrow \delta(E_{fi} \pm \hbar\omega_k)$

($V = H_{int}$ time indep so use FGR - const pot'l.
(energy conserved!))

$$V_{\alpha\beta} = \frac{e}{m} \sum_{k'\lambda'} \sqrt{\frac{2\pi\hbar}{V\omega_{k'}}}\, \hat{e}_{k'\lambda'} \langle f | \sum_j \vec{p}_j e^{i\vec{k}'\cdot\vec{r}_j} | i \rangle$$

$$\cdot \langle n_{k\lambda} \pm 1 | a_{k'\lambda'} + a^\dagger_{k'\lambda'} | n_{k\lambda} \rangle$$

<u>Rad ME:</u> $+ : \sqrt{n_{k\lambda}+1}\, \delta_{k'k}\, \delta_{\lambda'\lambda}$

$- : \sqrt{n_{k\lambda}}\, \delta_{k'k}\, \delta_{\lambda'\lambda}$

let $\hat{\sigma} = \hat{e}_{k\lambda} = \hat{e}_{-k\lambda}$

keep mode k ~~chang~~ fixed here by choice of $|\alpha\rangle$ & $|\beta\rangle$

$$\cdot\ V_{\alpha\beta} = \frac{e}{m}\sqrt{\frac{2\pi\hbar}{V\omega_k}}\left(\begin{Bmatrix} n_{k\lambda}+1 \\ n_{k\lambda} \end{Bmatrix}\right)^{1/2} \hat{e}\cdot\langle f|\sum_j \vec{p}_j\, e^{\mp i\vec{k}\cdot\vec{r}_j}|i\rangle$$

· ~~particle~~ atomic ME : electric dipole approx : $e^{\mp i\vec{k}\cdot\vec{r}_j} \approx 1$

$$\langle f|\underbrace{\vec{p}}_{\sum_j \vec{p}_j}|i\rangle = -\frac{im}{e}\omega_{fi}\langle f|\vec{d}|i\rangle$$

$$= \pm\frac{im}{e}\omega_k\, \vec{d}_{fi}$$

$$\Longrightarrow \boxed{\Gamma(\beta\to\alpha)_{i\to f,\ n_{k\lambda}\to n_{k\lambda}\pm 1} = \frac{4\pi^2}{V}\omega_k\,|\hat{e}\cdot\vec{d}_{fi}|^2 \begin{Bmatrix} n_{k\lambda}+1 \\ n_{k\lambda} \end{Bmatrix}\delta(\epsilon_{fi}\pm\hbar\omega)}$$

+ : emission

− : absorption

$n_{k\lambda}$ part : stimulated em & abs (background field b/c $n_{k\lambda}\neq 0$)

+1 : spontaneous absorption!

· See it is small in limit of $n_{k\lambda}\to\infty$ (macro field)

· comes out automatically w/ a quantized field! missing in semiclassical approach

· comes from quantum SHO of radiation field!

More on Spontaneous Emission Rate

$$\Gamma_{spont} = \frac{4\pi^2}{L^3}\,\omega^2\,|\hat{e}\cdot\vec{d}_{fi}|^2\,\rho_{ph}(\hbar\omega)$$

δ fn $\rightarrow$ density of states

$$\rho_{ph} = \frac{L^3\omega^2}{\pi^2\hbar}$$

(ref back to ρ for neutrinos in β-decay example)

(ρ only depends on energy-momentum dispersion relation!)

(includes factor of 2 for photon polarizations... more proper treatment: sum over $\hat{e}$. Being a little sloppy here.)

$$\boxed{\Gamma_{spont} = \frac{4\omega^3}{3}|\vec{d}_{fi}|^2}$$

(avg over emitted γ direction & polarization)

agrees w/ detailed balance! (see HW)

<u>Examples:</u> $n=2 \rightarrow n=1$ in Hydrogen "Lyman α"

<u>careful: selection rules!</u>

$|i\rangle$ & $|f\rangle$ opp parity $\Rightarrow$ $l=1$ only allowed

$2s \rightarrow 1s$ forbidden (in fact occurs thru 2γ transition) much slower, $\tau \sim 0.12\,s$

$n=2$

focus on $f = 210 \leftarrow m=0$ for simplicity (same answer for $m=\pm 1$ $\rightarrow$ rotational invariance!)

$i = 100$

Then only Q_z contributes by sel rules

$$\Gamma_{spnt}^{2p\to 1s} = \frac{4}{3}\omega^3 \left|\langle 100|ez|210\rangle\right|^2$$

$c = 3\times10^{17}\ nm/s$

$a_0 \approx 0.053\ nm$

$$\hookrightarrow \frac{2^7}{3^5}\sqrt{2}\, e\, a_0$$

$e^2 = \frac{1}{137}$

$\omega = \frac{2\pi}{\lambda} = \frac{2\pi}{(121.6nm)}$

$$= 0.74\, \alpha\, a_0^2 \omega^3 \quad \text{units of } \frac{1}{nm}$$

$$\times c \longrightarrow \text{to get units of } 1/s$$

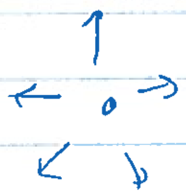
$$\simeq 0.628\times10^{9}\ s^{-1}$$

$$\tau_{2p} = \frac{1}{\Gamma_{spnt}^{2p}} \approx 1.59\times10^{-9}\ s \longrightarrow \text{excellent agreement w/ exp't!}$$

$\Longrightarrow$ 1st principles calc of lifetime
a major triumph of QM/QFT!

$\Longrightarrow$ much more info here (e.g. angular dist'ns !!, polarizations)

How to describe a background EM field?

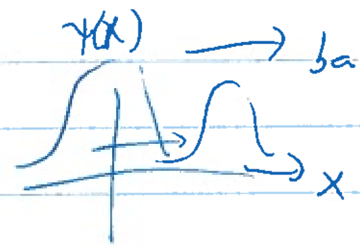


Φ taken care of, part of H_{part}.

What about $A(\vec{x},t)$? e.g. laser w/ $Q_k = e^{i\omega_k t}$ for some k

$$A = e^{i(\vec{k}\cdot\vec{r} - \omega_k t)}\hat{e}_{k\lambda}$$

Have $\langle \{n_{k\lambda}\} | \vec{A} | \{n_{k\lambda}\} \rangle = 0 \;\forall\; n_{k\lambda}$ ($A \sim Q$ of SHO)



→ background field has non definite photon #

superposition of states!

<u>what is min uncertainty state?</u>

"gaussian wavepacket for SHO"

"coherent states"

<u>Recall from SHO</u>: eigenstates of ann. op

a let $|\alpha\rangle$ be s.t. $a|\alpha\rangle = \alpha|\alpha\rangle$

$$\boxed{|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum \frac{\alpha^{n}}{\sqrt{n!}} |n\rangle}$$

Then

$X = \sqrt{\frac{\hbar}{2m\omega}}(a + a^\dagger)$

$P = -i\sqrt{\frac{m\omega\hbar}{2}}(a - a^\dagger)$

- $\langle \alpha | X | \alpha \rangle = \sqrt{\frac{\hbar}{2m\omega}}(\alpha + \alpha^*)$ $\qquad \langle \alpha | P | \alpha \rangle = -i\sqrt{\frac{m\omega\hbar}{2}}(\alpha - \alpha^*)$

$a|\alpha\rangle = \alpha|\alpha\rangle$

$\langle\alpha|a^\dagger = \alpha^* \langle \alpha|$

$$\langle \alpha | X^2 | \alpha \rangle = \frac{\hbar}{2m\omega}\langle \alpha | a^2 + (a^\dagger)^2 + 2a^\dagger a + 1 | \alpha \rangle$$

$$= \frac{\hbar}{2m\omega}(\alpha + \alpha^*)^2 + \frac{\hbar}{2m\omega}$$

$$\langle \alpha | P^2 | \alpha \rangle = -\frac{m\omega\hbar}{2}(\alpha - \alpha^*)^2 + \frac{m\omega\hbar}{2}$$

$(\Delta X)=\frac{\hbar}{2m\omega}$ $(\Delta P)=\frac{m\omega\hbar}{2}$ $\leftarrow$ same as g.s.!

$$\boxed{(\Delta X)(\Delta P)=\frac{1}{4}\hbar^2}$$ min uncert.!

- Time evolution

$\omega_n=(n+\frac{1}{2})\omega$

$$|\alpha,t\rangle=e^{-\frac{|\alpha|^2}{2}}\sum\frac{\alpha^n}{\sqrt{n!}}e^{-i\omega_n t}|n(0)\rangle$$

$$=e^{-\frac{i\omega t}{2}}e^{-\frac{|\alpha|^2}{2}}\sum\frac{(\alpha e^{-i\omega t})^n}{\sqrt{n!}}|n\rangle$$

irrelev phase

$$=e^{-\frac{i\omega t}{2}}|\alpha(t)\rangle$$

$\alpha(t)=\alpha e^{-i\omega t}$.

State stays coherent w/ time, keeps min uncert!

Summary $\Rightarrow$ coh. state basically g.s. shifted in time by classical oscillator posit. & moment. moves as cl. ptcl, keeps its shape fixed.

"most classical state"

so $X(t)\approx\cos\omega t$ ✓ $P(t)\sim\sin\omega t$ like classical osc.!

basis of quant optics $\leftarrow$ "glauber state"

nobel 1/2 Roy Glauber 2005

$\Downarrow$

repeat for photons $X\to Q_{k\lambda}\to A, B$

$P\to P_{k\lambda}\to\Pi, E$

- Excellent description of laser (stays coherent over long distances)

- More generally, any macro EM field (like wavepacket descr of cl. ptcls)

Dirac Electrons (Ch 20 of Shankar)

- Motivation:
  - relativistic electrons (e.g. high Z)
  - orig. of spin
  - magnetic moment ($g=2$)
  - spin-orbit
  - fine structure $\longrightarrow$ recall $(E_{SO})_{rel} + (E_{rel})_{rel} = E_{nj}$! magic! $\to$ automatic w/ Dirac eqn
  - precursor to QFT.

- Free nonrel ptcl, $V(\vec{r})=0$, $H=\frac{p^2}{2m}$ (?)

How

Time evo: $i\hbar\frac{\partial}{\partial t}|\psi\rangle = \underbrace{\frac{p^2}{2m}|\psi\rangle}_{H|\psi\rangle}$

How to relativise?

Try #1: $H_{rel} = \sqrt{p^2+m^2} \longrightarrow i\hbar\frac{\partial}{\partial t}|\psi\rangle = \sqrt{p^2+m^2}|\psi\rangle$;

Problems: highly nonlinear when p is not small (got away w/ it for F.S.)

must keep all the many derivatives space & time on unequal footing.