Physics 417: Problem Set 8 (*DUE ON THURSDAY 11/14*)

Problem 1: Spherical harmonics as angular momentum eigenfunctions

As we discussed in class, the spherical harmonics $Y_\ell^m(\theta,\phi)$ are eigenstates of angular momentum (or more precisely, they are the overlaps of these eigenstates with the position eigenkets on the sphere, $Y_\ell^m(\theta,\phi) = \langle \theta,\phi|\ell,m\rangle$). So they satisfy the differential equations

$$\begin{aligned} L^2 Y_\ell^m(\theta,\phi) &= \hbar^2 \ell(\ell+1) Y_\ell^m(\theta,\phi) \\ L_z Y_\ell^m(\theta,\phi) &= \hbar m Y_\ell^m(\theta,\phi) \end{aligned} \tag{1}$$

where L_z and L^2 are in their representations as differential operators. Demonstrate (1) explicitly for $Y_2^1(\theta,\phi)$ and $Y_3^2(\theta,\phi)$.

Problem 2: More on orbital angular momentum

The wave function of a particle subjected to a spherically symmetrical potential $V(r)$ is given by

$$\psi(\vec{x}) = (x + 2y + 3z) f(r) \tag{2}$$

for some function $f(r)$.

(a) Is ψ an eigenfunction of L^2? If so, what is the ℓ-value? If not, what are the possible values of ℓ we may obtain when L^2 is measured?

(b) What are the probabilities for the particle to be found in various m_ℓ states?

(c) Suppose it is known that $\psi(x)$ is an energy eigenfunction with eigenvalue E. Derive a formula for $V(r)$ in terms of $f(r)$.

Problem 3: Griffiths 4.31

Problem 4: Griffiths 4.33

Problem 5: Addition of angular momentum

Consider the addition of angular momentum with $j_1 = 1$ and $j_2 = 1$.

(a) What are the allowed values of m_1 and m_2?

(b) What are the allowed values of j and m?

(c) Use table 4.8 of Griffiths to relate $|j=2, m=1\rangle$ and $|j=2, m=0\rangle$ to the $|m_1, m_2\rangle$ eigenstates. (Be careful to use the table correctly! Read the caption carefully!) In other words, in the notation of Griffiths, find the RHS of

$$|j, m\rangle = \sum_{m_1, m_2} C^{112}_{m_1 m_2 m} |m_1, m_2\rangle \tag{3}$$

for $|j=2, m=1\rangle$ and $|j=2, m=0\rangle$.

(d) Check your result in part (c) by applying J_- to the RHS of (3) for $|j=2, m=1\rangle$ and verifying that you obtain the RHS of (3) for $|j=2, m=0\rangle$.

EXTRA CREDIT WORTH ONE PROBLEM: Using the recursion relations, *derive* the $j_1 = 1$, $j_2 = 1$ entry of table 4.8 in Griffiths.