Physics 417: Problem Set 9 (*DUE ON THURSDAY 11/21*)

Problem 1: The delta function bump

(a) Griffiths 6.1

(b) Griffiths 6.4(a)

Problem 2: Perturbation theory in two and three state systems

(a) In class we also did the example of the two state system with $H^{(0)} = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$ and $\epsilon H^{(1)} = \begin{pmatrix} 0 & \epsilon V \\ \epsilon V & 0 \end{pmatrix}$ (with $E_1 \neq E_2$). We calculated the first and second order energies $E_n^{(1)}$, $E_n^{(2)}$. Compute the first-order energy eigenstates and compare with the exact result.

(b) Griffiths 6.9

Problem 3: Perturbation theory in the harmonic oscillator system

In class we did the example of the harmonic oscillator $H^{(0)} = \frac{1}{2m}P^2 + \frac{1}{2}m\omega^2 X^2$ with perturbation $\epsilon H^{(1)} = \frac{1}{2}\epsilon m\omega^2 X^2$. We did the first-order energies $E_n^{(1)}$, the second-order ground state energy $E_0^{(2)}$, and the first-order ground state wavefunction $\psi_0^{(1)}(x)$.

(a) Use second-order perturbation theory to derive a formula for $E_n^{(2)}$ for all energy levels n and compare with the exact result.

(b) Derive the first-order wavefunction $\psi_n^{(1)}(x)$ for all energy levels n and compare with the exact result. [You will need the Rodrigues formula for this.]

Problem 4: The anharmonic oscillator

Consider a perturbation $\epsilon H^{(1)} = \epsilon X^4$ to the harmonic oscillator $H^{(0)} = \frac{1}{2m}P^2 + \frac{1}{2}m\omega^2 X^2$. This is an example of an anharmonic oscillator with a nonlinear restoring force.

(a) Show that the first-order shift in the energy is given by

$$E_n^{(1)} = \frac{3\hbar^2\epsilon}{4m^2\omega^2}(1 + 2n + 2n^2) \qquad (1)$$

(b) Argue that no matter how small ϵ is, the perturbation expansion will break down for sufficiently large n. What is the physical reason?

EXTRA CREDIT WORTH HALF OF A PROBLEM: Compute the second order shift in the energies and confirm your result in part (b).