## 1. Syllabus / Topics for Final Exam

**Time independent perturbation theory**

- Non-degenerate case
  - Examples: SHO + E field. Quadratic Stark Effect.
  - Selection rules and symmetries
- Degenerate case (Sakurai 5.2 and lecture notes)
  - Examples: Linear Stark Effect. (Sakurai 5.2 and Shankar 17.3)
  - Hydrogen Fine Structure. (Sakurai 5.3 and Shankar 17.3)
    - Relativistic kinetic energy
    - Spin-orbit coupling
    - Answer as expansion in $\alpha$, dimensional analysis
    - Miraculous answer: $\ell$, $s$, $m_\ell$, $m_s$ drops out, answer depends only on $j$!

**Time dependent perturbation theory** (Shankar 18 and Sakurai 5.5-5.6)

- Sudden approximation
- Adiabatic approximation
- Time-dependent perturbation theory
  - Schrodinger, Heisenberg, Interaction pictures
  - Dyson series for time-evolution operator in Interaction picture
  - 1st order solution to time-dependent perturbation theory (transition coefficients)
  - FERMI GOLDEN RULE
    - Monochromatic case
    - Constant case
    - Delta function vs density of states
    - Example: nuclear beta decay

**Scattering Theory**

- General setup, definition of cross section
- From FGR to 1st-order Born Approximation
- Elastic Scattering
  - Central potential
  - Rutherford scattering
  - Charge distribution – nuclear radius
- Inelastic scattering

Energy loss / stopping power

General (Elastic) Scattering Theory

Setup: incoming plane wave, outgoing spherical waves

Application of time-independent perturbation theory

Born approximation, formula for differential cross section in terms of $f(\theta, \phi)$

Beyond first order: Green's function solution to TISE (Lippmann-Schwinger equation)

Large $r$: multipole expansion

Recovering Born approximation

Partial wave expansion

Partial wave cross sections, S-matrix, unitarity bound, phase shifts

Ramsauer Townsend effect

Optical theorem

Example: Hard sphere

Bound states and Resonances

quasi-bound states, resonant scattering

zero-energy bound states

Resonance in terms of partial wave xsec and phase shift

Breit Wigner shape

S-matrix "complex energy" of resonance, im part is width (lifetime)

zero energy bound states in finite square well

**Absorption & Emission of EM Radiation**

Charged particles in oscillating E field (unrealistic)

Application of FGR for stimulated emission and absorption rate

Correct treatment of E & B fields via gauge potential formalism

Coulomb/Radiation/Transverse gauge

Minimal coupling $p + eA$

N electron Hamiltonian: paramagnetic, diamagnetic, Zeeman terms

Multipole expansion

Electric dipole approximation, FGR for abs and stim em rate

Magnetic dipole, FGR for abs and stim em rate

Electric quadrupole, FGR for abs and stim em rate

Expansion in $\alpha$

Selection Rules

Quantizing the EM Field

Warmup: scalar field theory

Canonical quantization

Maxwell Lagrangian for free EM field

Canonical quantization

Hamiltonian, mode (oscillator) expansion

Coupling to matter

Atomic transitions: rederiving stimulated emission and absorption

Spontaneous emission

Example: Hydrogen $n = 2 \to n = 1$, first principles calculation of lifetime

Coherent states

**Dirac Particles**

Dirac equation

Relativistic time-dependent Schrödinger equation

Need for multiple component wavefunction

Dirac $\alpha$ and $\beta$ matrices

Minimum 4 components – spin 1/2 particle & antiparticle!

Problem of negative energy solutions

Dirac sea, particles and holes

Angular momentum

only $J$ conserved

See spin 1/2 explicitly

Dirac particles + EM field

2-component spinor formalism

Dirac Hamiltonian in non-relativistic limit

Leading order

$A^2$ term

Zeeman term

$g = 2$ (comment on QED corrections)

Next to leading order: fine structure revisited

Relativistic KE, SO coupling, Darwin term

Explaining the miracle (dependence only on $j$)

**Path integral... NOT ON THE FINAL**